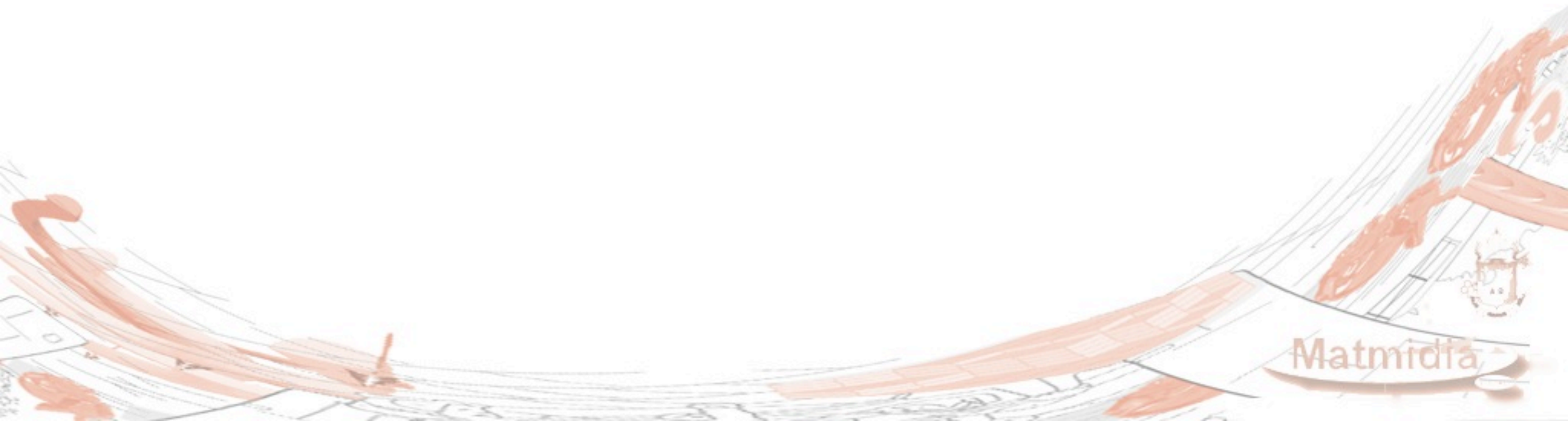
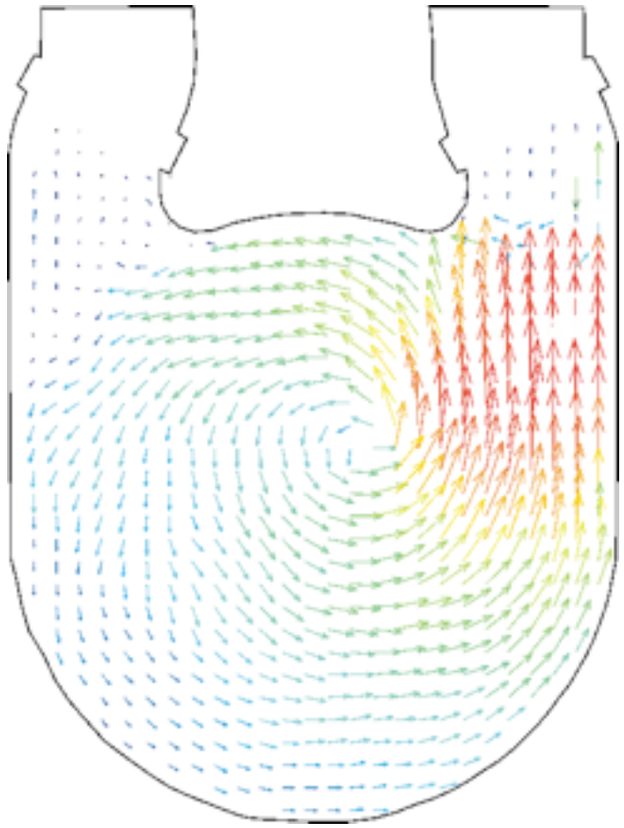
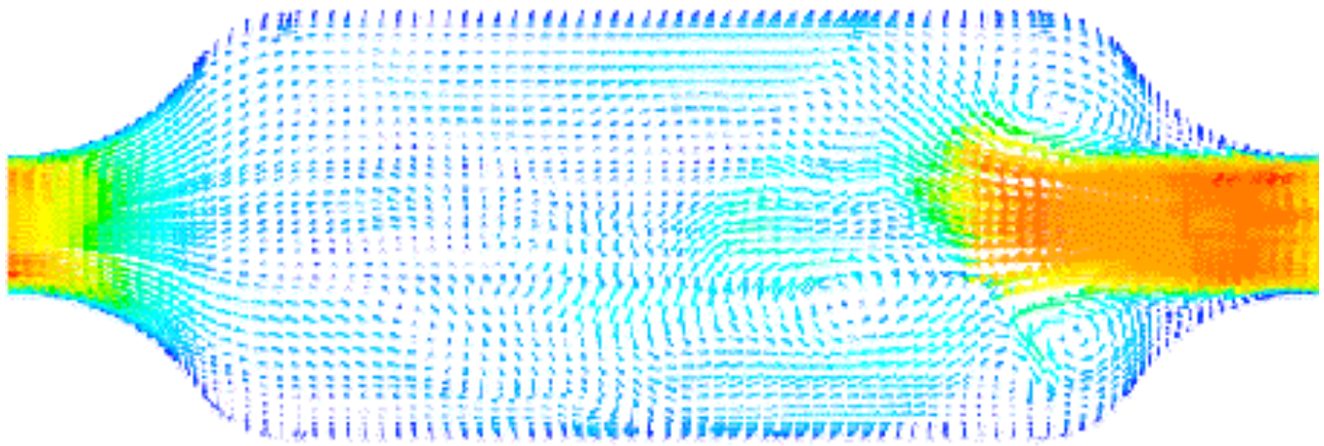
Topology Aware Vector Field Denoising

Renata Nascimento, João Paixão,
Hélio Lopes and Thomas Lewiner
Mathematics - PUC-Rio

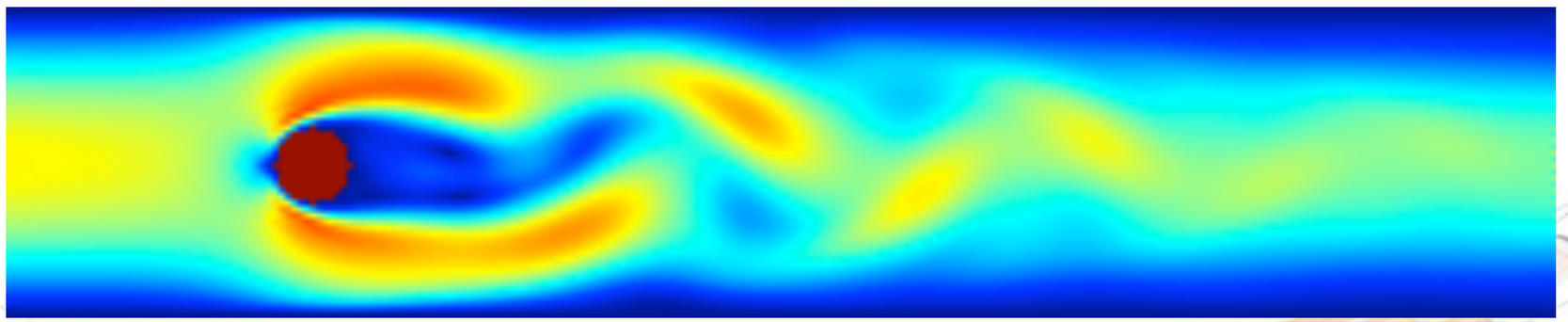


Fluid Dynamics

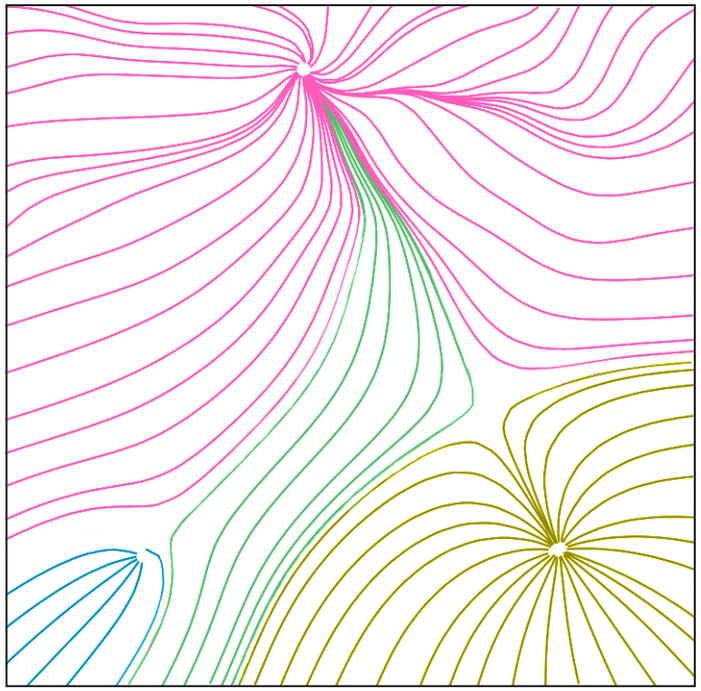


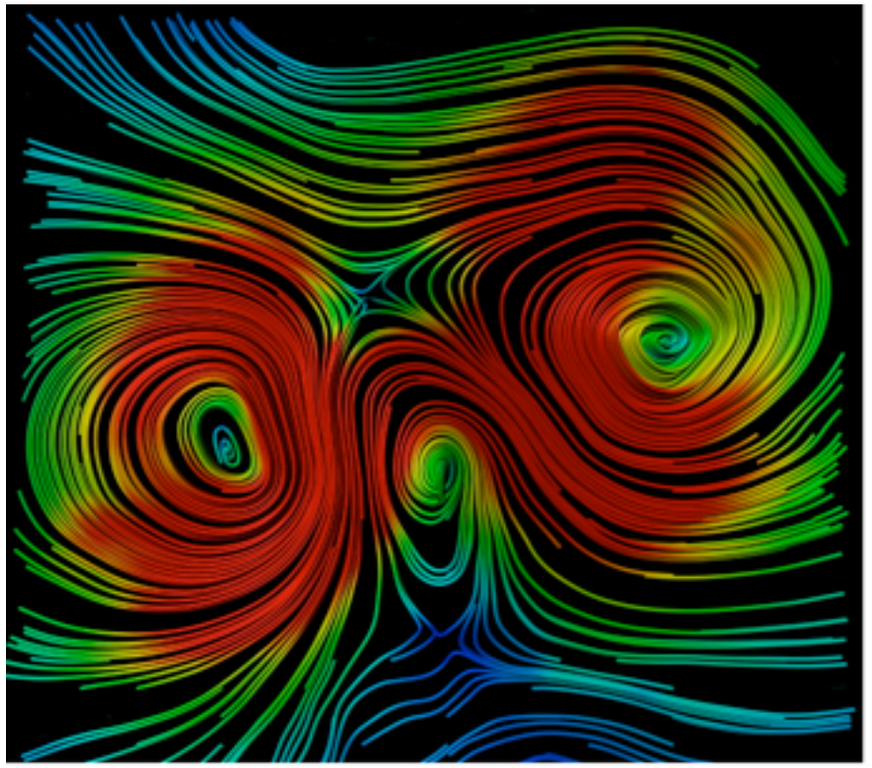


Movement ⇨ Velocity vector field



Analysis

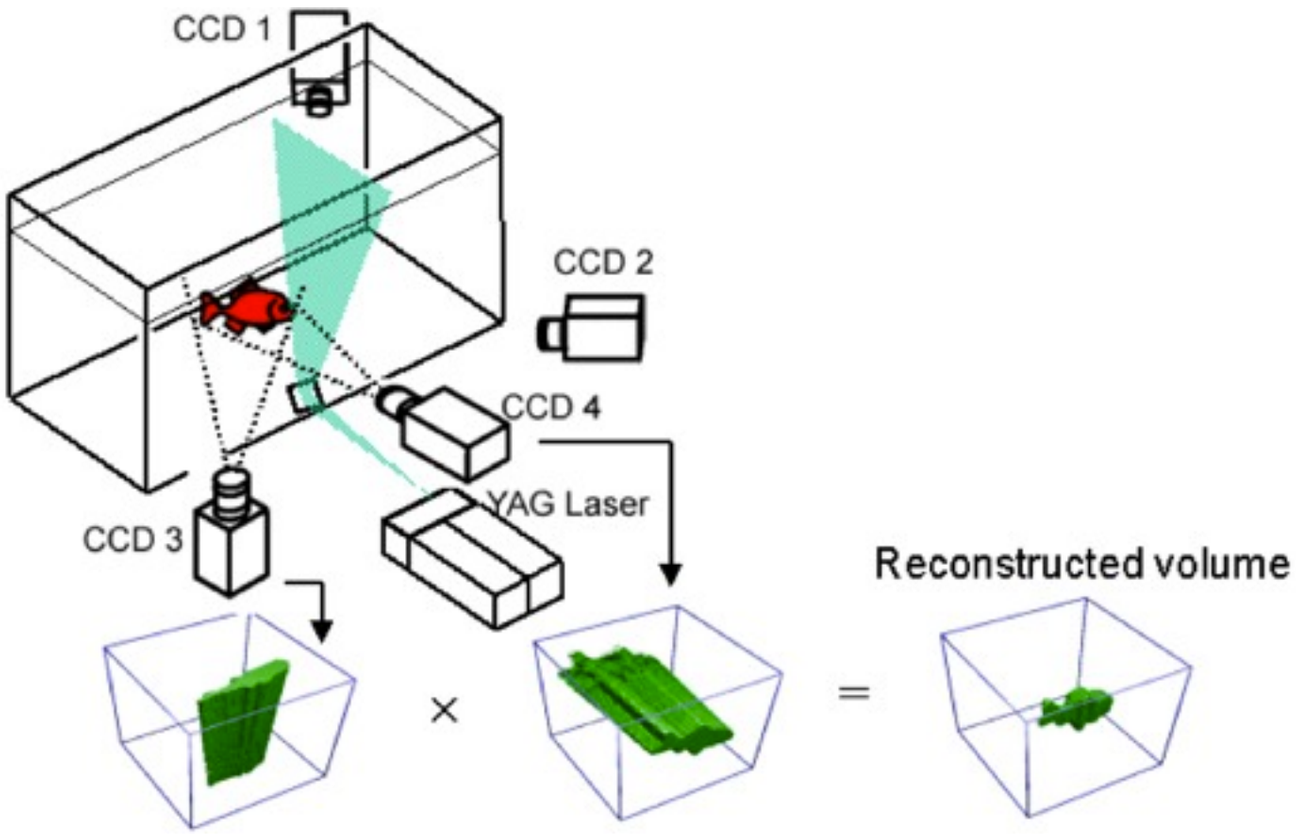


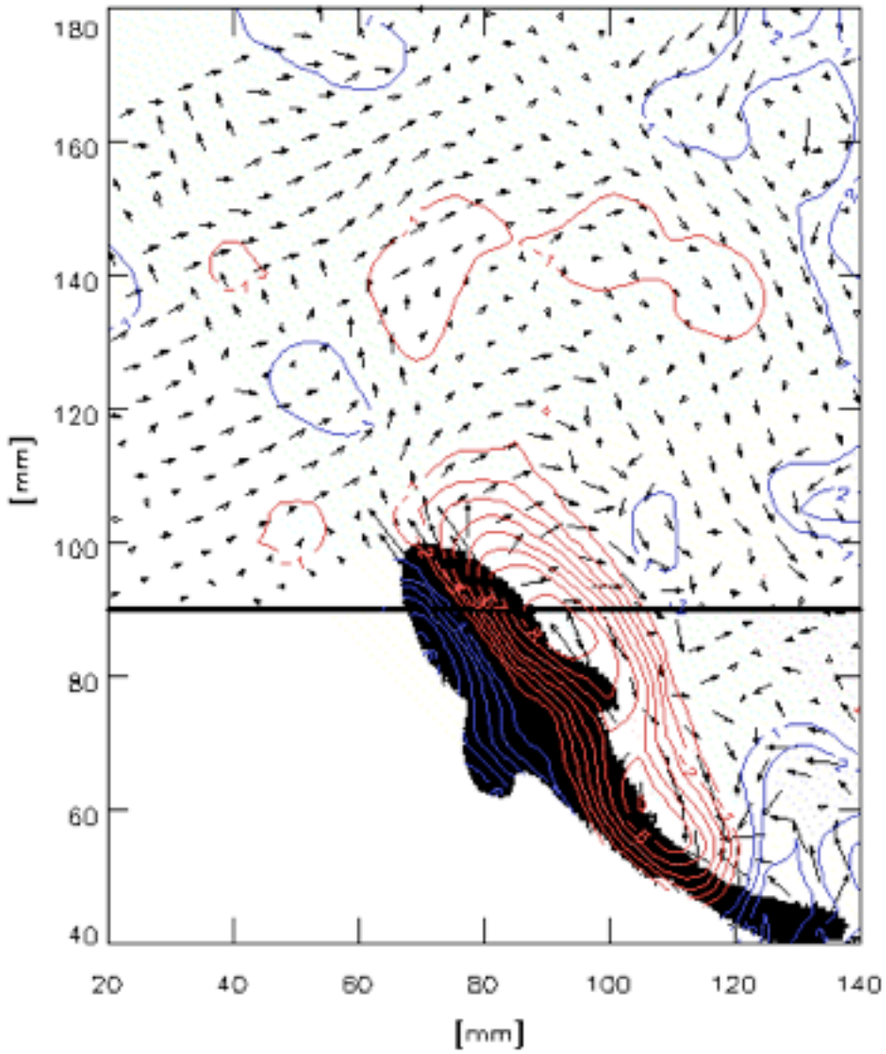


Global streamline behavior = topology

Real Velocity Measures

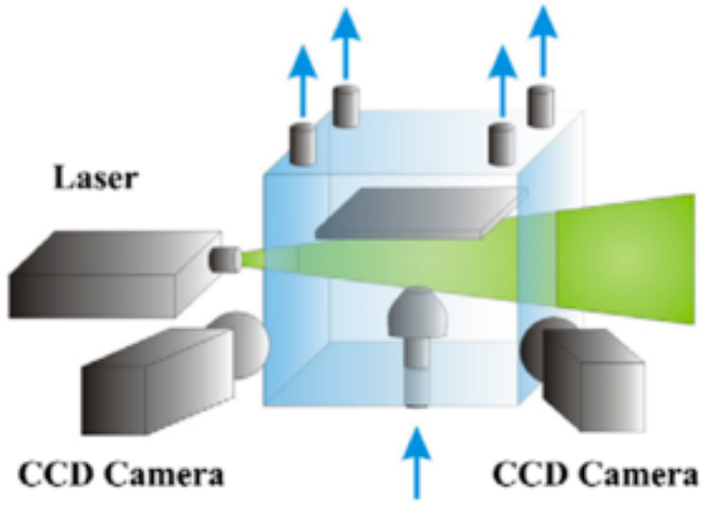
Particle Image Velocimetry - PIV

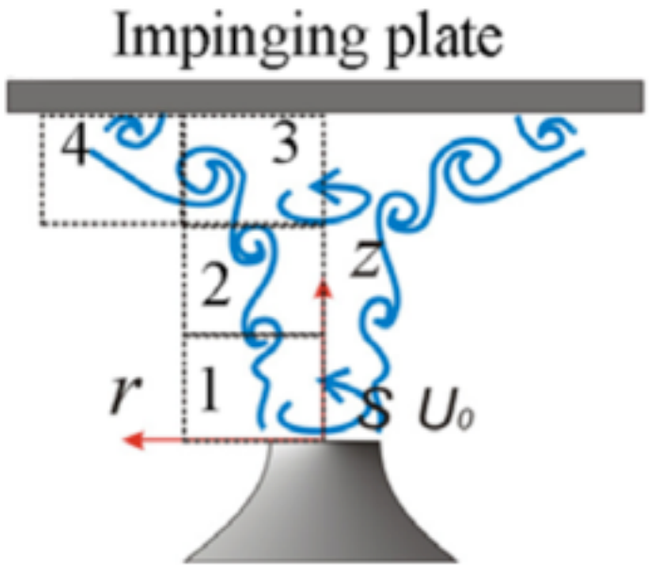


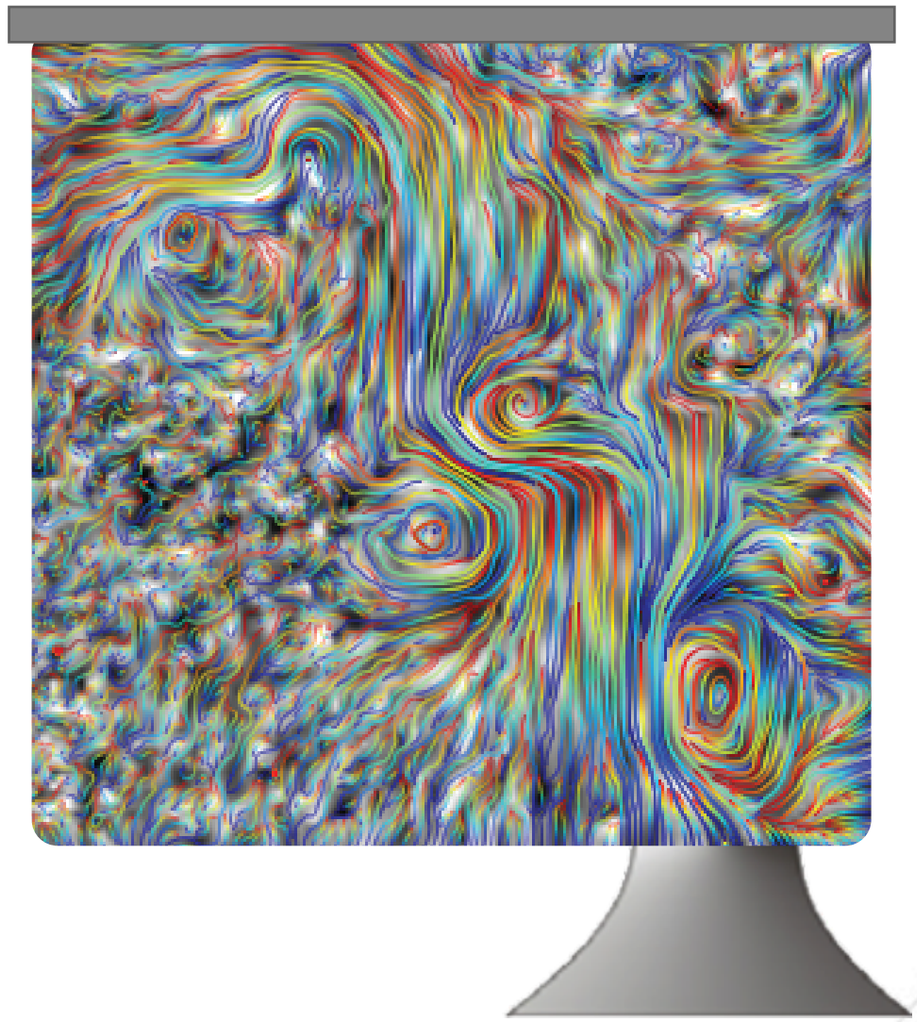


Data: Sakakibara Lab
Department of Engineering Mechanics and Energy
University of Tsukuba

