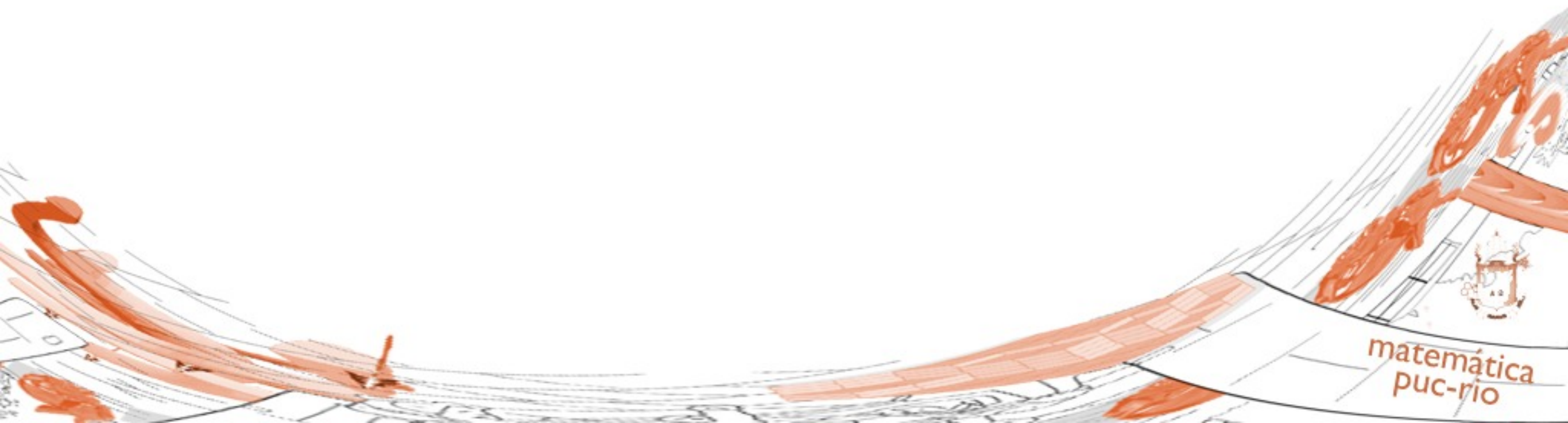


Curvas e Superfícies Implícitas: Noções de Geometria Diferencial e Discreta



Geometrias...

Ge.o.me.tri.a

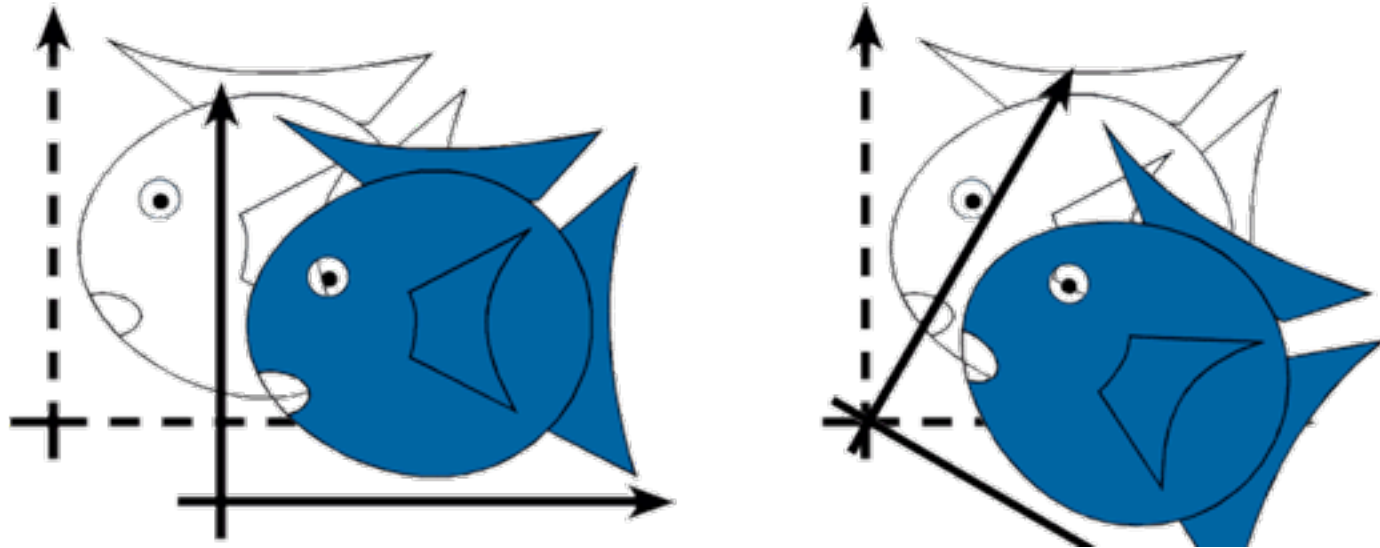
sf (geo+metro2+ia)