Real Velocity Measures

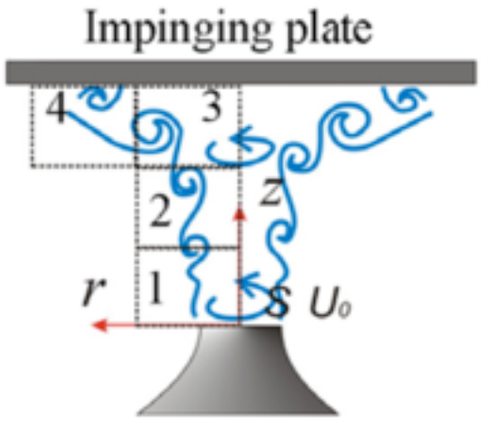


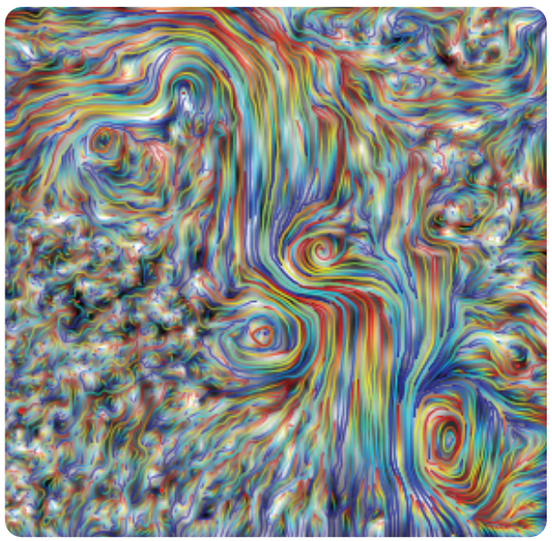




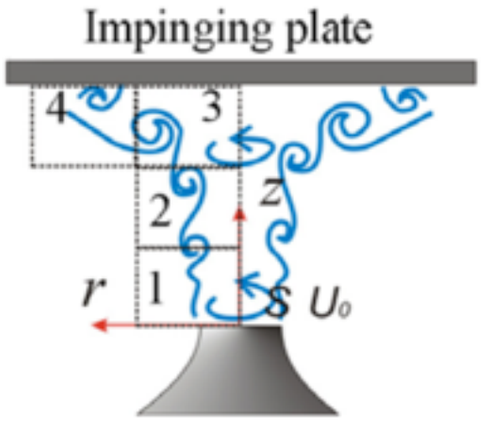
Data: Alzuguir, L. and Abrantes, J.
(Dept. of Mechanical Enginering, PUC - Rio)

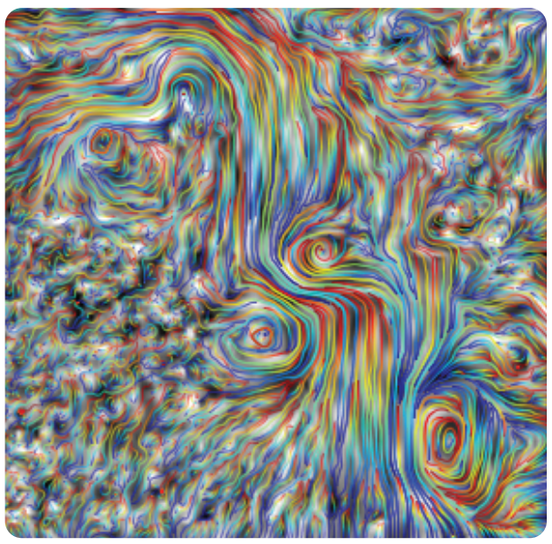
Real Data



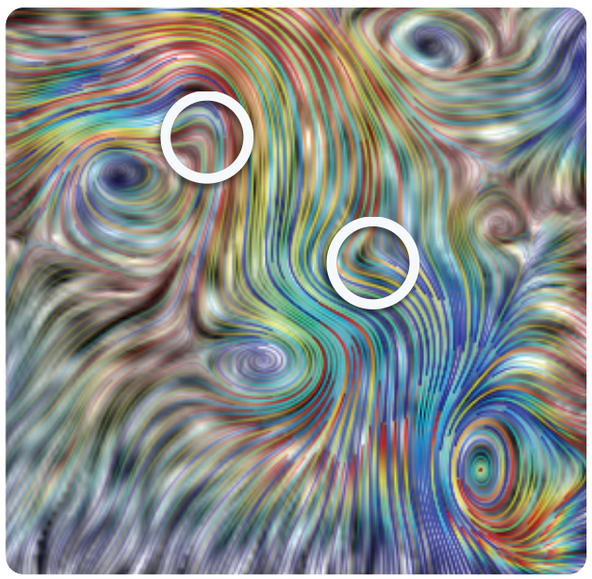


Real Data

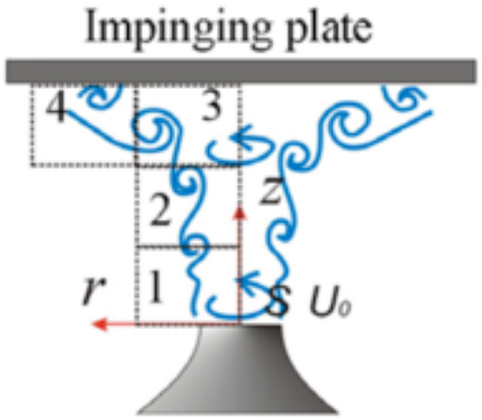


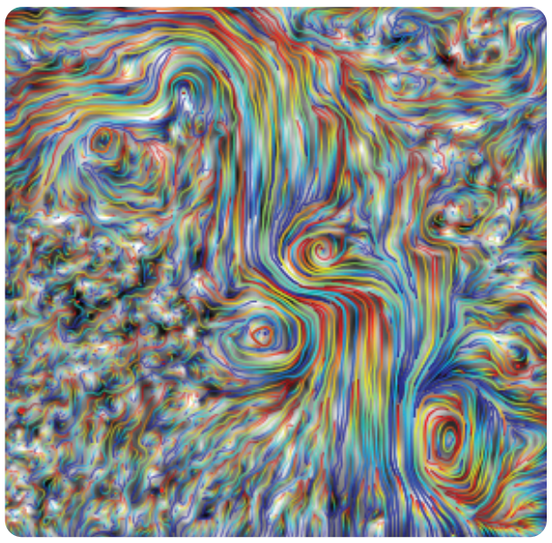


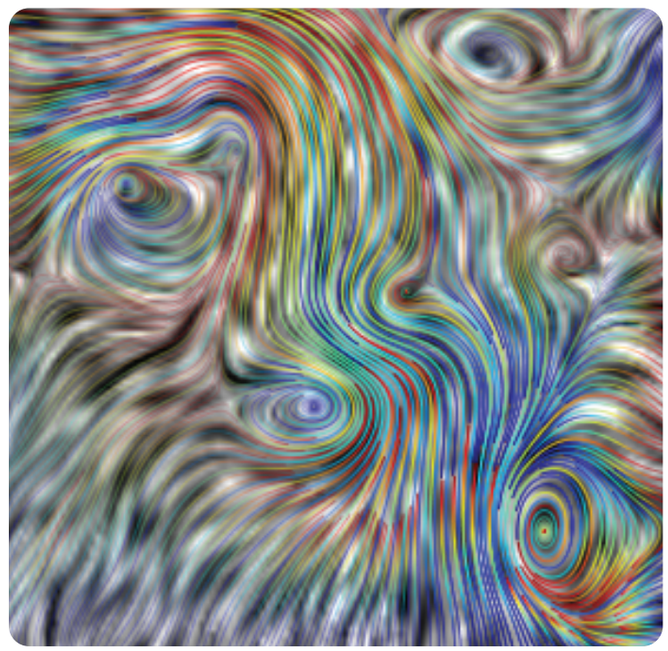
Denoising



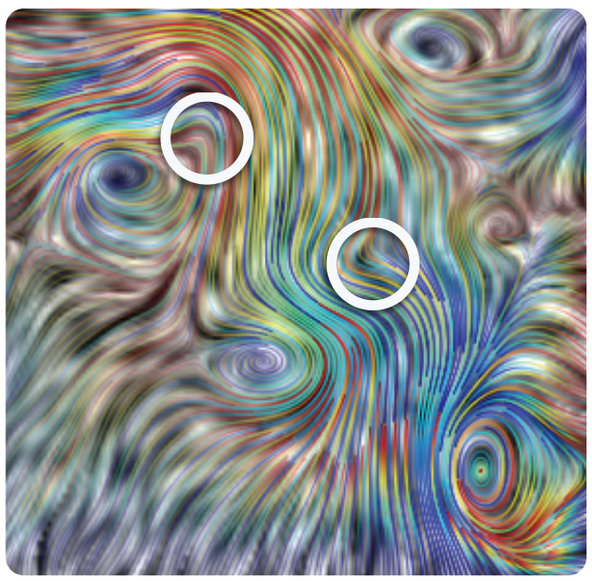
Real Data



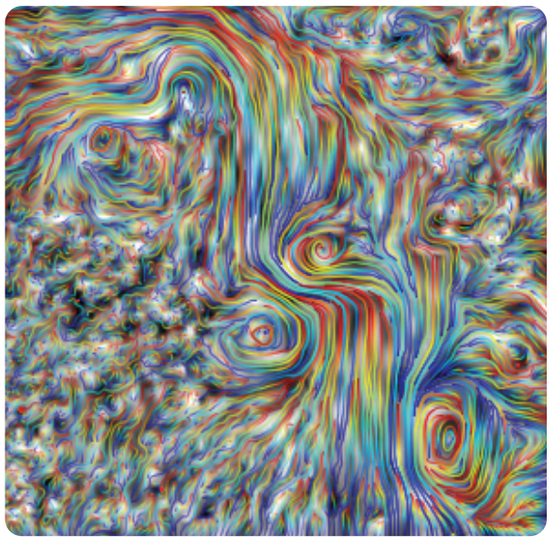


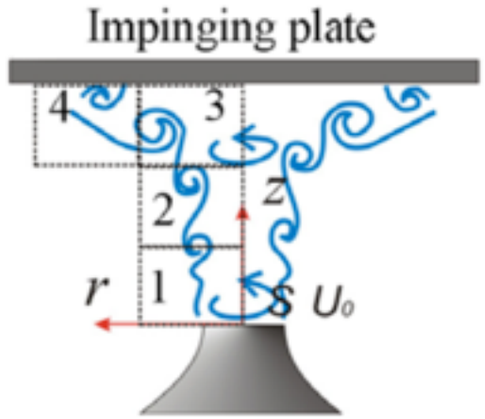


Denoising

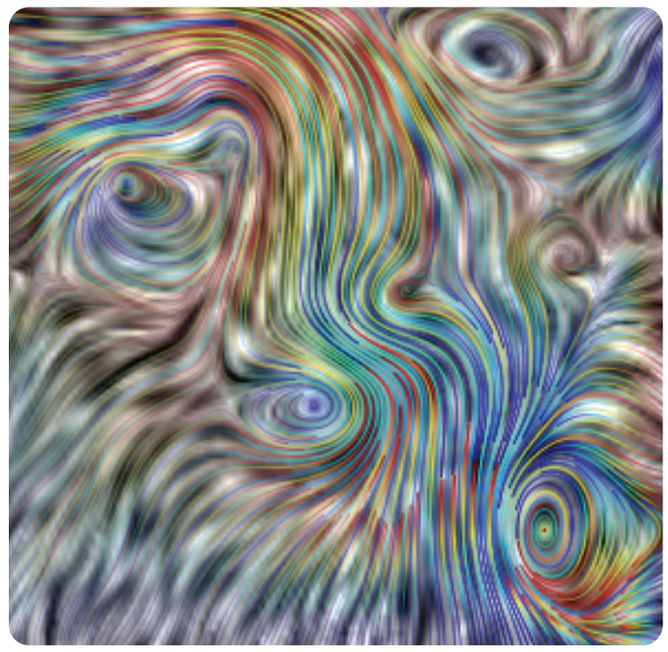


Contribution

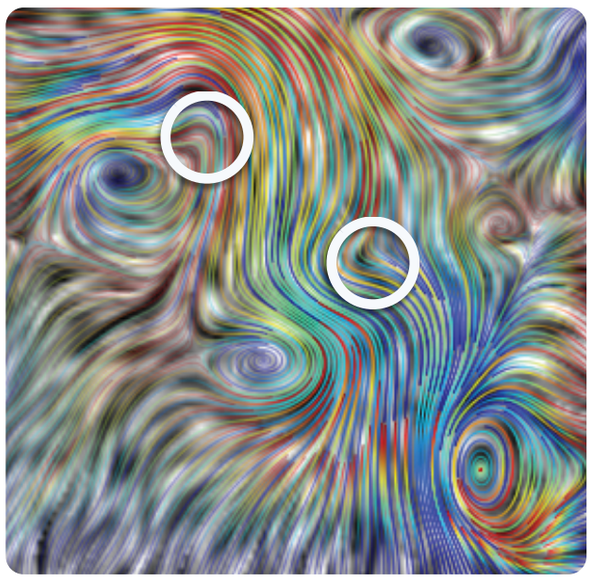




User



Denoising

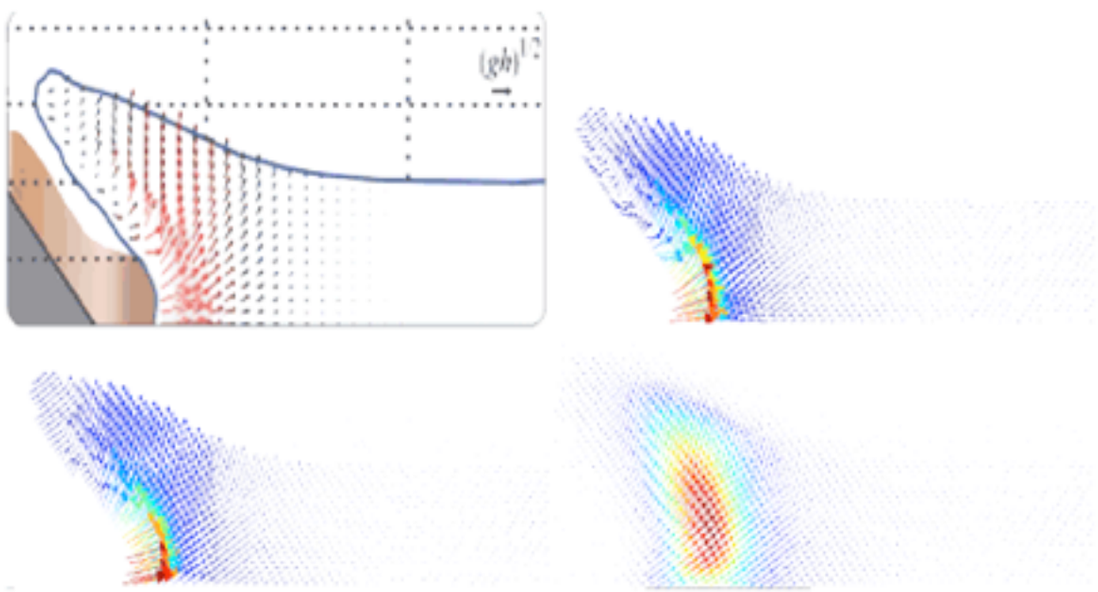


Related Work

- Local - Filter

- M. A. Westenberg and T. Ertl, “Denoising 2D vector fields by vector wavelet thresholding”.

- J. Paixão, et al., “Random walks for vector field denoising”.



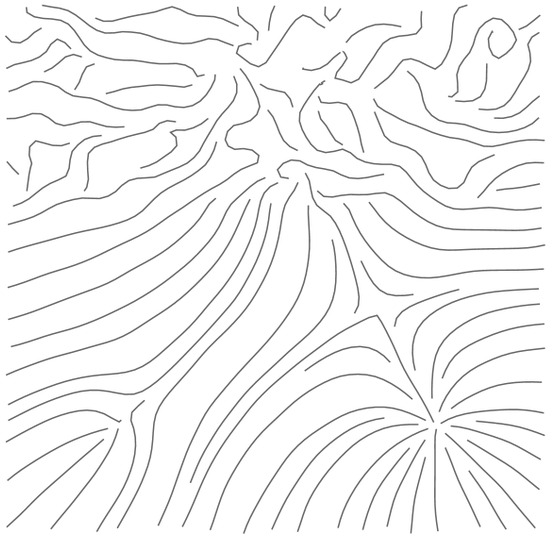
- Global - Topology

- X. Tricoche, “Vector and tensor field topology simplification, tracking, and visualization”.

- A. Sharf et al., “Interactive topology-aware surface reconstruction”.