Parte das matemáticas que trata das propriedades e medidas da extensão nos seus três aspectos, como linha, superfície ou volume (sólido).

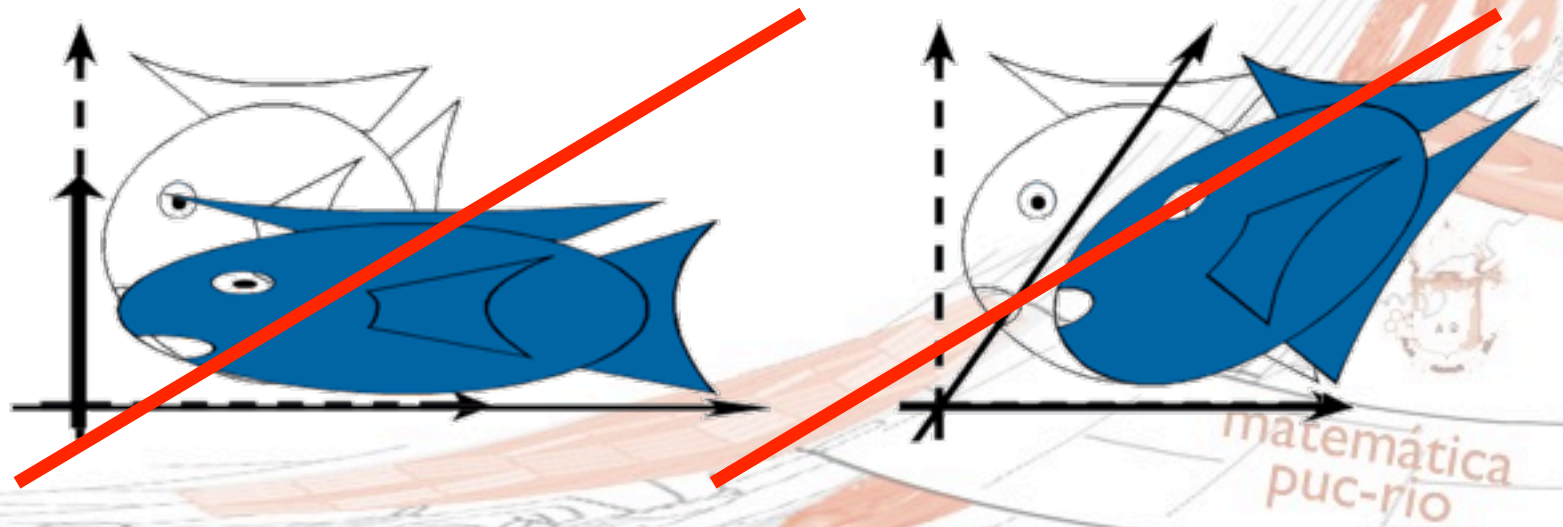
Michaelis



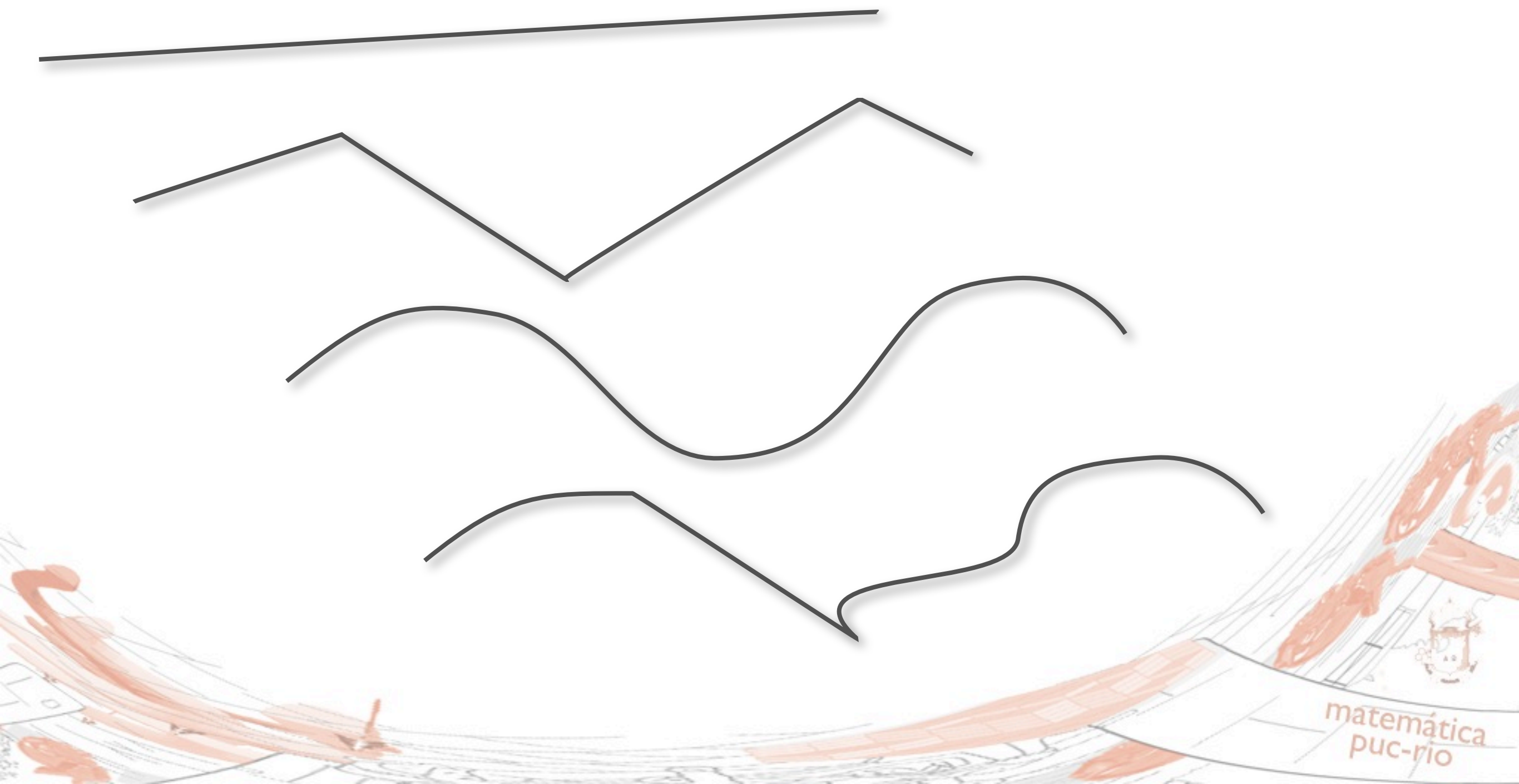
Medidas invariantes



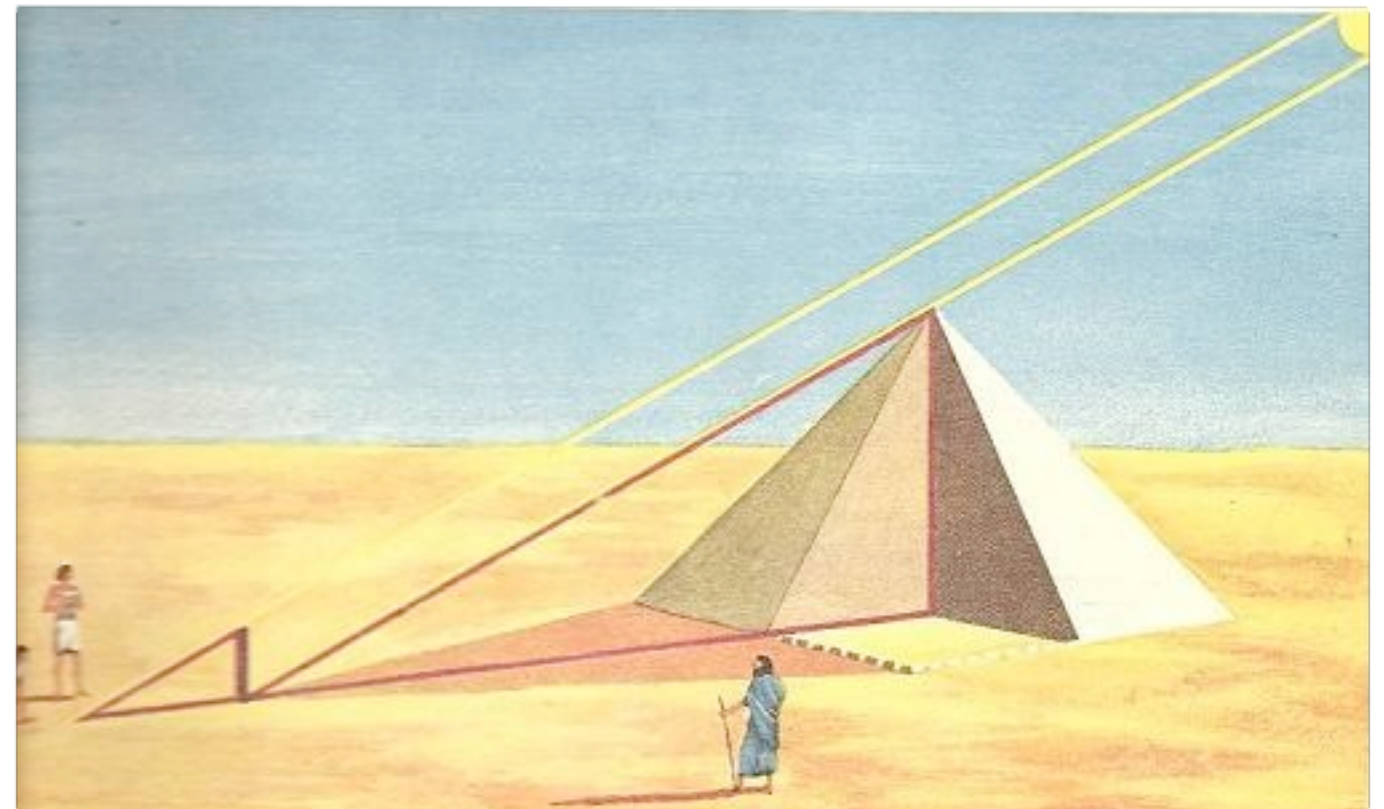