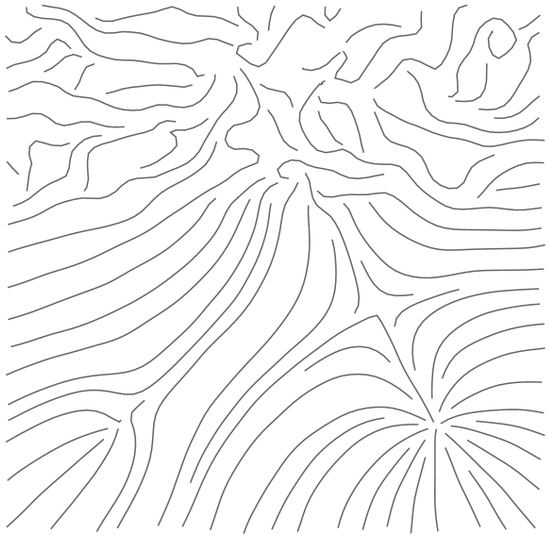
Topology-aware Vector Field Filter

Original field

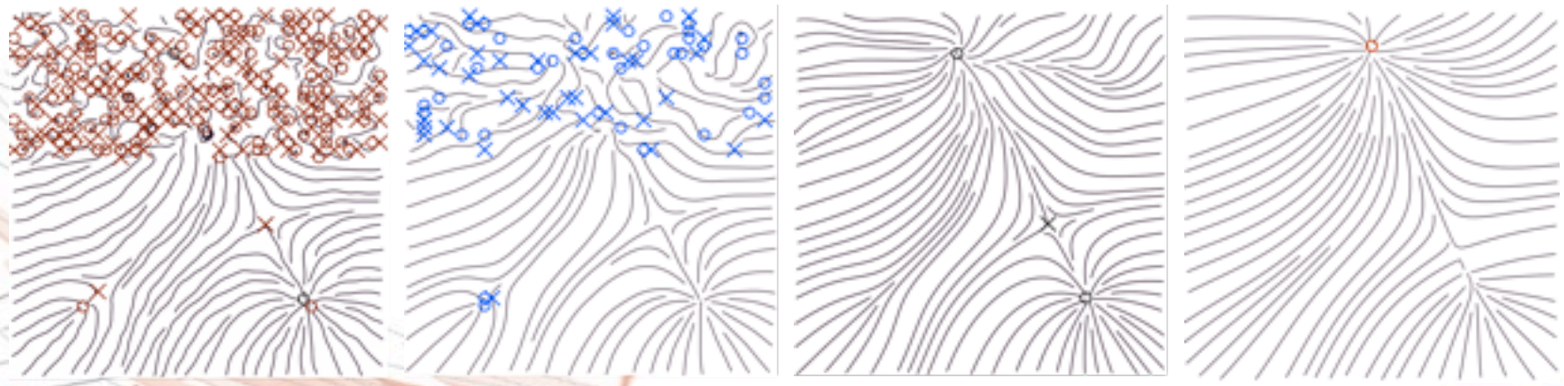


Topology-aware Vector Field Filter

Original field



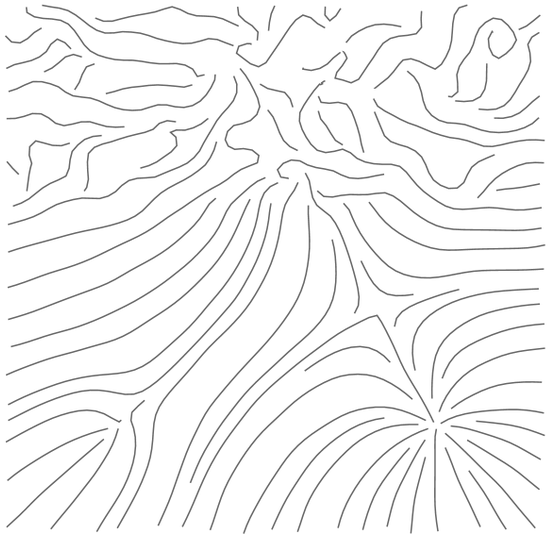
Denoising



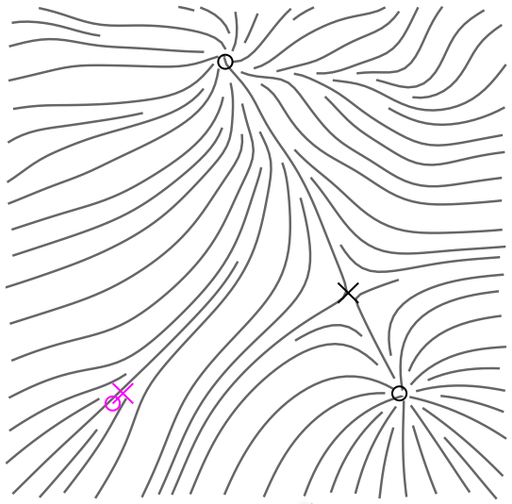
Scale space

Topology-aware Vector Field Filter

Original field

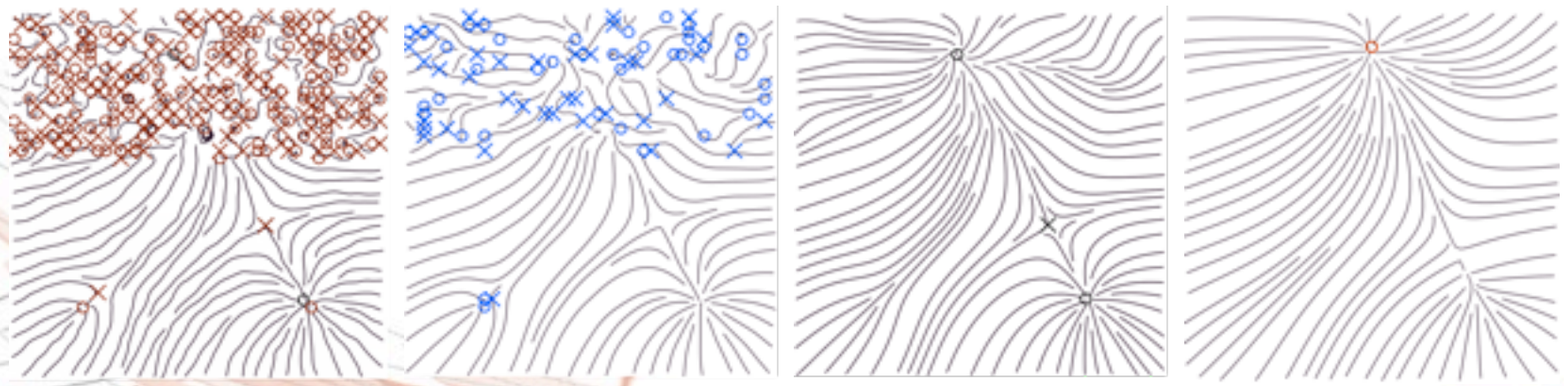


Denoising



Topology information

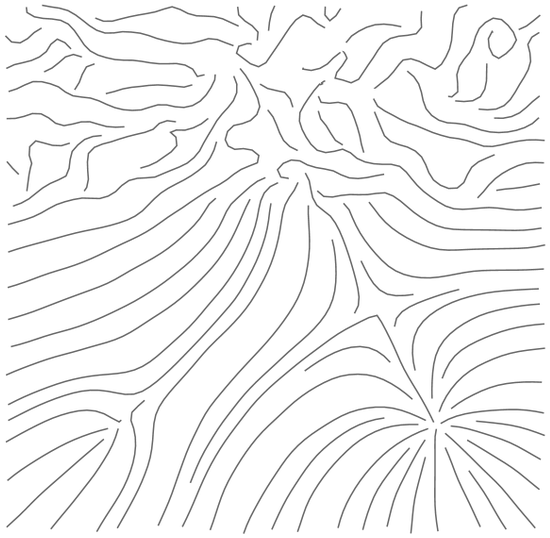
User



Scale space

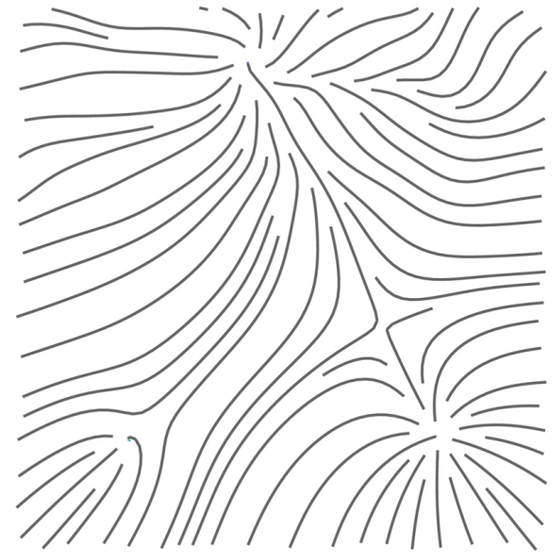
Topology-aware Vector Field Filter

Original field

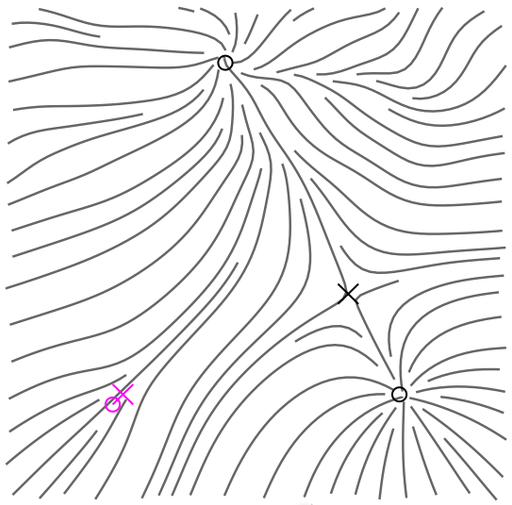


Denoising

Reconstruction

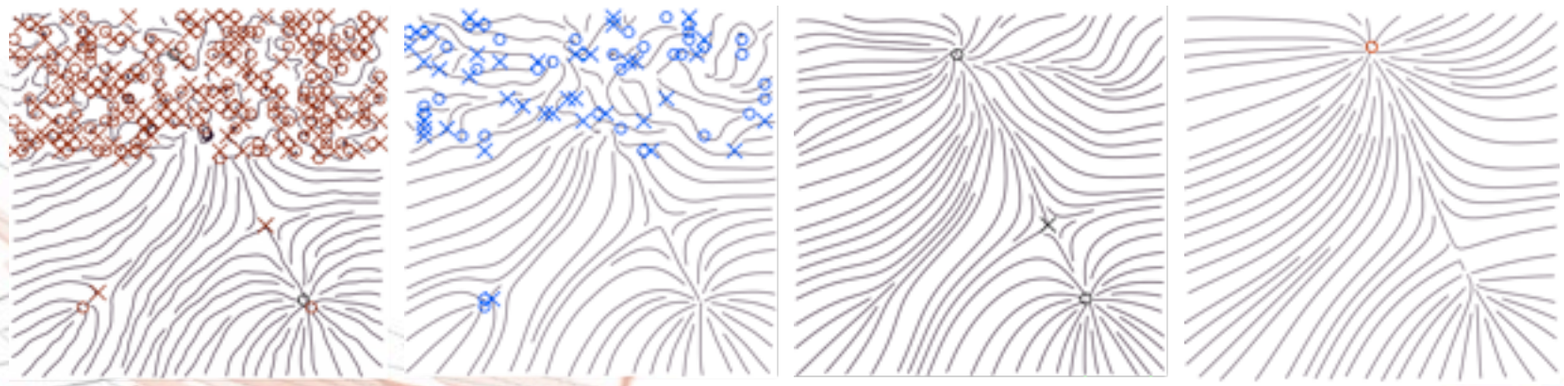


Final field



Topology information

User

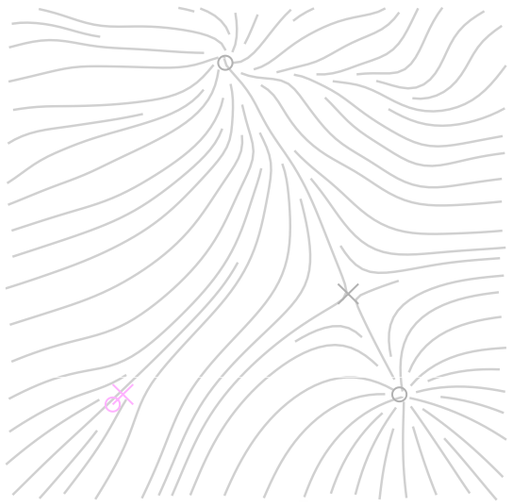


Scale space

Topology-aware Vector Field Filter

Original field

Reconstruction

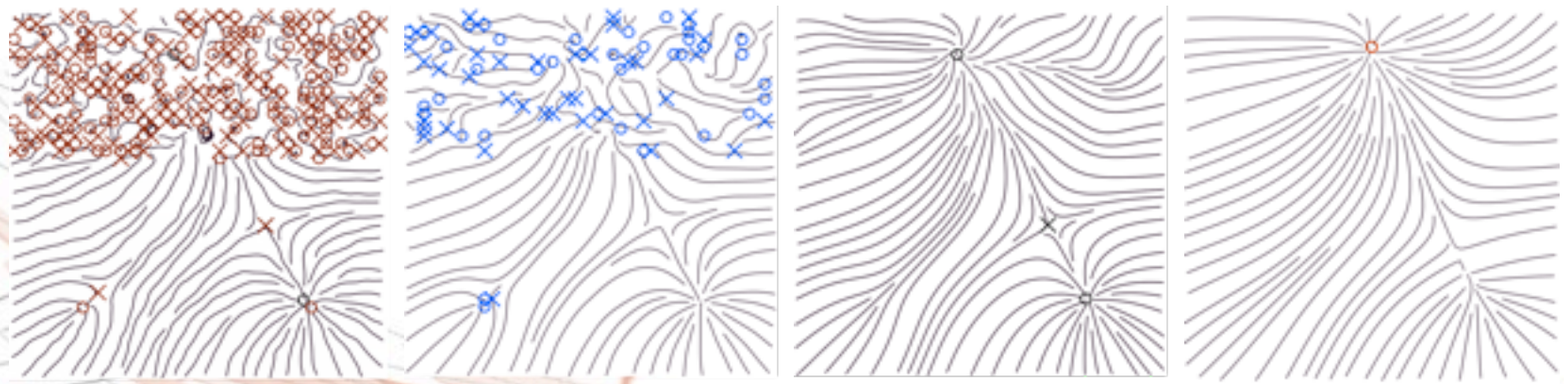


Final field

Denoising

Topology information

User



Scale space

Scale Space

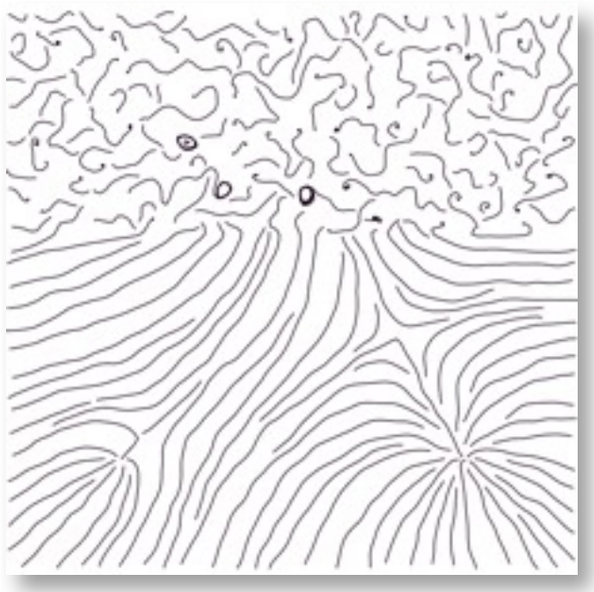
Scale Space

S

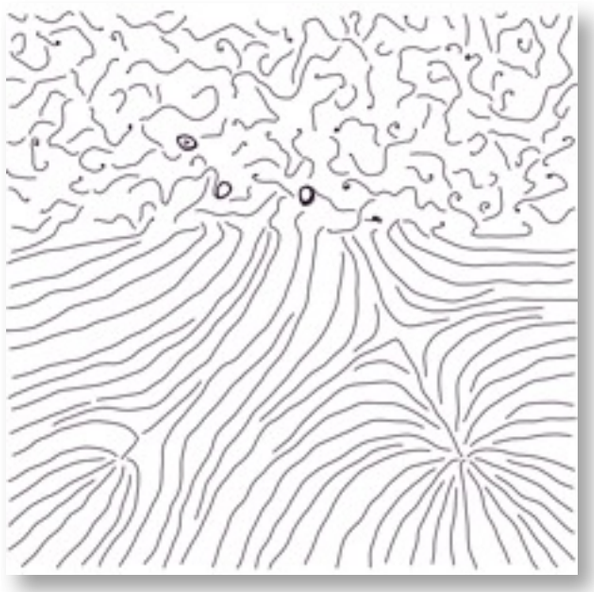
Scale Space

S

Scale Space



Scale Space



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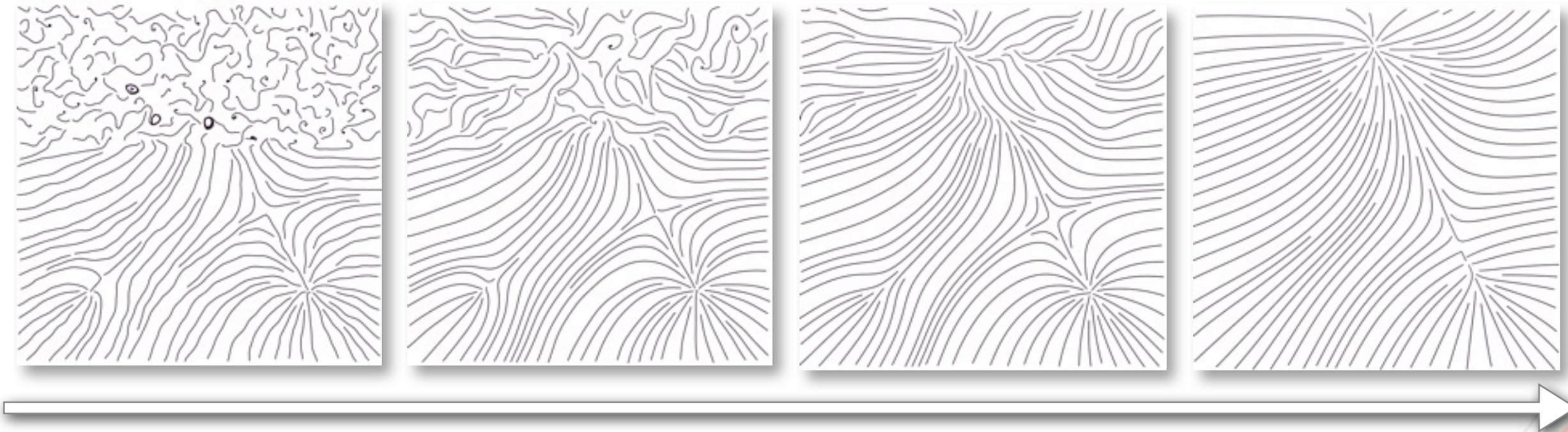


Scale Space

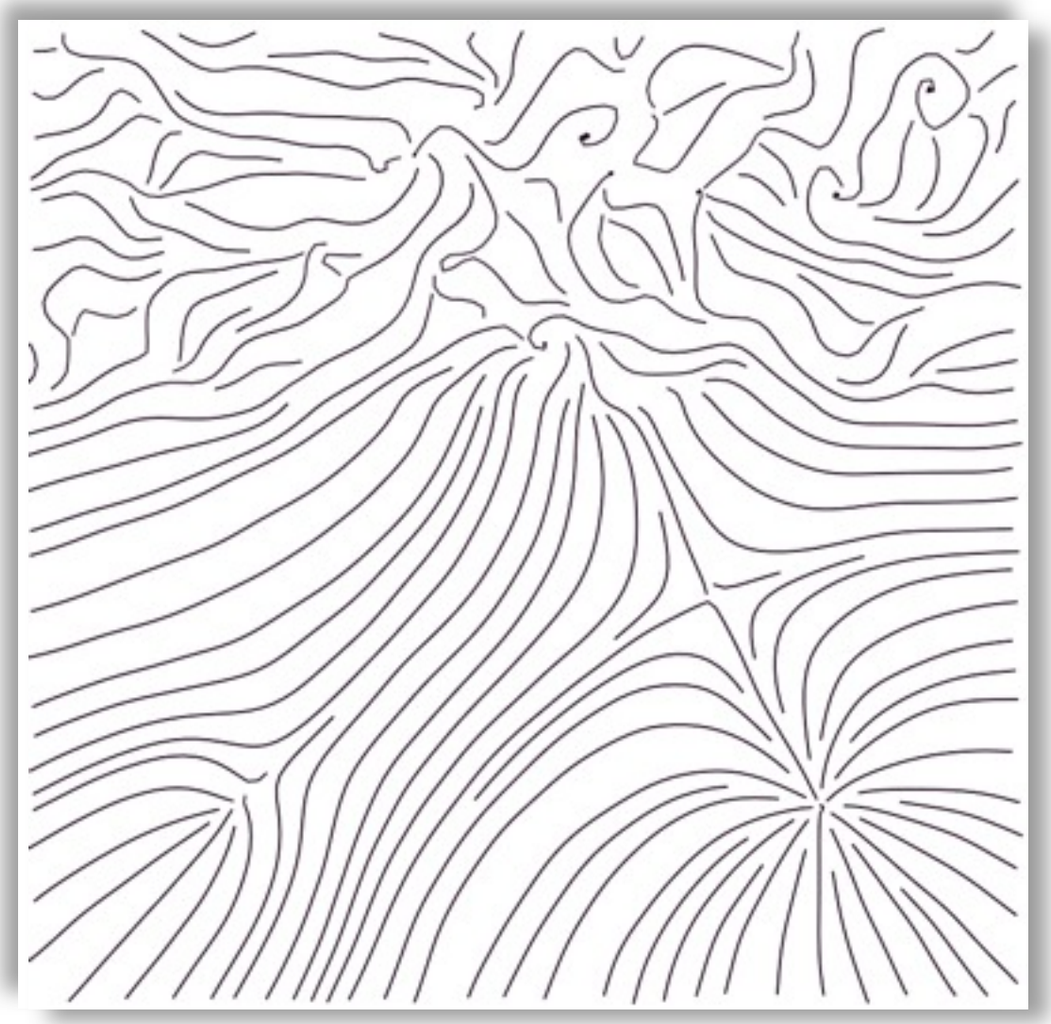
Denoising at a Single Scale?

Scale Space

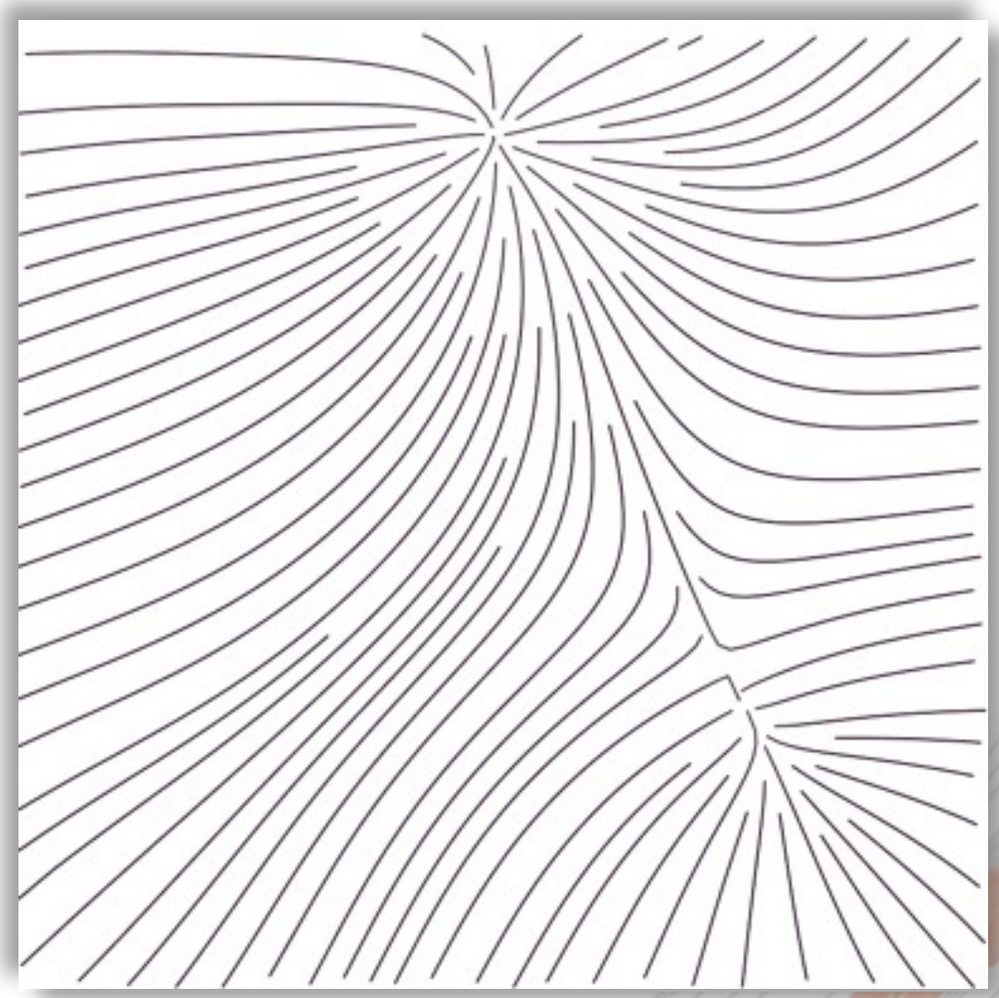
Single Scale?

Scale Space

Single Scale?



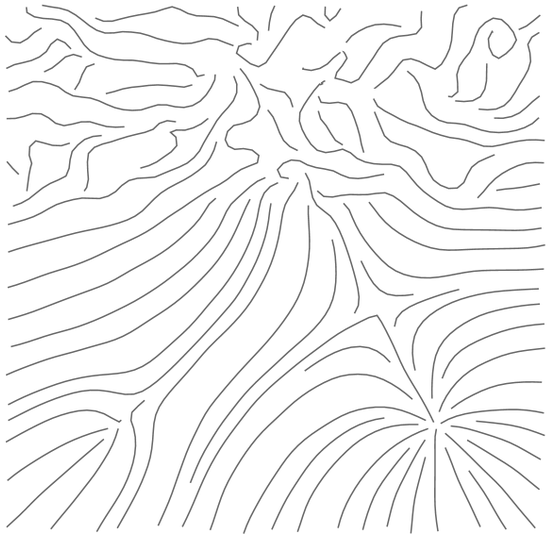
s = 10



s = 70

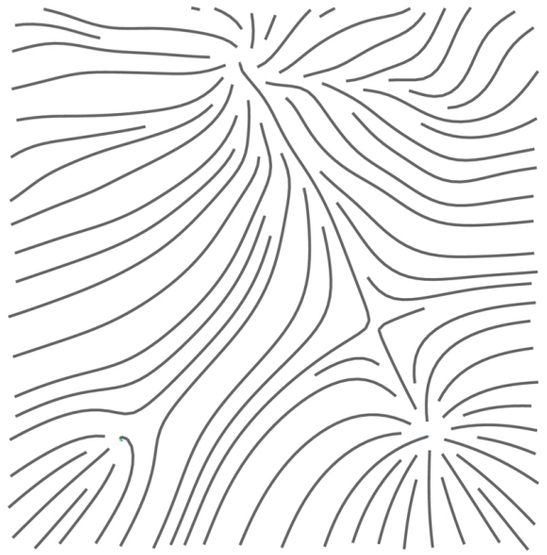
Big Picture

Original field

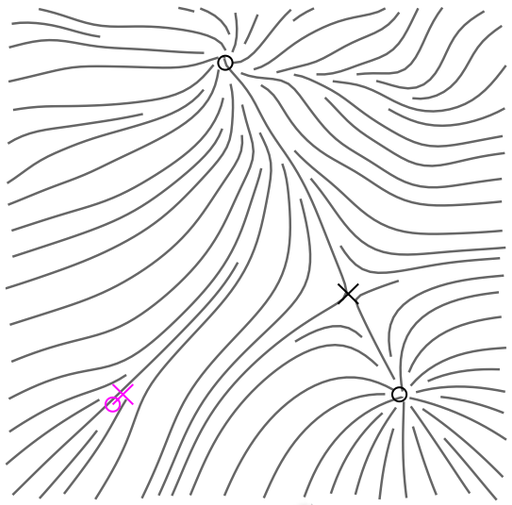


Denoising

Reconstruction

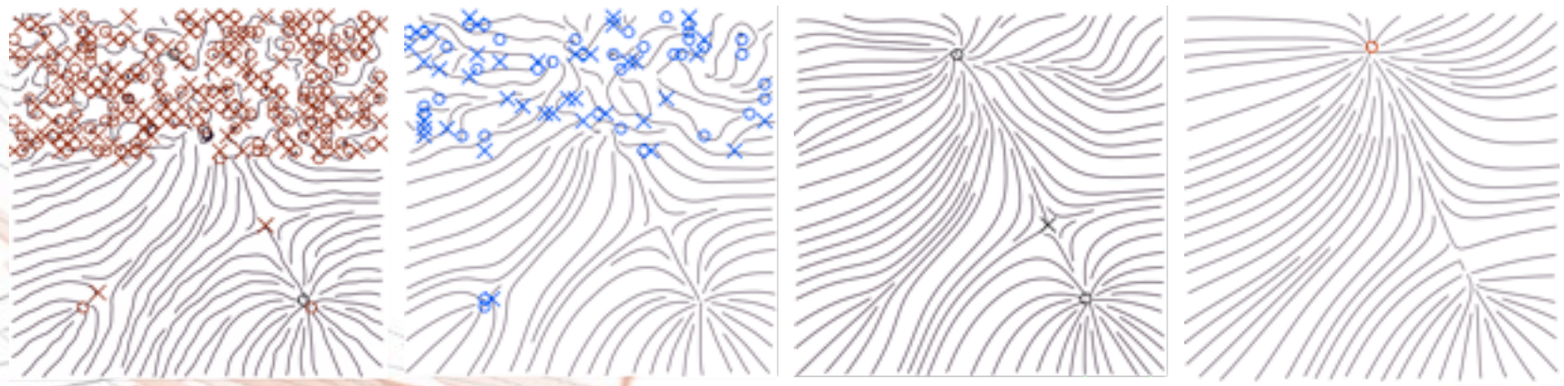


Final field



Topology information

User



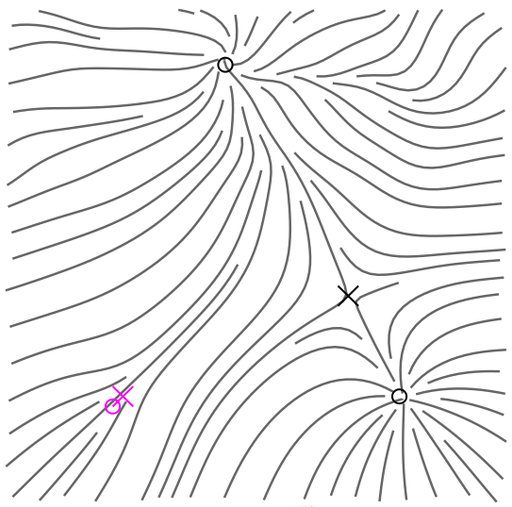
Scale space

Big Picture

Original field

Denoising

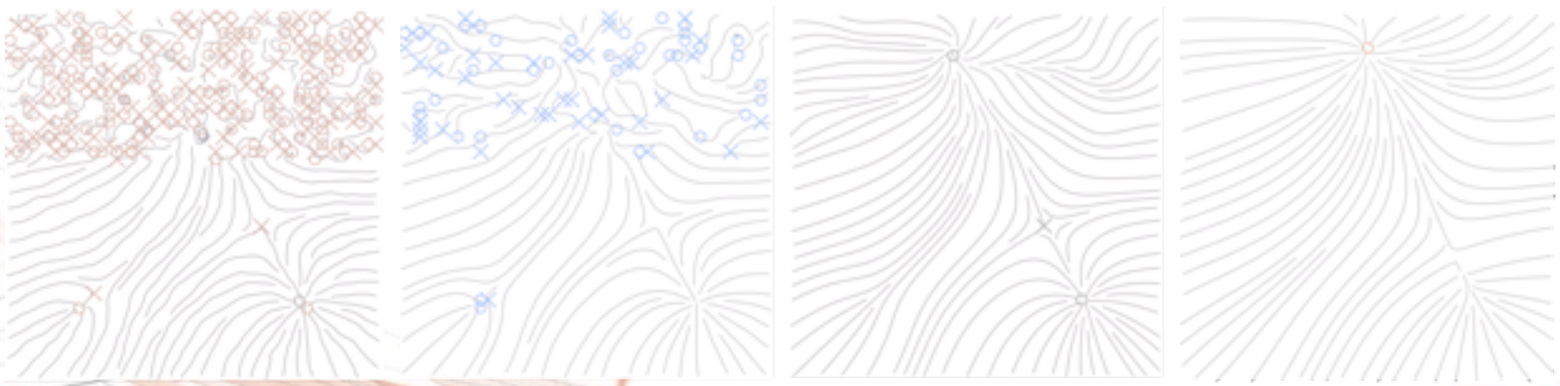
Reconstruction



Topology information

User

Final field



Scale space

Detection of Singularities

$$\boxed{\mathbf{v}(x_i, y_i) = 0}$$

Detection of Singularities

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- Bilinear interpolation
- Winding numbers
- Weak regions

Detection of Singularities

$$\boxed{\mathbf{v}(x_i, y_i) = 0}$$

- Bilinear interpolation

- Winding numbers

$$w_\Gamma(\mathbf{v}) = \frac{1}{2\pi} \oint_\Gamma d\theta(\mathbf{v})$$

- Weak regions

$$\boxed{min \, \|\mathbf{v}(x_i, y_i)\|_2 \leq \varepsilon}$$

Detection of Singularities

$$\boxed{\mathbf{v}(x_i, y_i) = 0}$$

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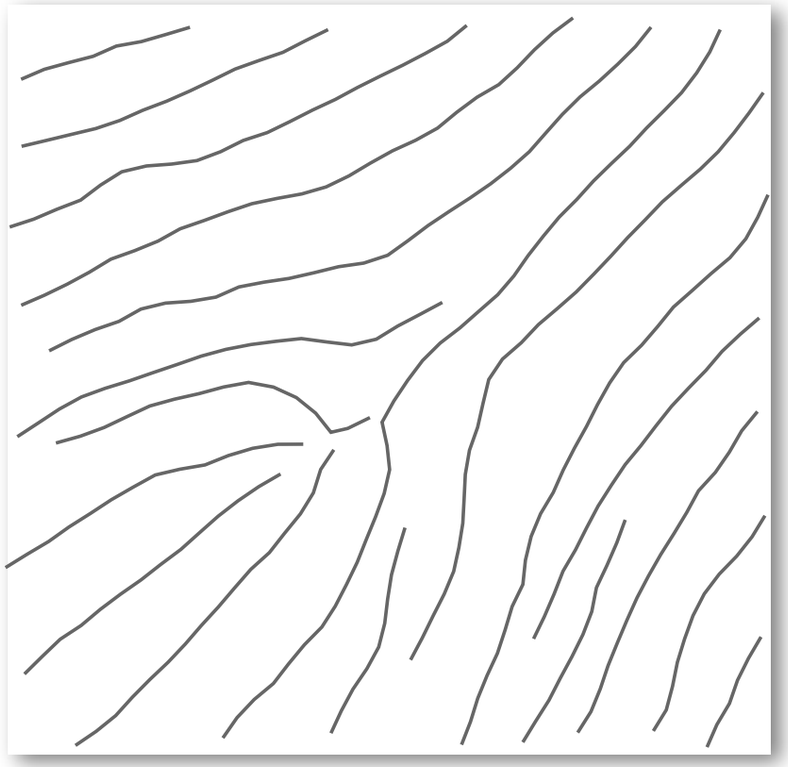
- Weak regions

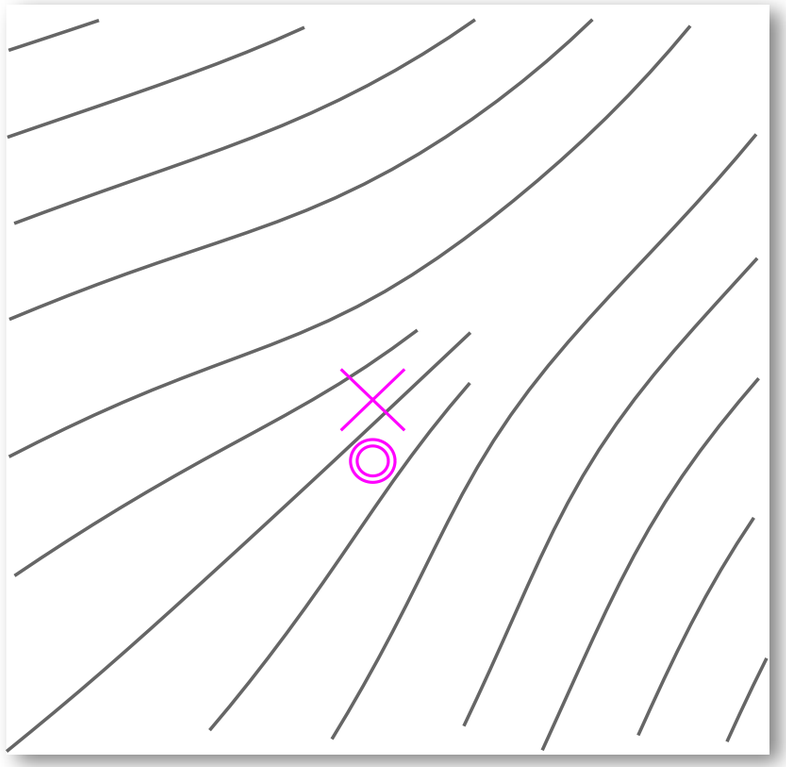
$$\boxed{min \left\| \mathbf{v}(x_i, y_i) \right\|_2 \leq \varepsilon}$$

Detection of Singularities

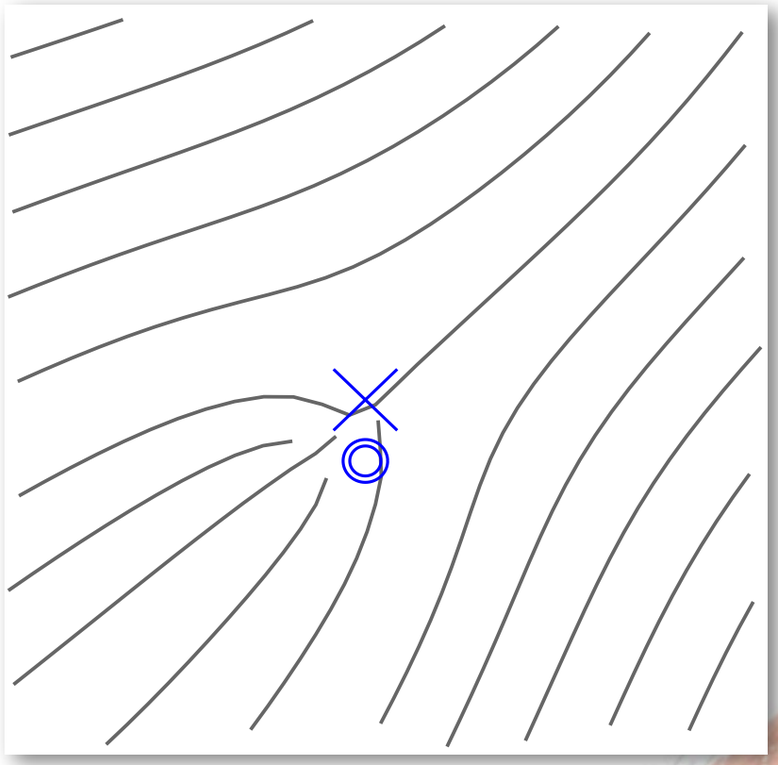
Weak regions



Original



Smoothed



Desired

Part1

5th Degree Polynomial

dnrmx :=

-8*Fx01^2*x^4*Fy10^4+2*Fx10^2*Fy00^4*x^5+4*Fx10^2*Fy10*Fy00^3-2*Fx00*Fx11*Fy10*Fy00^3+2*Fx01^2*x^5*Fy10^4-20*Fx10^2*Fy00^4*x^2-2*Fx00^2*Fy10*Fy11*Fx11^2*x^2+2*Fx00^4*Fy10*Fy11+2*Fx00*Fx01*Fy10^4+2*x*Fy10^4*Fx01^2-8*Fx11^2*x^4*Fy00^4-4*Fx00*Fx01^2*Fy10^2*x*Fx10-8*Fy01*x^5*Fx11^2*Fy10*Fx00^2-20*Fy10^2*Fx00^4*x^2+2*x^5*Fy11^2*Fx01^4+2*x^5*Fy11^4*Fx01^2+2*x*Fx00^4*Fy11^2+2*x*Fy00^4*Fx11^2+2*x*Fy01^2*Fx10^4+2*x^5*Fy01^4*Fx11^2-20*Fx00^2*Fy10^4*x^2+2*Fx00*Fx01^2*Fy10^2*x^2*Fx11-8*Fx01^2*x^2*Fy10^4-8*Fx11^2*x^2*Fy00^4+8*Fy00*Fx10^3*x^5*Fy01*Fx00-8*Fy00*Fx10^3*x^5*Fy01*Fx01+2*Fx10^2*Fy01^4*x^5+20*Fx00^2*Fy10^4*x^3-4*Fx00^2*Fy10*Fy11^2*Fy00*x

+12*Fx00^2*Fy10*Fy11*Fx01^2*x^2-2*Fx00^2*Fy10*Fy11*Fy01^2*x^2-40*Fx00^2*Fy10^3*x^3*Fy00+16*Fx00^2*Fy10^3*x^3*Fy01+40*Fx00^2*Fy10^3*x^2*Fy00-4*Fx00^2*Fy10^3*x^2*Fy01-36*Fx00^2*Fy10^2*x^4*Fy11^2+4*Fy00^2*Fx00*Fx10^2*Fx01*x

+28*Fy00^2*Fx00^2*Fx10*x^2*Fx11-20*Fx00^2*Fy10^3*x*Fy00+36*Fx00^2*Fy10^2*x^3*Fy11^2+4*Fx11^2*x^5*Fy01^3*Fy10+2*x^2*Fy00^2*Fx11^2*Fx00*Fx01+20*Fy00*Fx11^2*x^4*Fy11*Fx00^2+8*Fx00^3*Fy10*Fy11*Fx01*x

+4*Fx00^2*Fy10^2*Fy11*Fy01*x-4*Fx11^2*x^5*Fy00^3*Fy10+2*Fy00^4*Fx10*Fx11+2*Fy00*Fx10^4*Fy01+12*Fy00*Fx00^3*Fx10*Fy11*x-28*Fy00*Fx00^3*Fx10*Fy11*x^2+8*x^3*Fy11^2*Fy00^2*Fx01*Fx11+8*x^3*Fy01*Fx01*Fx11^2*Fy10*Fx00-10*Fx10^2*Fy00^4*x^4+2*Fx00^2*Fy10^4*x^5-2*Fx00*Fx11*Fy10*Fy00*Fy11^2*x^2-8*Fx11^2*x^5*Fy01^3*Fy00+2*Fx00^2*Fy11^4*x^5+64*Fy00*Fx00*Fx10^3*Fy01*x^3-20*Fy00*Fx00^3*Fx10*Fy10*x

+2*Fx00^2*Fy10*Fy11^2*Fy01*x^2+32*Fx00^2*Fy10^3*x^2*Fy11-4*Fx00^2*Fy11^3*x^5*Fy01+4*Fx00^2*Fy11^3*x^5*Fy00-8*Fx00^2*Fy11^3*x^5*Fy10+16*Fx00^2*Fy11^3*x^4*Fy10-8*Fx00^2*Fy11^3*x^3*Fy10-10*Fx00^2*Fy11^3*x^4*Fy00+8*Fx00^2*Fy11^3*x^3*Fy00+4*Fx00^2*Fy11^3*x^4*Fy01-8*Fx00^2*Fy10^3*x^5*Fy11-4*Fx00^2*Fy10^3*x^5*Fy00+4*Fx00^2*Fy10^3*x^5*Fy01-24*Fy01*x*Fx10^2*Fy10*Fx00^2+12*Fy01^2*x^3*Fx10^4+12*Fx00^2*Fy10^2*x^5*Fy11^2-4*Fy00*Fx10^3*x^5*Fy10*Fx00-14*Fx00^2*Fy10^3*x^4*Fy01+20*Fx00^2*Fy10^3*x^4*Fy00+32*Fx00^2*Fy10^3*x^4*Fy11+4*Fy00^2*Fx00*Fx10^2*Fx01*x^2-12*Fy00^2*Fx00^2*Fx10*Fx11*x-48*Fx00^2*Fy10^3*x^3*Fy11-12*Fx00^2*Fy11^2*x^2*Fy10^2-10*x^2*Fy11*Fy01*Fx01*Fx10*Fx00^2+72*Fx01*x^2*Fy10^2*Fx00*Fy00*Fy11-20*Fx00^2*Fy10^2*Fy00^2*x^2+32*Fx01*x^3*Fy10^2*Fx00*Fy01*Fy11+10*Fx00^2*Fy10^2*Fy00^2*x-2*Fx00^2*Fy01^2*x^4*Fy11^2-16*Fx00*Fx01*Fy10^2*Fy00*Fy11*x

+4*Fx00*Fy10*Fy00^3*Fx10+20*Fx00^2*Fy10^2*Fy00^2*x^3-4*x^5*Fy11*Fy01*Fx01*Fx11^3-4*Fy10^2*Fx01^3*x^5*Fx11+20*Fy00*Fx10^2*x^4*Fy11*Fx01^2-50*Fy00^2*Fx10^2*x^4*Fx00*Fx11+16*Fy00*Fx10^3*x^4*Fy11*Fx01-28*Fy00*Fx10^3*x^2*Fy11*Fx00-8*Fy10^2*Fx01^3*x^5*Fx00+16*Fy10^2*Fx01^3*x^4*Fx00+12*Fy00^2*Fx11^2*x^5*Fx01*Fx10