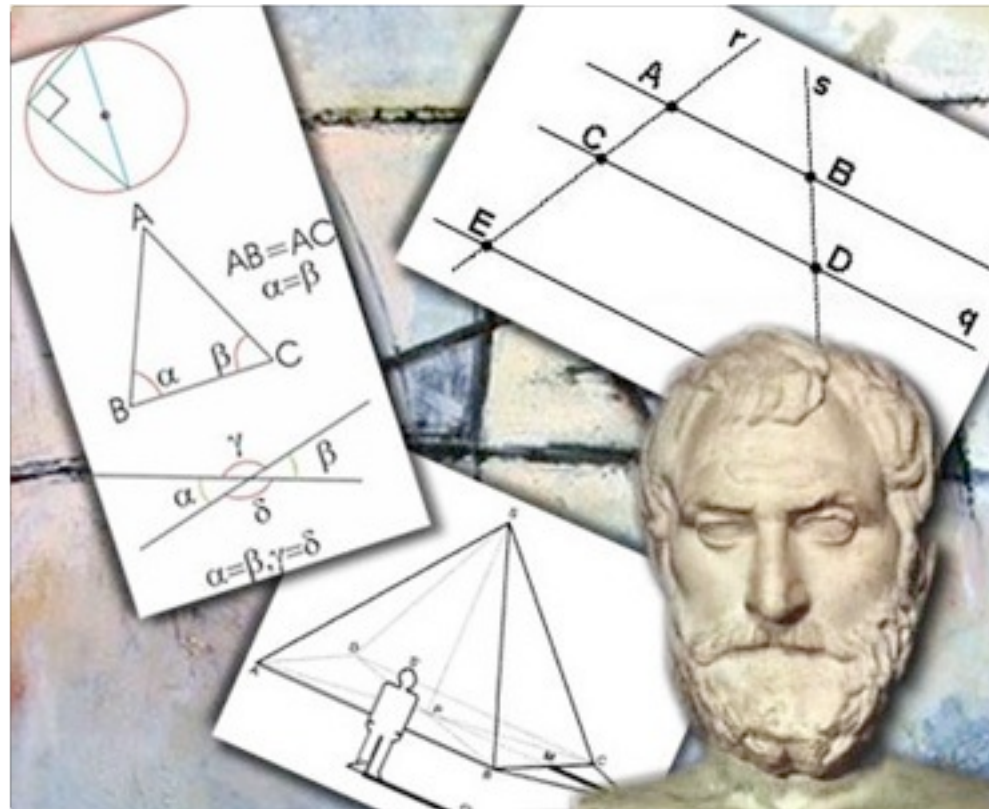
Geometria euclideana:
Invariante por translação e rotação
Medidas de comprimento e
curvatura