Part2

5th Degree Polynomial

-8*Fy00*Fx11^3*x^5*Fy01*Fx00-10*Fy10^2*Fx01^3*x^4*Fx10+4*Fy10^2*Fx01^3*x^4*Fx11-4*Fx00^2*Fy10*Fy11*Fx01*
x^2*Fx11+4*Fx00^2*Fy01^2*x^2*Fy10^2-4*Fx00*Fx01*Fy10^2*Fy01*x^2*Fy11+2*Fx00^2*Fy01^2*x^5*Fy11^2-6*x^2*F
y11*Fy01*Fx01*Fx10^3-4*x^5*Fy01^3*Fx11^2*Fy11-12*Fy10^2*Fx01^2*x^2*Fx00^2+4*Fy10^2*Fx01^3*x^5*Fx10-24*F
y11^2*x^3*Fx00^3*Fx01-2*Fy00*Fx10*Fy10*Fx01*Fy11^2*x^2-4*Fy00^2*Fx10*Fx11*Fy01*x^2*Fy11-16*Fy00^2*Fx10*F
x11*Fy01*x*Fy10-4*Fy00*Fx10^2*Fy01*Fx01*x^2*Fx11-16*Fy00*Fx10^2*Fy01*Fx00*Fx11*x-10*Fy11^2*x^4*Fx00^3*Fx
11-16*Fx00^2*Fy10*Fy11*Fx01*x*Fx10-24*Fy11^2*x^3*Fx00^3*Fx10-24*Fy11^2*x^4*Fx00^2*Fx01^2-4*Fx00^2*Fx10*F
y01*Fy10*Fx01*x+8*x^3*Fy01^2*Fx01*Fx11*Fy10^2-4*Fy00^2*Fx10*Fy10*Fx01*Fy01*x
+4*Fy11^2*x^5*Fx00^3*Fx11+12*Fy11^2*x^5*Fx00^2*Fx01^2+4*Fy00*Fx10*Fy10*Fx01^2*Fx00*x
+8*Fy00*Fx10*Fy10^2*Fx01*Fy01*x
+8*Fy00^2*Fx10*Fy10*Fx01*Fy11*x-4*Fy00*Fx10^2*Fy10*Fx01*Fx11*x-4*Fy00*Fx10*Fy10^2*Fx01*Fy11*x-2*Fy00*Fx1
0*Fy10*Fx01*Fy01^2*x^2-12*Fy11*x*Fx00^4*Fy10-2*Fy00*Fx10*Fy10*Fx01*Fx11^2*x^2+16*Fy11^2*x^2*Fx00^3*Fx10
+24*Fy11^2*x^4*Fx00^3*Fx01+8*Fy11^2*x^3*Fx00^3*Fx11+16*Fy11^2*x^4*Fx00^3*Fx10+2*Fy00^2*Fx00^2*x^5*Fx1
0^2-10*Fy00^2*Fx00^2*x^4*Fx10^2-2*Fy00*Fx10^2*Fy01*Fx01^2*x^2-8*Fy11^2*x^5*Fx01^3*Fx00+4*Fy11^2*x^5*Fx0
1^3*Fx10-40*Fy01*x^2*Fx10^3*Fy00*Fx11-8*Fy11^2*x^5*Fx00^3*Fx01+4*Fx11^2*x^5*Fy00^3*Fy11+12*Fy00*Fx10^2*
Fy01*Fx11^2*x^2-2*Fy00*Fx10^2*Fy01*Fy11^2*x^2-4*Fy11^2*x^5*Fx00^3*Fx10+2*Fy01^2*x^5*Fx00^2*Fx10^2-8*Fy0
1^2*x^4*Fx00^2*Fx10^2+2*Fy01^2*x^5*Fx01^2*Fx10^2-2*Fy01^2*x^4*Fx01^2*Fx10^2-8*Fy00^2*Fx10^3*Fx11*x-8*Fy
00^2*Fx00^2*Fx11^2*x^2+2*Fy00^2*Fx01^2*x^5*Fx10^2+2*Fy00^2*Fx01^2*x^5*Fx11^2-2*Fy00^2*Fx01^2*x^4*Fx11^
2+8*Fy11^2*x^5*Fx01*Fx10*Fx00*Fx11-16*Fy11^2*x^4*Fx01*Fx10*Fx00*Fx11+12*Fy00^2*Fx00^2*x^3*Fx11^2-4*Fy00
^2*Fx01*x*Fx10^3-4*Fy00^2*Fx01^2*x^4*Fx10^2+4*Fy00^2*Fx01^2*x^2*Fx10^2-8*Fy00^2*Fx00^2*x^4*Fx11^2+2*Fy0
0^2*Fx00^2*x^5*Fx11^2+4*Fy00^2*Fx10^2*Fy01*Fy11*x
+8*Fy00*Fx10^3*Fy01*Fx11*x-4*x^5*Fy11^2*Fx01^3*Fx11-6*Fy01^2*x^4*Fx00^2*Fx11^2+56*Fy00*Fx00^2*Fx10^2*Fy
11*x^2-64*Fy00*Fx00^2*Fx10^2*Fy11*x^3+2*Fy01^2*x^5*Fx00^2*Fx11^2+12*Fx11^2*x^5*Fy00^2*Fy01^2-8*Fx11^2*x
^5*Fy00^3*Fy01+12*Fy01^2*x^3*Fx00^2*Fx10^2-4*Fy00*Fx10^2*Fy01^2*x*Fy10+8*Fy01*x^2*Fx10^3*Fy00*Fx01-10*F
y00^2*Fx10^4*x^4-8*Fy01^2*x^2*Fx00^2*Fx10^2-10*Fy10^2*Fx00^2*x^4*Fx10^2+2*x*Fy10^2*Fx01^2*Fy00^2-8*Fy10
^2*Fx01^2*x^2*Fx10^2+2*Fy00*Fx10^2*Fy01^2*x^2*Fy11+8*Fy10*Fx01^2*x^5*Fx11*Fy11*Fx10-8*Fy10*Fx01*x^4*Fx1
1^2*Fy11*Fx00+16*Fy01*x^2*Fx10^3*Fy10*Fx01-28*Fy01*x^2*Fx10^3*Fy10*Fx00-2*Fy00^2*Fx11^4*x^4-4*Fy10^2*Fx
00^2*x^4*Fx11^2+2*Fy10^2*Fx00^2*x^5*Fx11^2+2*Fy10^2*Fx00^2*x^5*Fx10^2-12*Fx11*x*Fy00^4*Fx10-24*Fy10^2*F
x00*x^4*Fx11*Fx01*Fx10-24*Fy10*Fx00^2*x^4*Fx11*Fy11*Fx10-2*Fy10^2*Fx01^2*x^4*Fx11^2+8*Fy00*Fx11^3*x^5*F

Part3

5th Degree Polynomial

+16*Fy00*Fx11^3*x^5*Fy01*Fx10-8*Fy10^2*Fx01^2*x^4*Fx10^2+32*Fy01*x^3*Fx01*Fx11*Fy10*Fx00^2-16*Fy10*Fx01^2*x^4*Fx10*Fy11*Fx11+12*Fy10^2*Fx01^2*x^3*Fx10^2-32*Fy10*Fx01*x^4*Fx10^2*Fy11*Fx00+24*Fy00*Fx10^2*x^3*Fy10*Fx01^2-40*Fy00*Fx10^2*x^4*Fy10*Fx00^2+8*Fy10*Fx00*x^5*Fx11^2*Fy11*Fx01+4*x^2*Fy00^2*Fx10*Fy11^2*Fx01+2*Fy10^2*Fx01^2*x^5*Fx11^2+16*Fx11^2*x^4*Fy00^3*Fy10+8*Fy10^2*Fx00*x^5*Fx10*Fx01*Fx11+8*Fy10*Fx00*x^5*Fx10^2*Fy11*Fx01+8*Fy10*Fx00^2*x^5*Fx10*Fy11*Fx11+24*Fy01*x^2*Fx01*Fx10*Fy10*Fx00^2-64*Fy01*x^3*Fx10^2*Fy10*Fx00^2-2*Fy00^2*Fx10*Fx11*Fx01^2*x^2+12*Fy00^2*Fx10*Fx11*Fy01^2*x^2-2*Fy00^2*Fx10*Fx11*Fy11^2*x^2-8*Fy11^2*x^2*Fx00^2*Fx10^2+60*Fy00*Fx10^2*x^4*Fy01*Fx11^2-2*Fy11^2*x^4*Fx00^2*Fx11^2-22*Fy00^2*Fx11^2*x^4*Fx01*Fx10+2*Fy11^2*x^5*Fx00^2*Fx11^2+12*Fy11^2*x^3*Fx00^2*Fx10^2+2*Fy11^2*x^5*Fx00^2*Fx10^2+4*Fy00*Fx11^3*x^4*Fy10*Fx01-8*Fy11^2*x^4*Fx00^2*Fx10^2+2*Fy11^2*x^5*Fx01^2*Fx10^2-6*Fy11^2*x^4*Fx01^2*Fx10^2-4*Fy10^2*Fx00^3*Fx11*x
+4*Fy10^2*Fx00^2*Fx11^2*x^2+2*Fy10^2*Fx01^2*x^5*Fx10^2+8*Fy01^2*x^5*Fx01*Fx10*Fx00*Fx11-16*Fy01^2*x^4*Fx01*Fx10*Fx00*Fx11+2*Fy11^2*x^5*Fx00^4-8*Fx00^2*Fy10^3*Fy11*x
+2*Fx00^2*Fy00^2*x^5*Fy10^2-8*Fy11^2*x^4*Fx00^4-8*Fx10^2*Fy00^3*Fy01*x
+2*Fy10^2*Fx00^4*x^5-2*Fy10^2*Fx01^4*x^4-10*Fx11^2*x^4*Fy00^3*Fy11+2*Fx01^2*x^5*Fy00^2*Fy11^2-6*Fx01^2*x^4*Fy00^2*Fy11^2+24*Fy00*Fx10^2*Fx11*x^2*Fy10*Fx01+2*Fx01^2*x^5*Fy00^2*Fy10^2+2*Fy10^2*Fx01^4*x^5-8*Fy11^2*x^2*Fx00^4+2*Fx01^2*x^5*Fy01^2*Fy10^2-40*Fy00*Fx00^3*Fx10*Fy10*x^3+32*Fy00*Fx10^2*Fx11*x^3*Fy11*Fx01+24*Fy00*Fx10^2*Fx11*x^2*Fy11*Fx00-2*Fx01^2*x^4*Fy01^2*Fy10^2-8*Fx01^2*x^4*Fy10^2*Fy00^2+12*Fx01^2*x^3*Fy10^2*Fy00^2-8*Fx01^2*x^2*Fy10^2*Fy00^2+8*x*Fx00*Fx01*Fy10*Fy11*Fy00^2+12*Fy11^2*x^3*Fx00^4-8*Fx11^2*x^2*Fy10^2*Fy00^2+2*Fx10^2*Fy00^2*x^5*Fy10^2+2*Fx10^2*Fy01^2*x^5*Fy10^2+20*Fy01^2*x^4*Fx10^2*Fx01*Fx11+40*Fy01^2*x^3*Fx10^2*Fx00*Fx11+10*Fy00^2*Fx10^4*x
+2*Fx10^2*Fy00^2*x^5*Fy11^2-4*Fx10^2*Fy00^3*Fy11*x-4*Fx10^2*Fy00^2*Fy11^2*x^4-4*Fy01*x^5*Fx10^3*Fy10*Fx01+4*Fx10^2*Fy00^2*Fy11^2*x^2-10*Fx10^2*Fy00^2*Fy10^2*x^4-8*Fx10^2*Fy01^2*x^4*Fy10^2+12*Fx10^2*Fy01^2*x^3*Fy10^2-8*Fx10^2*Fy01^2*x^2*Fy10^2-2*Fx10^2*Fy01^2*x^4*Fy11^2+2*Fx10^2*Fy01^2*x^5*Fy11^2+2*Fx11^2*x^5*Fy00^2*Fy11^2-2*Fx11^2*x^4*Fy00^2*Fy11^2+12*Fy01*x*Fx10^3*Fy10*Fx00+56*Fy01*x^2*Fx10^2*Fy10*Fx00^2+2*Fx11^2*x^5*Fy00^2*Fy10^2+8*Fy00*Fx00*x^5*Fx11^2*Fy01*Fx01+2*Fx11^2*x^5*Fy01^2*Fy10^2-6*Fx11^2*x^4*Fy01^2*Fy10^2-8*Fx11^2*x^4*Fy10^2*Fy00^2+8*Fy00*Fx01^2*x^5*Fx11*Fy01*Fx10+12*Fx11^2*x^3*Fy10^2*Fy00^2+8*Fy00*Fx10^2*x^5*Fy10*Fx00^2+40*Fy00^2*Fx00*Fx10^3*x^2-20*Fy00^2*Fx00*Fx10^3*x
+8*x^3*Fy00*Fx11^3*Fy01*Fx10+2*x^4*Fy10*Fx01^2*Fy11*Fy01^2+4*Fy00*Fx00^3*Fx10*Fy10-20*Fy00^2*Fx00^2*Fx

Part4

5th Degree Polynomial

+20*Fy00^2*Fx00^2*Fx10^2*x^3-12*Fx00*Fy10^2*x^5*Fx11*Fy00*Fy11-8*Fy00*Fx00^2*Fx10^2*Fy10-40*Fy00^2*Fx00
*Fx10^3*x^3-24*Fy00*Fx01*x^4*Fx10^2*Fy01*Fx00+8*x*Fy00*Fx10*Fy01*Fx11*Fy10^2-4*x*Fy00^3*Fx01*Fx11*Fy10+
4*Fy00*Fx10^3*x^5*Fy10*Fx01+4*Fy00*Fx10^3*x^5*Fy11*Fx00-36*Fy00^2*Fx10^2*x^4*Fx11^2-8*Fy00*Fx01^2*x^4*F
x10*Fy01*Fx11+36*Fy00^2*Fx10^2*x^3*Fx11^2+16*Fy00^2*Fx10^3*x^3*Fx01-24*Fx00^2*Fy01*x^4*Fy11*Fy00*Fy10-3
2*Fy00*Fx00^2*x^4*Fx11*Fy01*Fx10-4*Fy00^2*Fx10^3*x^2*Fx01-8*Fx00*Fy01^2*x^4*Fy11*Fx01*Fy10+20*Fy00^2*Fx
10^3*x^4*Fx00+18*Fy00*Fx10^4*x^4*Fy01-32*Fy00*Fx10^4*x^3*Fy01+28*Fy00*Fx10^4*x^2*Fy01-4*Fy00^2*Fx10*Fx1
1^2*Fx00*x-48*Fy00^2*Fx10^3*x^3*Fx11+32*Fy00^2*Fx10^3*x^2*Fx11-4*Fy00*Fx10^4*x^5*Fy01+12*Fy00^2*Fx10^2*
x^5*Fx11^2+32*Fy00*Fx00^3*Fx10*Fy11*x^3+12*Fx10*Fy00^2*x^5*Fx01*Fy01*Fy11+6*x^2*Fy00*Fx11^2*Fy01*Fx00^
2-14*Fy00^2*Fx10^3*x^4*Fx01+32*Fy00^2*Fx10^3*x^4*Fx11-14*x^4*Fy11*Fy01*Fx01*Fx11^2*Fx10-4*Fy00^2*Fx10^3
*x^5*Fx00-14*x^2*Fx00^2*Fx01*Fy11^2*Fx10-20*x^2*Fy10*Fx01^2*Fy11*Fx00*Fx10+4*Fy00^2*Fx10^2*Fx11*Fx01*x
+2*Fy00^2*Fx10*Fx11^2*Fx01*x^2-8*Fy00^2*Fx10^3*x^5*Fx11-10*Fy00^2*Fx11^3*x^4*Fx00+4*Fy00^2*Fx11^3*x^5*F
x00-40*Fx11*x^2*Fy00^2*Fx00*Fy10*Fy11+12*Fy01^2*x^5*Fx10^2*Fx00*Fx11-12*Fy01^2*x^5*Fx10^2*Fx01*Fx11+2*F
y01^2*x^5*Fx10^4+8*Fy00*Fx00^2*x^5*Fx10*Fy01*Fx11-24*Fy00^2*Fx00*x^4*Fx11*Fx01*Fx10-4*Fy00*Fx11^4*x^5*F
y01+4*x^2*Fx00^2*Fx11^2*Fy01*Fy10+4*Fy00^2*Fx10^3*x^5*Fx01+40*Fy00*Fx10^3*x^2*Fy10*Fx00+8*Fx00^2*Fx10*
Fy00*Fy11*Fx11*x
+8*Fy00^2*Fx11^3*x^3*Fx00+4*Fy00^2*Fx11^3*x^4*Fx01-16*x^3*Fx00*Fx01^2*Fy11^2*Fx10-2*x^4*Fy01^3*Fx01*Fx1
1*Fy10+6*x^2*Fy00*Fx11^2*Fy01*Fy10^2-8*Fy00^2*Fx11^3*x^5*Fx10+16*Fy00^2*Fx11^3*x^4*Fx10-16*Fx00*Fy01*x^
4*Fy11^2*Fx01*Fy00-8*Fy00^2*Fx11^3*x^3*Fx10+8*Fy00^2*Fx00*x^5*Fx10*Fx01*Fx11+8*Fy00*Fx00*x^5*Fx10^2*Fy0
1*Fx01+2*Fy00*Fx10*Fy10*Fx01^3*x^2+8*Fy00^3*Fx10*Fx11*Fy01*x-24*Fy01^2*x^4*Fx10^2*Fx11^2-40*Fx00*Fy10^2
*Fy00*Fx11*x^2*Fy01-12*Fy00^2*Fx11^2*x^5*Fx00*Fx10+4*Fx00*Fx11*Fy10^2*Fy00^2-12*Fy00^2*Fx11^2*x^2*Fx10^
2-4*Fy00^2*Fx11^3*x^5*Fx01-8*Fy00*Fx11^2*x^4*Fy10*Fx01^2+12*Fy11*x^5*Fx01^2*Fy01*Fx00*Fx11-16*Fy11*x^5*F
x01^2*Fy01*Fx10*Fx11+20*Fy11*x^4*Fx01^2*Fy01*Fx00*Fx10+8*Fy01^2*x^3*Fx10^3*Fx01-2*Fx10^2*Fy10^2*Fy00^2-
24*Fy01^2*x^3*Fx10^3*Fx11+8*x^3*Fy11*Fy01*Fx01^2*Fx10^2-10*Fy01^2*x^4*Fx10^3*Fx01-18*Fy00*Fx10^3*x^4*Fy
11*Fx00+16*Fy01^2*x^4*Fx10^3*Fx00-6*x^2*Fy01*Fx01*Fx11*Fy10^3-12*Fy11*x^5*Fx01^2*Fy01*Fx00*Fx10-12*Fy00
*Fx00^2*Fx10^2*Fy01*x-24*Fy01^2*x^3*Fx10^3*Fx00+12*Fy01^2*x^5*Fx10^2*Fx11^2-80*Fy00*Fx00^2*Fx10^2*Fy10*
x^2-12*Fy01*x*Fx10^4*Fy00+16*Fy01^2*x^2*Fx10^3*Fx00+24*Fy01^2*x^4*Fx10^3*Fx11+16*Fy11*x^5*Fx01^2*Fy00*
Fx10*Fx11-4*Fy01^2*x^5*Fx10^3*Fx00+12*Fx11*x^5*Fy01^2*Fx01*Fy00*Fy11+4*Fy01^2*x^5*Fx10^3*Fx01+4*Fy01^2
*x^5*Fx11^3*Fx00+12*Fy00^2*Fx10^2*x^5*Fx00*Fx11-8*Fy01^2*x^5*Fx10^3*Fx11-8*Fy01^2*x^5*Fx11^3*Fx10+16*x

Part5

5th Degree Polynomial

+20*Fy00*Fx10^3*x^4*Fy10*Fx00-28*Fy00*Fx10^3*x^2*Fy10*Fx01+12*Fy11*x^5*Fx01^2*Fy00*Fx00*Fx10+20*Fy11*x^4*Fx01^2*Fy00*Fx00*Fx11-24*Fy11*x^4*Fx01^2*Fy00*Fx10*Fx11+40*Fy10^2*Fx00^3*Fx10*x^2+8*Fx00*Fx11*Fy10^2*Fy00*Fy01*x-12*Fy11*x^5*Fx01^2*Fy00*Fx00*Fx11-26*Fy11*x^4*Fx01^2*Fy00*Fx00*Fx10-40*Fy10^2*Fx00^3*Fx10*x^3-2*x^4*Fy11*Fy01^3*Fx01*Fx10-2*x^2*Fx00^3*Fy11^2*Fx11-14*x^4*Fy11^2*Fy01*Fx01*Fx11*Fy10-4*Fy10^2*Fx00^3*x^2*Fx11+32*Fy10^2*Fx00^3*x^2*Fx01+12*Fx00*Fy11^2*x^5*Fx10*Fy01*Fy10+64*Fx00*Fy10^2*x^4*Fx10*Fy00*Fy01+16*Fy10^2*Fx00^3*x^3*Fx11-34*Fx11*x^4*Fy00^2*Fx00*Fy01*Fy11+8*Fx00*Fy01*x^5*Fy10^2*Fx01*Fy00-14*Fy10^2*Fx00^3*x^4*Fx11-34*Fx00*Fy10^2*x^4*Fx11*Fy01*Fy11+32*Fx00*Fy11^2*x^4*Fx10*Fy00*Fy10+32*Fy11*x^3*Fx00^2*Fy10*Fx01*Fx11-48*Fy00*Fx11^2*x^3*Fy01*Fx10^2+16*Fy11*x^5*Fx00^2*Fy00*Fx10*Fx11+12*Fy11*x^5*Fx00^2*Fy00*Fx01*Fx11-12*Fy11*x^5*Fx00^2*Fy00*Fx01*Fx10+20*Fy10^2*Fx00^3*x^4*Fx10+72*Fx11*x^3*Fy00^2*Fx00*Fy10*Fy11-12*Fy11*x^5*Fx00^2*Fy01*Fx01*Fx11+18*Fy10*Fx00^4*x^4*Fy11-24*Fx11^2*x^3*Fy00^3*Fy10-32*Fy10*Fx00^4*x^3*Fy11+28*Fy10*Fx00^4*x^2*Fy11+40*Fx00*Fy10^2*x^4*Fx11*Fy00*Fy11+12*Fx11*x^5*Fy00^2*Fx00*Fy01*Fy11-36*Fy10^2*Fx00^2*x^4*Fx01^2+36*Fy10^2*Fx00^2*x^3*Fx01^2+32*Fx11*x^3*Fy00^2*Fx10*Fy01*Fy11+4*Fy10*Fx00*Fx10^3*Fy00+12*Fy11*x^5*Fx00^2*Fy01*Fx01*Fx10-16*Fy11*x^5*Fx00^2*Fy01*Fx10*Fx11-20*Fy10^2*Fx00^2*Fx10^2*x^2+10*Fy10^2*Fx00^2*Fx10^2*x

+20*Fy10^2*Fx00^2*Fx10^2*x^3+8*Fx11^2*x^3*Fy00^3*Fy11-34*Fy11*x^4*Fx00^2*Fy00*Fx01*Fx11+72*Fx00*Fy10^2*x^3*Fx11*Fy00*Fy01-8*Fy10^2*Fx01^3*x^3*Fx00-32*Fx00*Fy10*Fy00^2*Fx01*x^2*Fy11+8*Fy10^2*Fx01^3*x^3*Fx10-4*Fy10*Fx01^4*x^5*Fy11+40*Fy11*x^4*Fx00^2*Fy00*Fx01*Fx10+32*Fy11*x^3*Fx00^2*Fy01*Fx01*Fx10+24*Fx11^2*x^4*Fy00^3*Fy01+28*Fy11*x^4*Fx00^2*Fy01*Fx01*Fx11-34*Fy11*x^4*Fx00^2*Fy01*Fx01*Fx10-48*Fy11*x^3*Fx00^2*Fy01*Fx10*Fx11+16*Fx00*Fy10^2*x^5*Fx11*Fy00*Fy01-8*Fy10^2*Fx00^3*x^5*Fx01+8*Fy01*x^5*Fx10^2*Fy11*Fx00^2+8*Fy01*x^5*Fx10^2*Fy11*Fx01^2+4*Fy10^2*Fx00^3*x^5*Fx11+2*Fy00*Fx10*Fy10*Fx01*Fx00^2+4*Fy00*Fx11^3*x^5*Fy10*Fx00-6*Fy00*Fx11^3*x^4*Fy10*Fx00-24*Fx00*Fy11^2*x^4*Fx11*Fy00*Fy01-4*Fy10^2*Fx00^3*x^5*Fx10-4*Fy10*Fx00^4*x^5*Fy11+48*Fx11*x^4*Fy00^2*Fx01*Fy10*Fy11+48*Fy11*x^4*Fx00^2*Fy01*Fx10*Fx11+12*Fx00*Fy10^2*x^5*Fx11*Fy01*Fy11+12*Fy10^2*Fx00^2*x^5*Fx01^2+32*Fy10^2*Fx00^3*x^4*Fx01-12*Fx10*Fy00^2*x^5*Fx01*Fy01*Fy10+28*Fx11*x^4*Fy00^2*Fx01*Fy01*Fy11-48*Fy10^2*Fx00^3*x^3*Fx01-56*Fy11*x^4*Fx00^2*Fy00*Fx10*Fx11+24*Fy00*Fx11^2*x^3*Fy10*Fx00^2-24*Fy00*Fx11^2*x^4*Fy10*Fx00^2-56*Fx11*x^4*Fy00^2*Fx00*Fy10*Fy11+8*x^2*Fy00*Fx10*Fy01*Fx11*Fx00*Fx01+16*Fx11^2*x^2*Fy00^3*Fy10+24*Fy10*Fx01^2*x^5*Fy11*Fx00*Fx10-2*Fy00^3*Fx10*Fy10*Fx01+32*Fy10*Fx01^2*x^4*Fy11*Fx00*Fx11+32*Fy11*x^3*Fx00^2*Fy00*Fx01*Fx11+4*Fx00^2*Fy10^3*Fy00+4*Fy00^2*Fx10*Fy10^2*Fx01+10*Fx10^2*Fy00^4*x-16*Fy10*Fx01^2*x^5*Fy00*Fx10*Fx11-2*Fx10^2*Fy01^4*x^4+12*Fx11*x^5*Fy01^2*Fx00*Fy00*Fy10-2*Fx00^2*Fx01*Fy10^2*Fx10+40*Fx10*Fy00^2*x^4*Fx00*Fy01*Fy11+24*Fx00*Fy10^2*Fy11*x^2

Part6

5th Degree Polynomial

*Fx10*Fy01-24*Fx11^2*x^4*Fy00^2*Fy01^2-4*Fx00*Fx01*Fy10^3*Fy00+2*Fx00*Fx01*Fy10^2*Fy00^2+2*Fx00^2*Fy10*
Fy11*Fx10^2-16*Fx10*Fy00^2*x^5*Fx00*Fy10*Fy11+20*Fx11*x^4*Fy01^2*Fx00*Fy00*Fy11+12*Fx11^2*x^3*Fy00^4-26
*Fy10*Fx01^2*x^4*Fy01*Fx00*Fx10-12*Fy10*Fx01^2*x^5*Fy01*Fx00*Fx11+16*Fx00*Fy11^2*x^5*Fx11*Fy00*Fy01-26*
Fx00*Fy11^2*x^4*Fx10*Fy01*Fy10+2*Fx11^2*x^5*Fy00^4+12*Fy10*Fx01^2*x^5*Fy01*Fx00*Fx10-4*Fx00^3*Fy10*Fy11
*Fx10-2*Fx00^2*Fy10^2*Fy11*Fy00+2*Fx00^2*Fy10*Fy11*Fy00^2+2*Fy00*Fx10^2*Fy01*Fy10^2-8*Fy01*x^5*Fx10^2*Fy
10*Fx00^2-8*Fy01^2*x^4*Fx10^4+20*Fy10*Fx01^2*x^4*Fy01*Fx00*Fx11-24*Fy10*Fx01^2*x^4*Fy01*Fx10*Fx11-4*Fy0
0*Fx10^3*Fy01*Fx00-12*Fx11*x^5*Fy01^2*Fx00*Fy00*Fy11-2*Fy00^2*Fx10^2*Fy01*Fy10+32*Fx00*Fy10^2*Fy11*x^3*F
x11*Fy01-10*Fx00^2*Fy10^4*x^4+10*Fx00^2*Fy10^4*x
+2*Fy00*Fx10^2*Fy01*Fx00^2+2*Fy00^2*Fx10*Fx11*Fy10^2+16*Fy10*Fx01^2*x^5*Fy01*Fx10*Fx11-24*Fx11^2*x^3*F
y00^3*Fy01+16*Fy10*Fx01^2*x^3*Fy01*Fx00*Fx10-68*Fy10*Fx01^2*x^4*Fy11*Fx00*Fx10+12*Fx10*Fy00^2*x^5*Fx00*
Fy01*Fy10-2*Fy00^2*Fx10^2*Fx11*Fx00+32*Fy00*Fx10^3*x^3*Fy10*Fx01-18*Fy00*Fx10^3*x^4*Fy10*Fx01-4*Fy00^3*F
x10*Fx11*Fy10-24*Fy10*Fx01^2*x^5*Fy11*Fx00*Fx11+32*Fy00*Fx10^3*x^3*Fy11*Fx00+2*Fy00^2*Fx10*Fx11*Fx00^2-
2*Fx00^2*Fy11^4*x^4-48*Fx11*x^3*Fy00^2*Fx01*Fy10*Fy11-2*Fy00*Fx10*Fy10^3*Fx01-2*Fy00*Fx10^3*Fy10*Fx01-2*
Fx00^2*Fy10^2*Fy00^2+6*x^4*Fy11^2*Fx01^2*Fx10*Fx11-12*Fx00*Fy11^2*x^5*Fx10*Fy00*Fy10-2*Fx00^3*Fx10*Fy01
*Fy10+2*Fx00*Fx01*Fy10^2*Fx10^2-2*Fx00*Fx10^3*Fy00*Fy11+4*Fx00^2*Fx10^2*Fy00*Fy11-10*x^2*Fy01*Fx01*Fx11
*Fy10*Fx10^2-16*Fx11*x^5*Fy01^2*Fx01*Fy10*Fy11-2*Fx00^3*Fx10*Fy00*Fy11-2*Fx00*Fx10*Fy00^3*Fy11+4*Fx10^2*
Fy01^3*x^5*Fy10-32*Fy01*x^2*Fx10^2*Fy11*Fx00^2-2*Fx00*Fx10*Fy01*Fy10^3-2*Fx00*Fx10^3*Fy01*Fy10-12*Fx10^2
*Fy01^2*x^2*Fy00^2+20*Fx11*x^4*Fy01^2*Fx01*Fy10*Fy00+20*Fx10^2*Fy00^4*x^3+4*Fx00^2*Fx10^2*Fy01*Fy10-8*F
x10^2*Fy01^3*x^5*Fy00+4*Fy01*x^5*Fx10^3*Fy11*Fx01+20*Fy01*x^4*Fx11^2*Fy10*Fx00^2+4*Fy10^2*Fx00^3*Fx10-
20*Fy00*Fx10^3*x*Fy10*Fx00-16*Fy01*x^4*Fx10^2*Fy11*Fx01^2+20*Fy00^2*Fx10^4*x^3-2*Fx00^3*Fx11*Fy10*Fy00-
16*Fy01*x^3*Fx10^2*Fy10*Fx01^2+4*Fy00^2*Fx00*Fx10^3+24*Fx11*x^2*Fy00^2*Fx00*Fy01*Fy10-46*Fx00*Fy10^2*x^
4*Fx10*Fy00*Fy11+12*Fx01^2*x^3*Fy10^4+40*Fy00*Fx00^3*Fx10*Fy10*x^2-24*Fy00*Fx10^3*x^3*Fy11*Fx01+8*x^3*F
y11^2*Fy01*Fx01*Fx10*Fy00+2*x^4*Fy00*Fx11^2*Fy01*Fy11^2-10*Fx10^2*Fy01^3*x^4*Fy10+8*x^2*Fx00^3*Fx01*Fy1
1^2+12*x^3*Fx00^2*Fx01^2*Fy11^2+8*x^4*Fy00*Fx11^2*Fy01^3+16*Fy00*Fx10^3*x^5*Fy01*Fx11-32*Fx00*Fy00^2*F
y10*x^4*Fx01*Fy11+40*Fy11^2*x^3*Fx00^2*Fx01*Fx10+20*Fy11^2*x^4*Fx00^2*Fx01*Fx11+24*Fy00*Fx00*Fx10^3*Fy
01*x
+28*Fy00*Fx00^2*Fx10^2*Fy01*x^2+8*Fy00*Fx10^2*x^5*Fy10*Fx01^2-8*Fy00*Fx10^2*x^5*Fy11*Fx00^2-4*Fx11^2*x^
5*Fy00^2*Fy10*Fy11+24*Fy00^2*Fx00*Fx10*Fx11^2*x^2-48*Fy00^2*Fx00*Fx10*Fx11^2*x^3-4*Fx10*Fy00^2*x^5*Fy10
^2*Fx11+48*Fy01*x^3*Fx10^2*Fy11*Fx00^2-32*Fy01*x^4*Fx10^2*Fy11*Fx00^2-16*Fx11^2*x^3*Fy10^2*Fy00*Fy01-8*

Part7

5th Degree Polynomial

*Fx11^2*x^3*Fy10*Fy00^2*Fy11+10*Fx11^2*x^4*Fy10*Fy00^2*Fy11+14*Fx11^2*x^4*Fy10^2*Fy00*Fy01-8*Fy00*Fx11^2*x^2*Fy10*Fx00^2-2*Fy00^2*Fx00^2*Fx10^2+12*Fx11*x*Fy10^3*Fy00*Fx00-4*Fx11^2*x^5*Fy01^2*Fy11*Fy10+8*Fx10^2*Fy01^3*x^3*Fy10-4*Fx11^2*x^5*Fy01*Fy11^2*Fy00-8*Fy01*x^5*Fx11^2*Fy10*Fx01^2-4*Fx11^2*x^5*Fy01*Fy10^2*Fy00+16*Fx11*x^2*Fy01*Fy10^3*Fx00+4*x^2*Fx00*Fx11*Fy01^2*Fy10^2-4*Fx10*Fy00^2*x^5*Fy11^2*Fx11-32*Fy00^2*Fx00^2*Fx10*x^3*Fx11-4*Fx10^2*Fy01*x^5*Fy10^2*Fy00-4*Fx10*Fy01^2*x^5*Fy10^2*Fx11-8*x^3*Fx00*Fx01*Fy11^3*Fy00-2*x^4*Fy11^3*Fy01*Fx01*Fx10+8*x^3*Fy01^2*Fx11*Fx10*Fy10*Fy11-2*Fy00^2*Fx10^2*Fx00*Fx01+14*Fx10*Fy01^2*x^4*Fy10^2*Fx11+12*Fx10*Fy01*x*Fy10^3*Fx00+14*Fx10^2*Fy00^2*Fy10*x^4*Fy11+4*Fx10^2*Fy01^3*x^4*Fy11+16*Fx10^2*Fy01^3*x^4*Fy00+18*Fx10*Fy00^2*Fy10^2*x^4*Fx11+6*Fx10*Fy00^2*Fy11^2*x^4*Fx11-4*Fx10^2*Fy00^2*x^5*Fy11*Fy10+8*x^2*Fy00^3*Fx11^2*Fy01+12*x^3*Fy01^2*Fx11^2*Fx10^2+18*Fx10^2*Fy01*x^4*Fy10^2*Fy00-16*Fx10*Fy01^2*x^3*Fy10^2*Fx11+16*Fy01*x^2*Fx10^3*Fy11*Fx00-10*x^2*Fy11*Fy00*Fx01*Fx11*Fy10^2-4*Fx00*Fx10^2*Fy01*Fy10*Fx11*x
+4*Fy11*x^5*Fx00^3*Fy00*Fx10-4*Fy11*x^5*Fx00^3*Fy00*Fx11+10*Fy10^2*Fx00^4*x-2*Fy00^2*Fx10^2*x*Fx01^2-16*Fy01*x^4*Fx11^2*Fy11*Fx00^2-4*Fx10*Fy01^2*x^5*Fy11^2*Fx11-8*x^3*Fy11*Fy00*Fx01*Fx11^2*Fx10+8*x^2*Fy00*Fx10*Fy01*Fx11*Fy10*Fy11+2*x^5*Fy11^2*Fx01^2*Fx11^2+16*Fx01*x^2*Fy00^3*Fy11*Fx10+10*Fx10^2*Fy01^2*x^4*Fy11*Fy10+6*Fx10^2*Fy01*x^4*Fy11^2*Fy00-18*Fy01*x^4*Fx10^3*Fy10*Fx00+2*Fy00^2*Fx10^4*x^5-4*Fx10^2*Fy01*x^5*Fy11^2*Fy00+8*Fy00^2*Fx11^2*x^3*Fx01*Fx10+40*Fy00^2*Fx11^2*x^4*Fx00*Fx10-4*Fx10^2*Fy01^2*x^5*Fy11*Fy10-8*Fx10^2*Fy01^2*x^3*Fy11*Fy10+16*Fy01*x^4*Fx10^3*Fy11*Fx00-14*Fy01*x^4*Fx10^3*Fy11*Fx01-8*x^3*Fy01^2*Fx01*Fx11*Fy10*Fy00-4*x^5*Fy11^3*Fx01^2*Fy01-14*x^2*Fy10^2*Fx01^2*Fy11*Fy00-10*x^2*Fy11*Fy01*Fx01*Fx10*Fy00^2+2*x^4*Fy01^4*Fx11*Fx10-24*Fy00*Fx10^2*x^5*Fy01*Fx11^2-24*Fy00*Fx10^3*x^3*Fy01*Fx01-4*Fy00*Fx11^3*x^5*Fy11*Fx00+24*Fx10*Fy01^2*x^5*Fx11*Fy00*Fy10+2*x^4*Fy00*Fx11^4*Fy01-12*Fx11*x^5*Fy01^2*Fx01*Fy00*Fy10+12*Fy11*x*Fx00*Fx10^3*Fy00-8*x^3*Fy11^2*Fy01*Fx01*Fx10*Fy10-2*Fx00*Fx11*Fy10*Fy00*Fy01^2*x^2-68*Fx10*Fy01^2*x^4*Fx11*Fy00*Fy10+16*Fx00*Fy11^2*x^3*Fx10*Fy01*Fy10-38*Fy01^2*x^4*Fx10^2*Fx00*Fx11+10*Fy11^2*x^4*Fx00^2*Fx10*Fx11-4*Fy11^2*x^5*Fx01*Fx11^2*Fx00+88*Fx10*Fy00^2*x^4*Fx11*Fy01*Fy10+2*Fx00*Fx10*Fy01*Fy10*Fy00^2-8*Fy11^2*x^3*Fx00^2*Fx11*Fx10-8*Fy00*Fx11^2*x^5*Fy11*Fx01^2-4*Fx10^2*Fy01^3*x^5*Fy11-24*Fy01*x^3*Fx10^3*Fy10*Fx01-2*Fx00*Fx11*Fy10*Fy00*Fx01^2*x^2+12*x^3*Fy10^2*Fx01^2*Fy11^2+6*x^2*Fy10*Fx01^2*Fy11*Fy00^2-2*x^4*Fx00*Fx11*Fy01^3*Fy11-10*x^2*Fx00*Fx11*Fy01*Fy11*Fy10^2-4*x^4*Fx00*Fx01*Fy11^3*Fy01-8*x^3*Fy11*Fy01^2*Fx01*Fx10*Fy00+8*x^3*Fy10*Fx01^2*Fy11*Fy00*Fy01-8*Fx10^2*Fy01^3*x^3*Fy00-24*Fy01*x^3*Fx10^3*Fy11*Fx00-8*Fy01^2*x^2*Fx10^4-4*Fy11^2*x^5*Fx00^2*Fx10*Fx11-2*x^2*Fy10^2*Fx01^3*Fx10-8*x^3*Fx00*Fx11^2*Fy0

Part8

5th Degree Polynomial

Fy00*Fx11^3*x^5*Fy11*Fx01+4*Fy00*Fx11^3*x^4*Fy11*Fx00-16*Fy11^2*x^3*Fx01*Fx10^2*Fx00-4*Fx10*Fy01^4*x^5*Fx11-38*Fy11^2*x^4*Fx00^2*Fx01*Fx10+10*Fx01^2*x^4*Fy10^2*Fy00*Fy01+12*Fx01*x*Fy10*Fy00^3*Fx10-4*Fx01^2*x^5*Fy01^2*Fy11*Fy10-4*Fx01^2*x^5*Fy01*Fy11^2*Fy00-4*Fx01^2*x^5*Fy01*Fy10^2*Fy00-4*Fx01^2*x^5*Fy00^2*Fy10*Fy11-32*Fy00*Fx00^2*Fx10^2*Fy01*x^3-20*x^2*Fy01^2*Fx11*Fx10*Fy10*Fy00+14*Fx01^2*x^4*Fy10*Fy00^2*Fy11-16*Fx01^2*x^3*Fy10*Fy00^2*Fy11-8*x^3*Fx00*Fx01*Fy11^2*Fy01*Fy10+8*x^2*Fx00*Fx01*Fy10*Fy11*Fy00*Fy01-4*Fx00*Fy00^2*x^5*Fy10^2*Fx01-40*Fy00*Fx10^3*x^3*Fy10*Fx00-8*Fx01^2*x^3*Fy10^2*Fy00*Fy01-14*x^4*Fy11*Fy01*Fx01^2*Fx11*Fx00+12*Fx00*Fy00^3*Fy11*x*Fx10-4*Fx00^2*Fy00^2*x^5*Fy11*Fy10-56*Fy00*Fx10^3*x^4*Fy01*Fx11-4*Fx00*Fy00^2*x^5*Fy11^2*Fx01-4*Fx00^2*Fy01*x^5*Fy10^2*Fy00-4*Fx00*Fy01^2*x^5*Fy10^2*Fx01+14*Fx00*Fy00^2*Fy11^2*x^4*Fx01+4*x^2*Fx00*Fx11*Fy00^2*Fy11^2-4*Fy11^2*x^5*Fx01*Fx10^2*Fx00+8*x^3*Fy00*Fx11^2*Fy01*Fx00*Fx01-24*Fy00*Fx10^2*x^4*Fy10*Fx01^2-48*Fx00*Fy10^2*x^3*Fx10*Fy01*Fy11-4*Fx11*x^5*Fy01^3*Fx00*Fy10+4*Fx11*x^5*Fy01^3*Fx01*Fy10+28*Fx11^2*x^4*Fy01^2*Fy10*Fy00-12*Fx11^2*x^5*Fy01^2*Fy00*Fy10-24*Fy00^2*Fx10^2*x^3*Fx01*Fx11+12*Fx11^2*x^5*Fy01^2*Fy00*Fy11-16*Fx11*x^3*Fy01^2*Fx00*Fy10^2-16*Fx00*Fy00^2*Fy11^2*x^3*Fx01-24*Fy00*Fx00^2*Fx10^2*Fy11*x-8*Fx11*x^5*Fy01^2*Fx00*Fy11^2-8*Fx11*x^5*Fy01^2*Fx00*Fy10^2+4*Fx11*x^5*Fy01^3*Fx00*Fy11+4*Fx11*x^4*Fy01^3*Fx00*Fy10-4*Fx10^2*Fy00^3*x^5*Fy10+4*Fx10^2*Fy00^3*x^5*Fy11-4*Fy01*x^5*Fx10^3*Fy11*Fx00-16*Fx00*Fy11^2*x^5*Fx10*Fy00*Fy01+4*Fx11*x^5*Fy00^3*Fx00*Fy10+4*Fx11*x^5*Fy00^3*Fx01*Fy11-4*Fx11*x^5*Fy00^3*Fx01*Fy10+16*Fy01*x^4*Fx10^3*Fy10*Fx01+16*Fy01*x^3*Fx10^3*Fy11*Fx01+2*Fy00^2*Fx11^4*x^5-12*Fx11^2*x^5*Fy00^2*Fy01*Fy11+12*Fx11^2*x^5*Fy00^2*Fy01*Fy10+20*Fx11*x^4*Fy01^2*Fx00*Fy10^2+14*Fx00^2*Fy01*x^4*Fy10^2*Fy00+6*Fx00*Fy01^2*x^4*Fy10^2*Fx01+18*Fx00^2*Fy00^2*Fy10*x^4*Fy11+18*Fx00*Fy00^2*Fy10^2*x^4*Fx01+16*Fx11*x^5*Fy00^2*Fx00*Fy10*Fy11-48*Fx00*Fy10^2*x^3*Fx11*Fy00*Fy11-8*Fx11*x^5*Fy00^2*Fx00*Fy11^2-12*Fx00*Fy10^2*x^5*Fx10*Fy01*Fy11-4*Fx11*x^5*Fy00^3*Fx00*Fy11-8*Fx10^2*Fy00^3*x^5*Fy01+2*x*Fy00^2*Fx11^2*Fx00^2+10*Fx00^2*Fy01*x^4*Fy11^2*Fy00+6*Fx00^2*Fy01^2*x^4*Fy11*Fy10-4*Fx00^2*Fy01*x^5*Fy11^2*Fy00+8*Fx00*Fy01^2*x^5*Fy11*Fx01*Fy10-4*Fx00^2*Fy01^2*x^5*Fy11*Fy10-4*Fx00*Fy01^2*x^5*Fy11^2*Fx01-2*Fx00*Fx10*Fy00*Fy11*Fy01^2*x^2-4*x*Fy00*Fx10^3*Fy11*Fx01-4*x*Fy01^2*Fx10^3*Fx00+12*Fx00*Fy11^2*x^5*Fx11*Fy00*Fy10-8*Fx00^2*Fy01*x^3*Fy11^2*Fy00+2*x^4*Fx00*Fx01*Fy11^4+8*Fx00*Fx10^2*Fy01*Fy10*Fx01*x-2*Fx00*Fx10*Fy01*Fy10*Fx01^2*x^2+2*x^2*Fy10^2*Fx01^2*Fx10*Fx11+8*x^5*Fy11*Fy01*Fx01^2*Fx11^2-2*x^2*Fy01^3*Fx10^2*Fy10-32*Fx10*Fy10*Fy00*Fx11*x^3*Fy01*Fy11+14*Fy11^2*x^4*Fx01*Fx10^2*Fx00-4*Fy11^2*x^5*Fx01^2*Fx10*Fx11-64*Fx10*Fy10*Fy00*Fx01*x^3*Fy01*Fy11-32*Fx10*Fy10*Fy00*Fx00*Fy01*x^2*Fy11-16*x^3*Fy11*Fy01*Fx01*Fx11*Fy10^2-10*x^4*Fy10*Fx01^2*Fy11^2*Fy01-12*Fx00*Fy10^2*Fy00^2*Fx01*x+28*Fx00*Fy10^2*Fy00^2*Fx01*x^2-32*

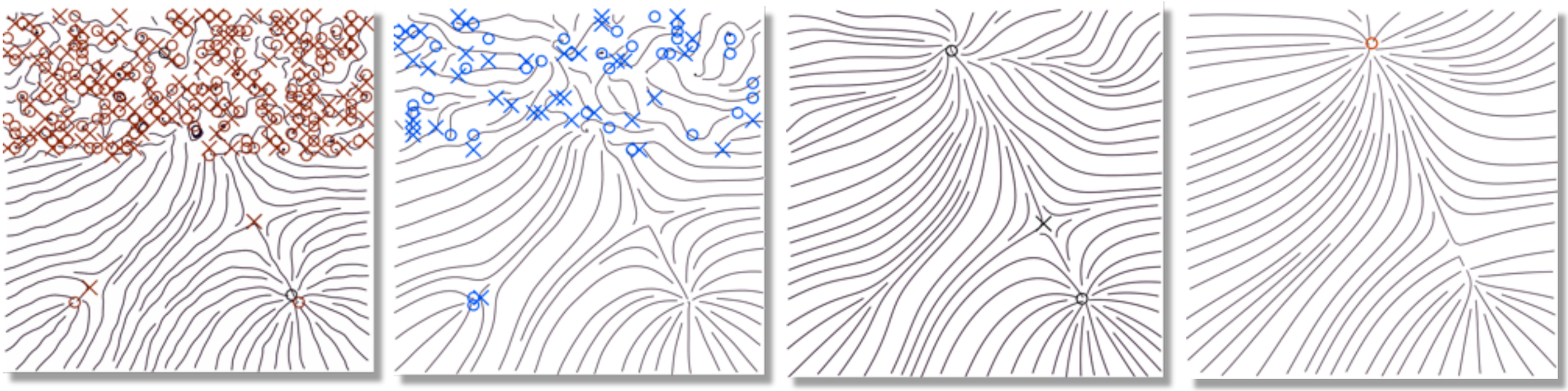
5th Degree Polynomial

(. . .)

5th Degree Polynomial

You got the idea ...

Filtering = Remove Singularities

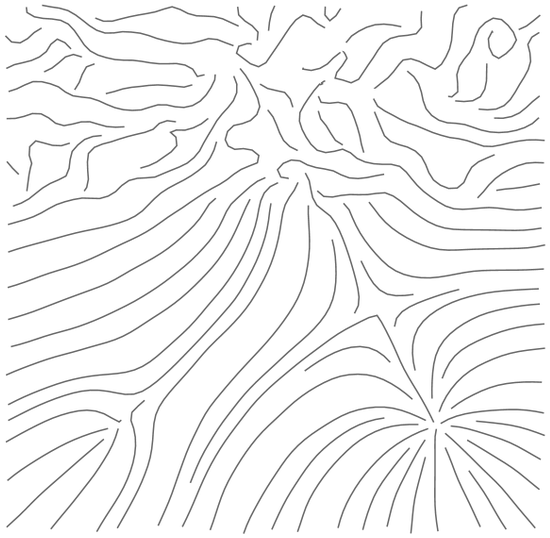


Control Scale with Singularities Information!

User Interaction

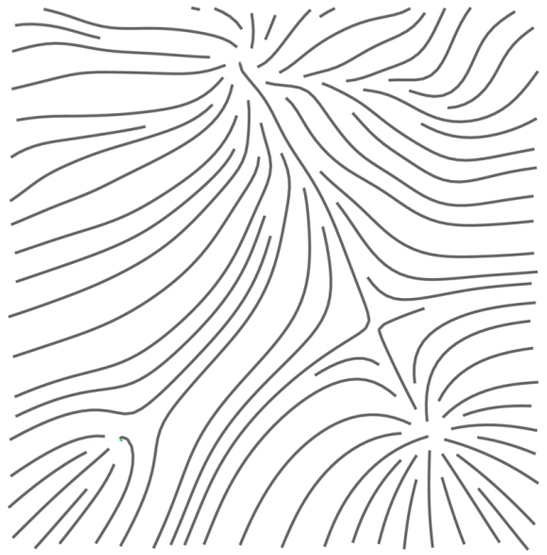
Big Picture

Original field

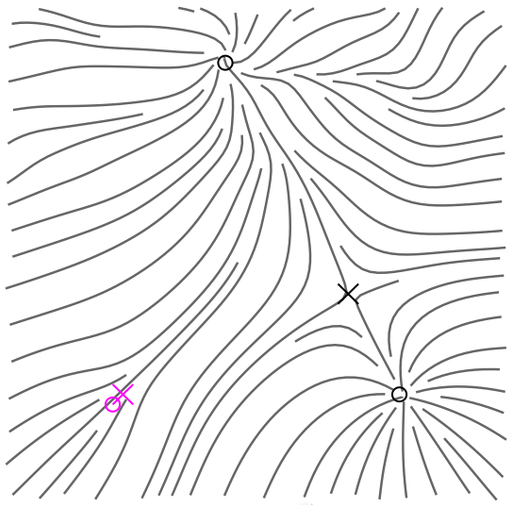


Denoising

Reconstruction

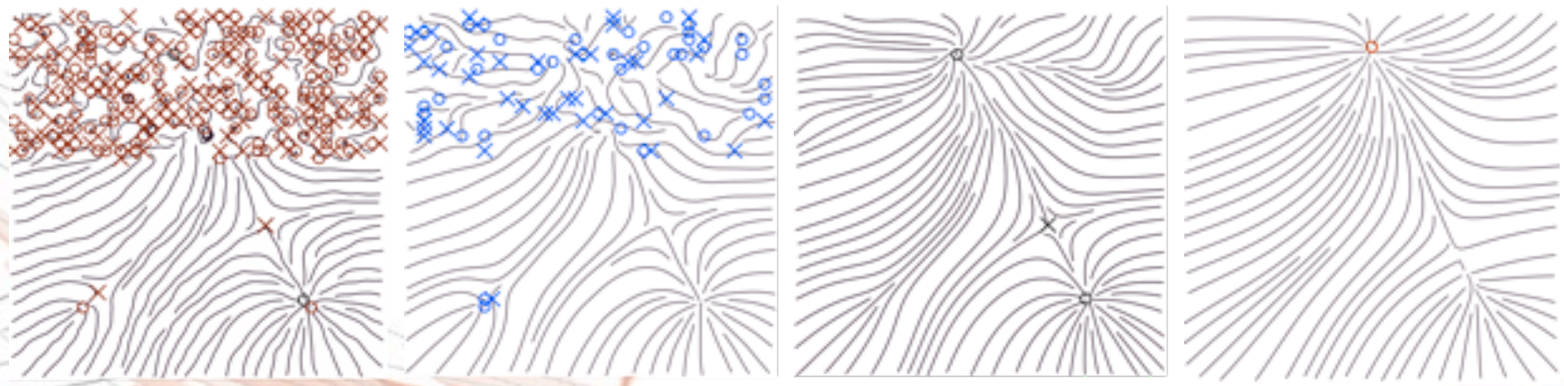


Final field



Topology information

User



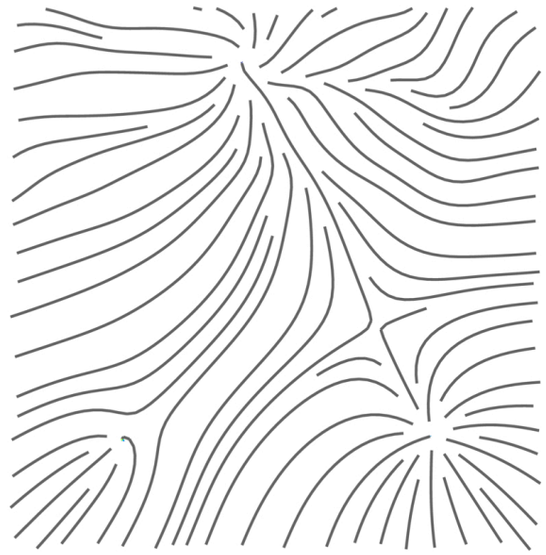
Scale space

Big Picture

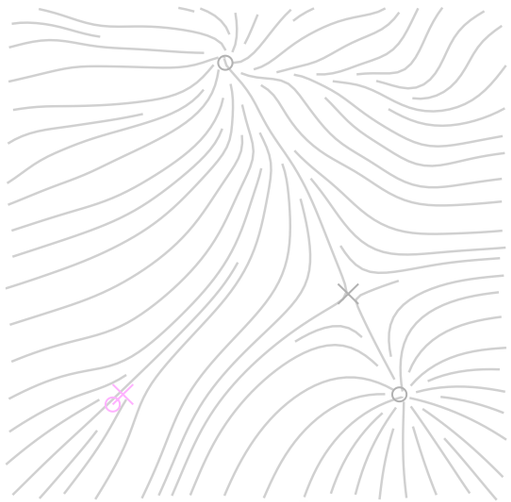
Original field

Denoising

Reconstruction

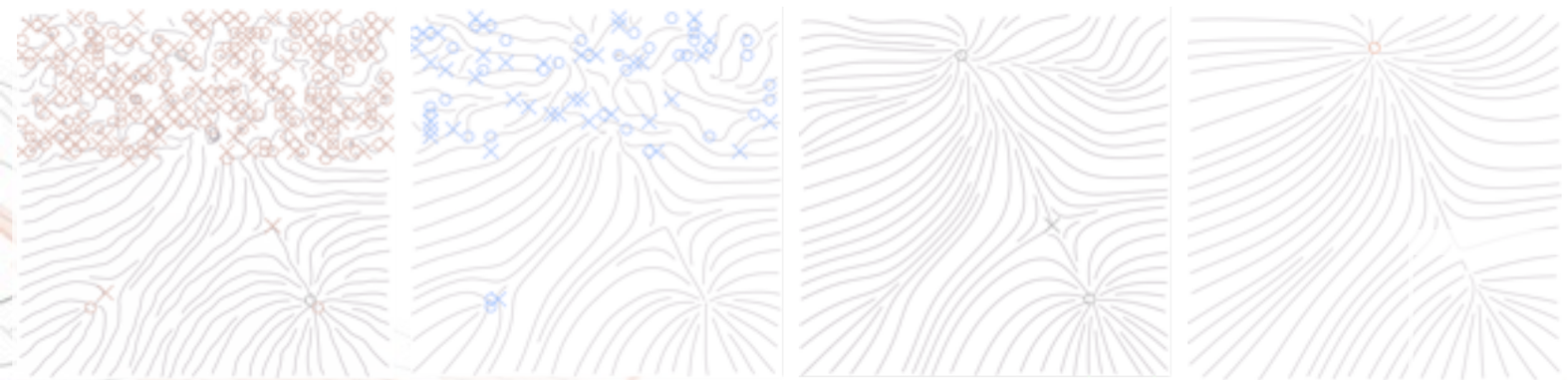


Final field



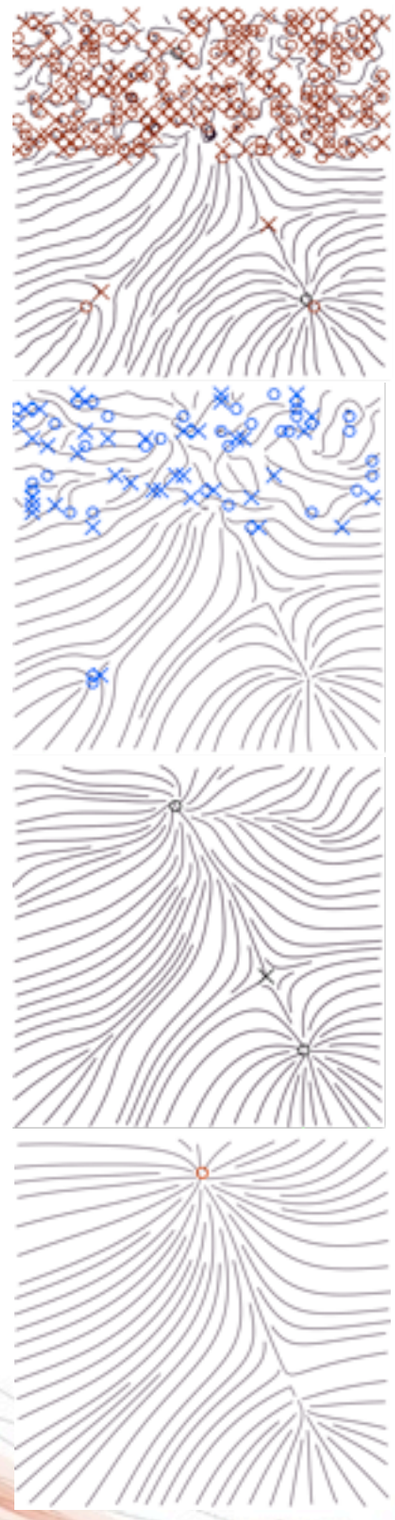
Topology information

User

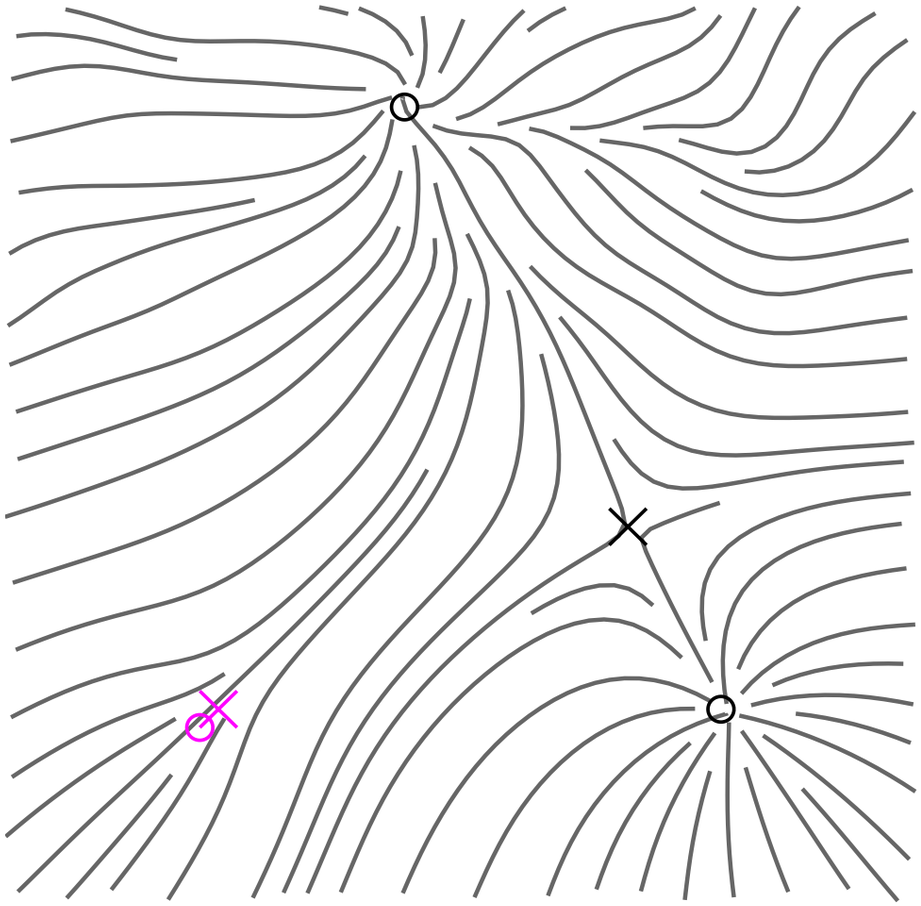


Scale space

Reconstruction

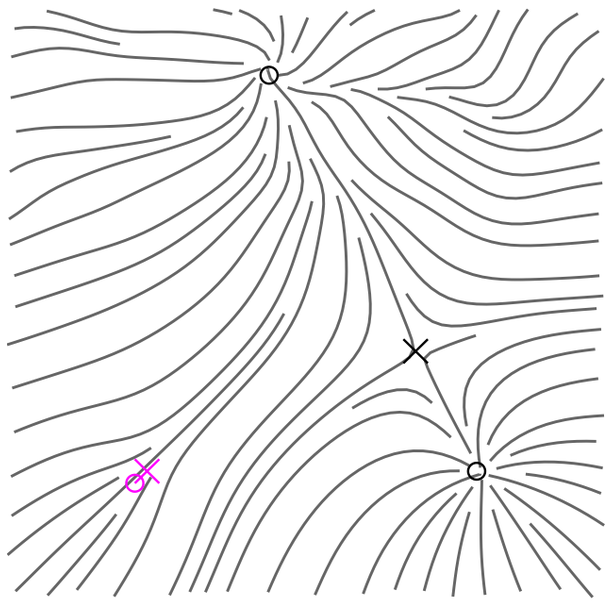


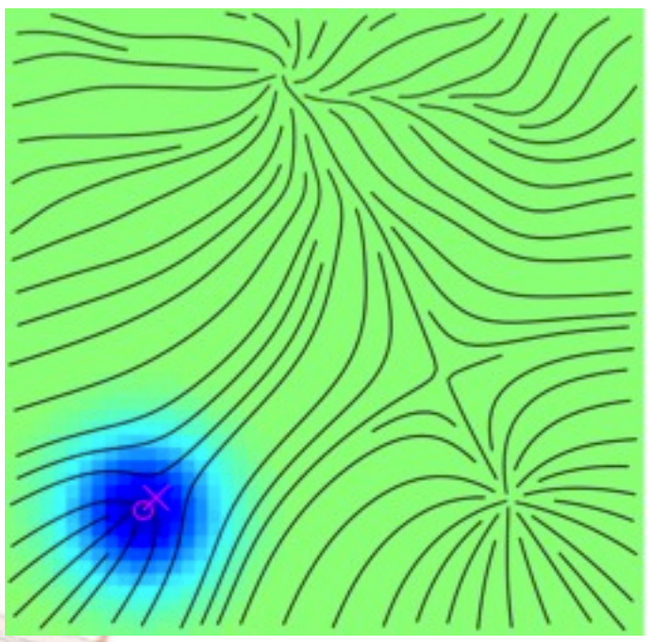
$$\tilde{\mathbf{v}}(x_i, y_i) = \mathbf{v}_{i,j}(s(x_i, y_i))$$

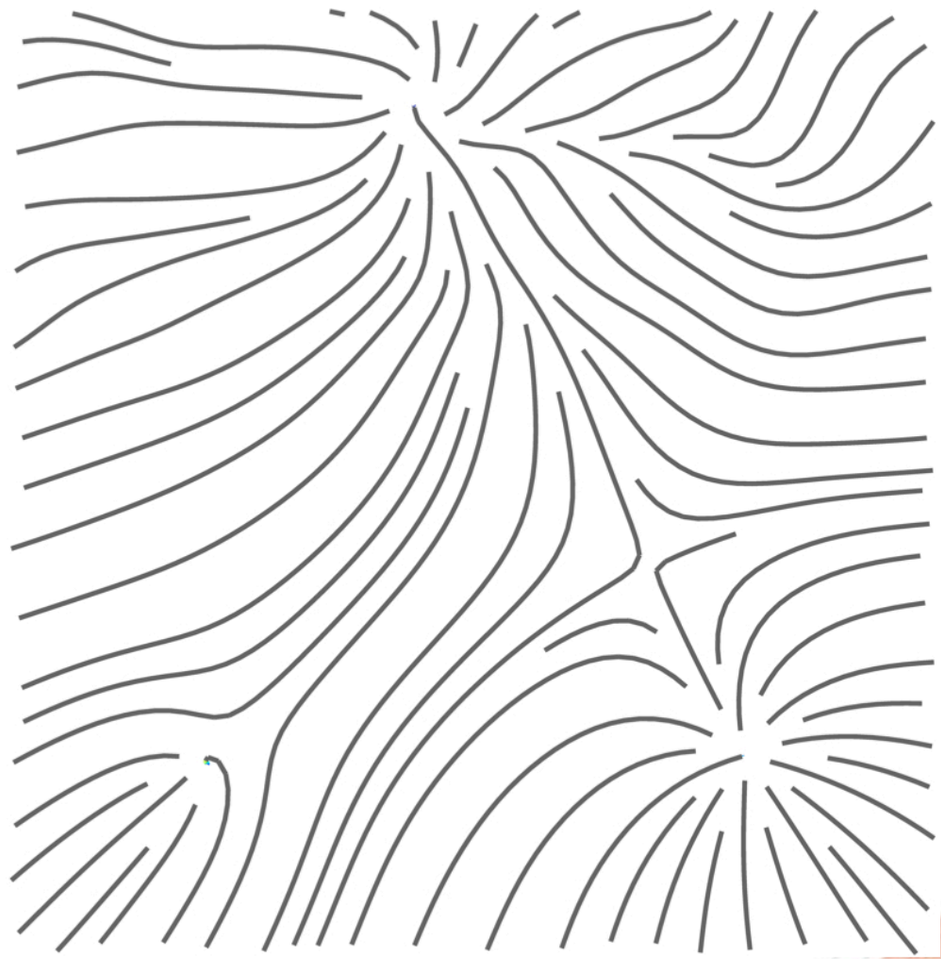


S

Reconstruction



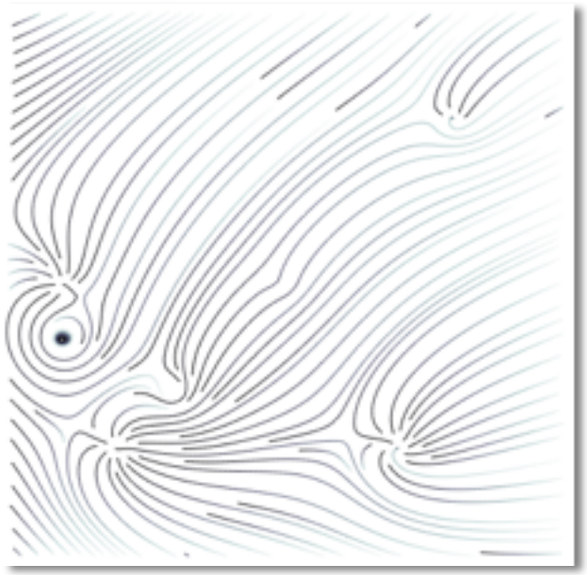




Final field

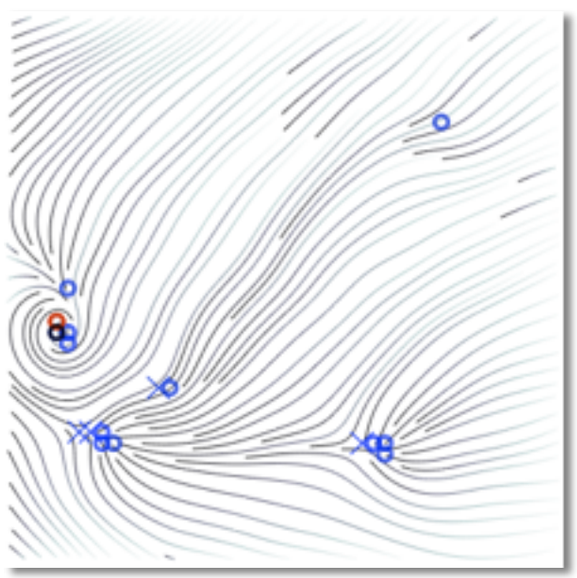
Results

Analytic field

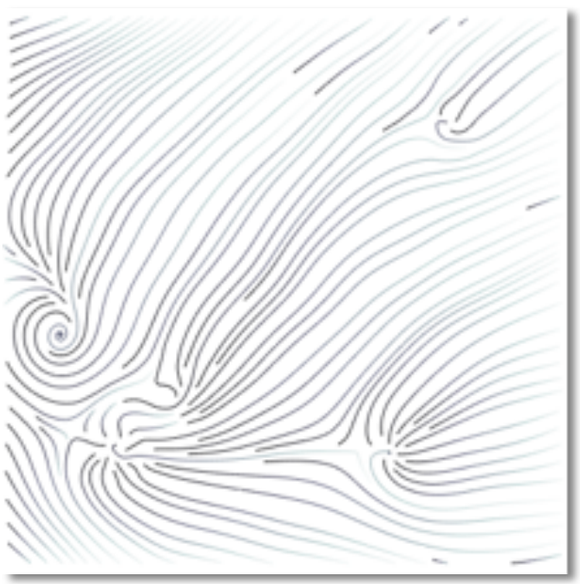


Original

Non-Gaussian Noise



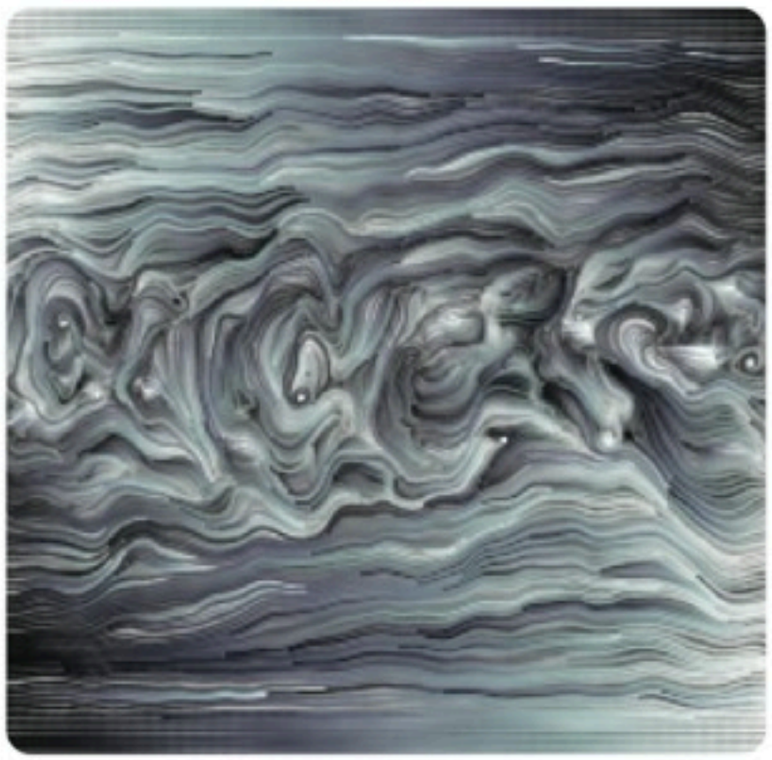
Smoothed



Reconstructed

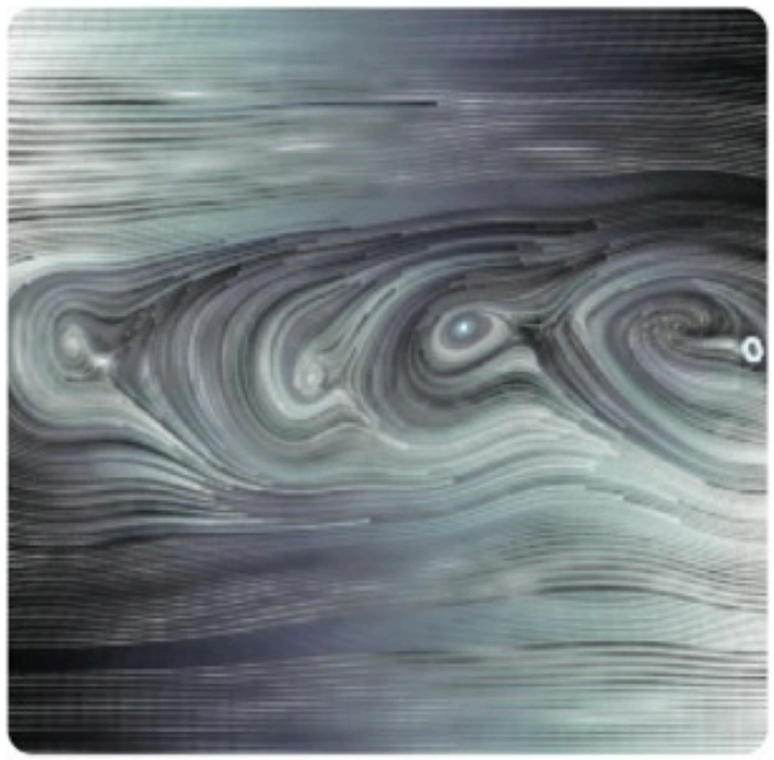
Results

Granular flow: shear band



Original

Smoothed

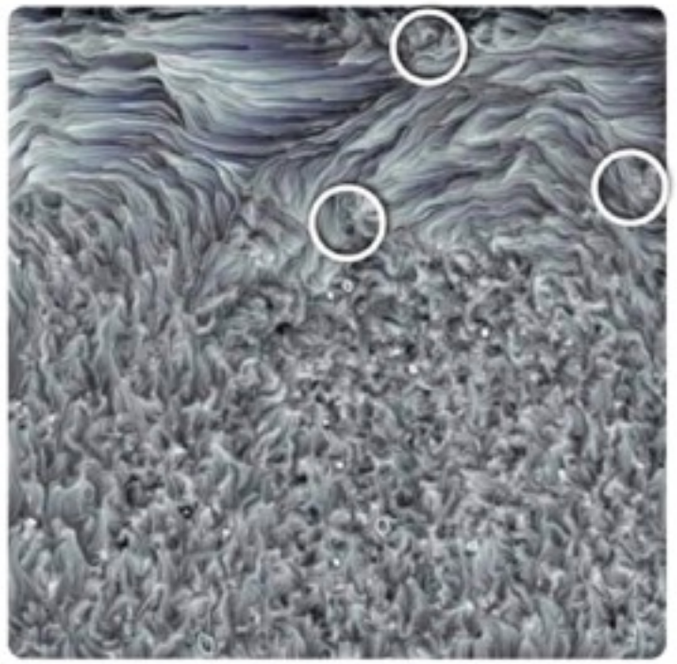


Reconstructed

Data: Bordignon, A.
(Dept. of Mathematics, PUC - Rio)

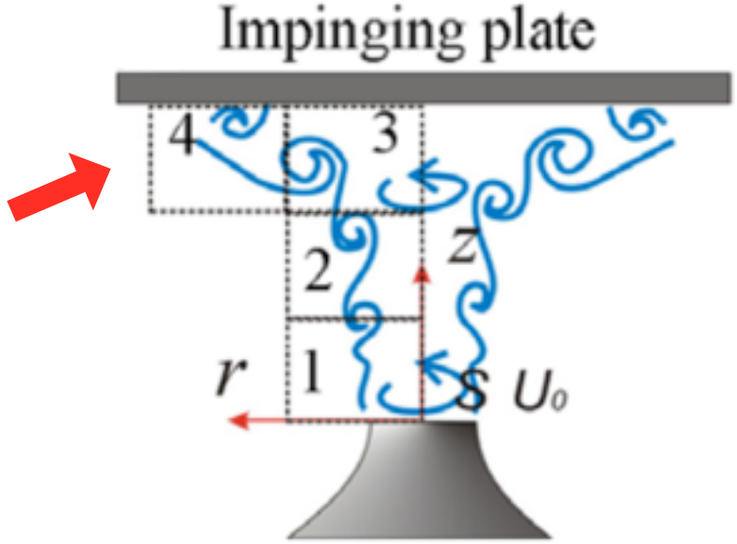
Results

Wall jet



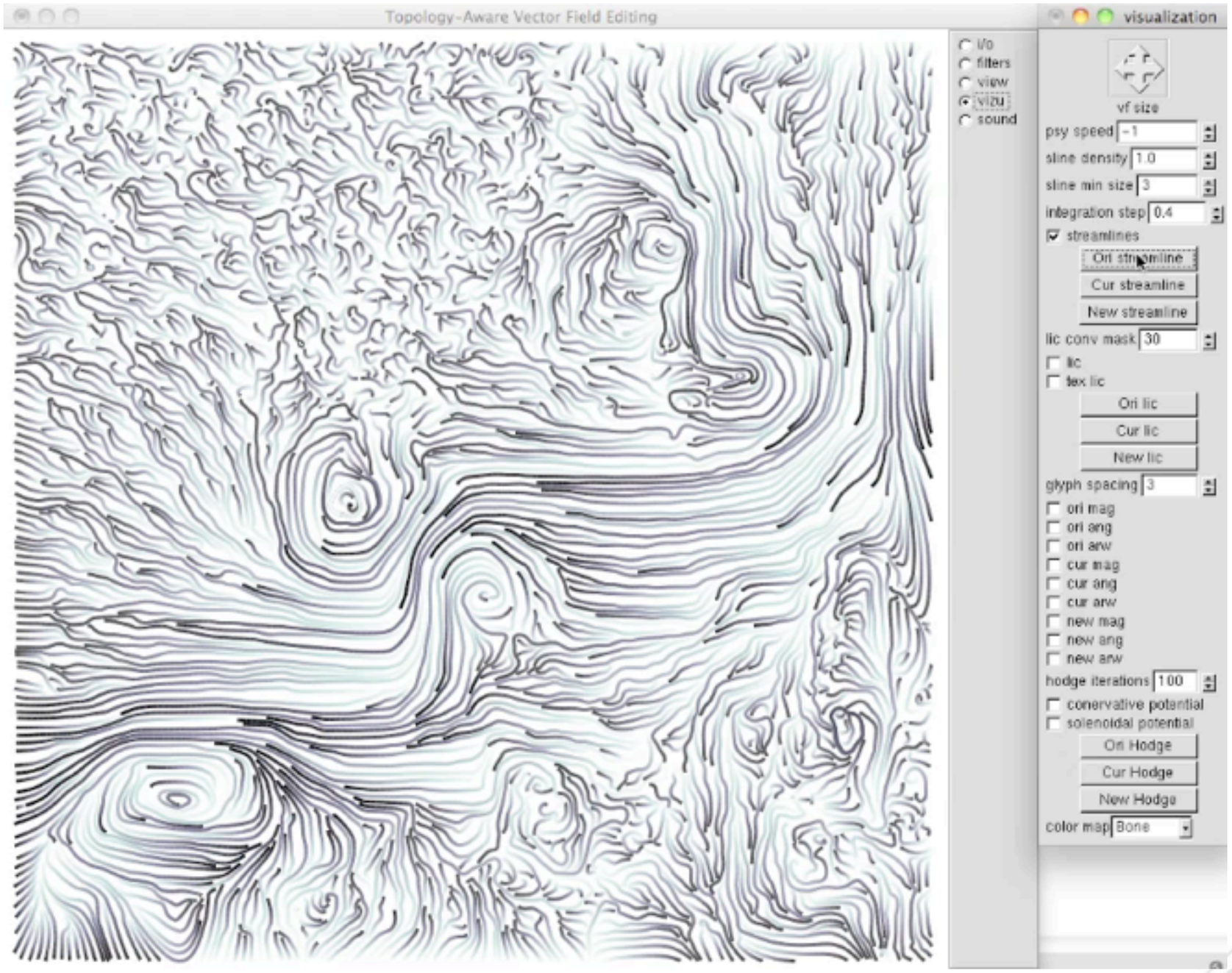
Original

Smoothed



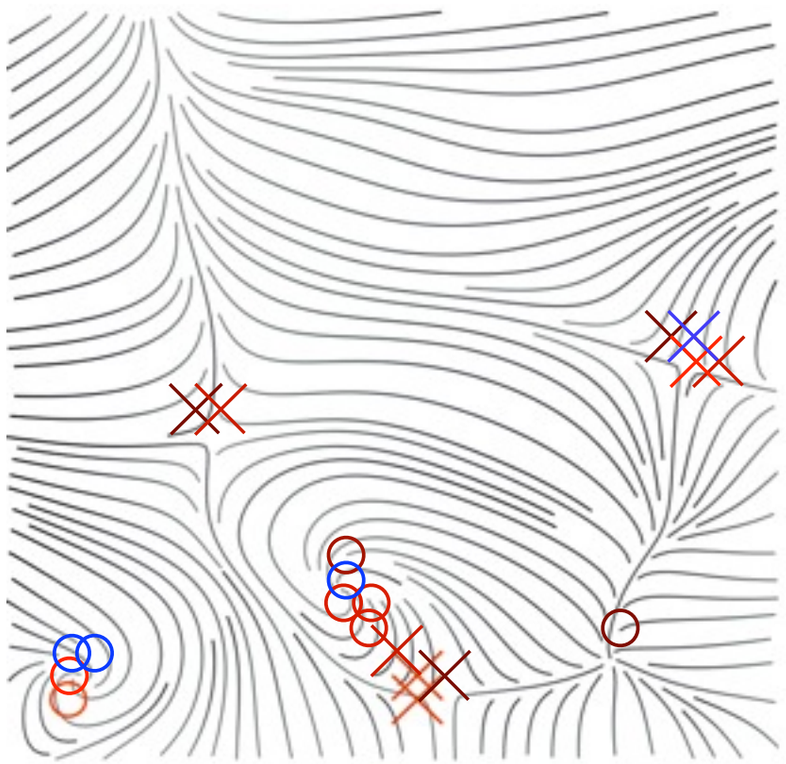
Reconstructed

Results



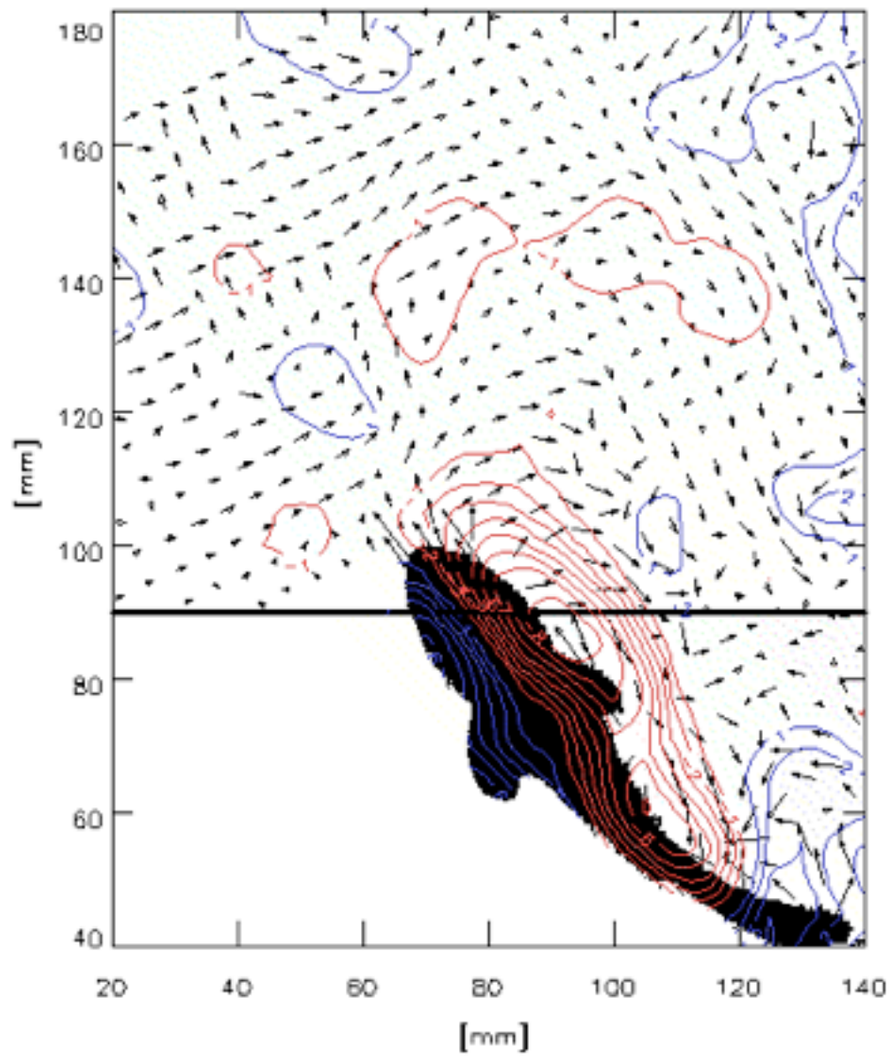
Limitations

- Detection of singularities is only done locally;
- The technique works on a structured grid;
- Large-scale denoising may displace the location of the singularity.



Thank you!

Questions?



NO FISH WAS HURT DURING THIS EXPERIMENT!!!!!