Breve histórico

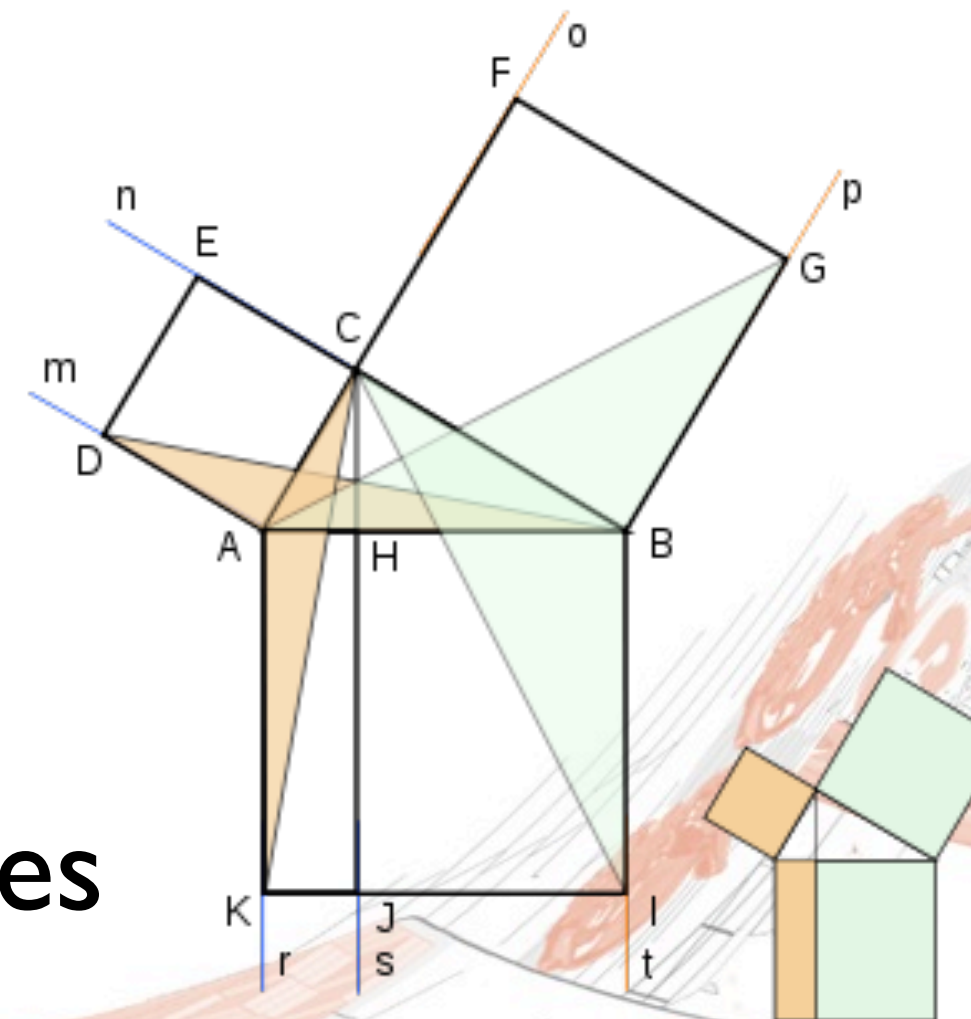
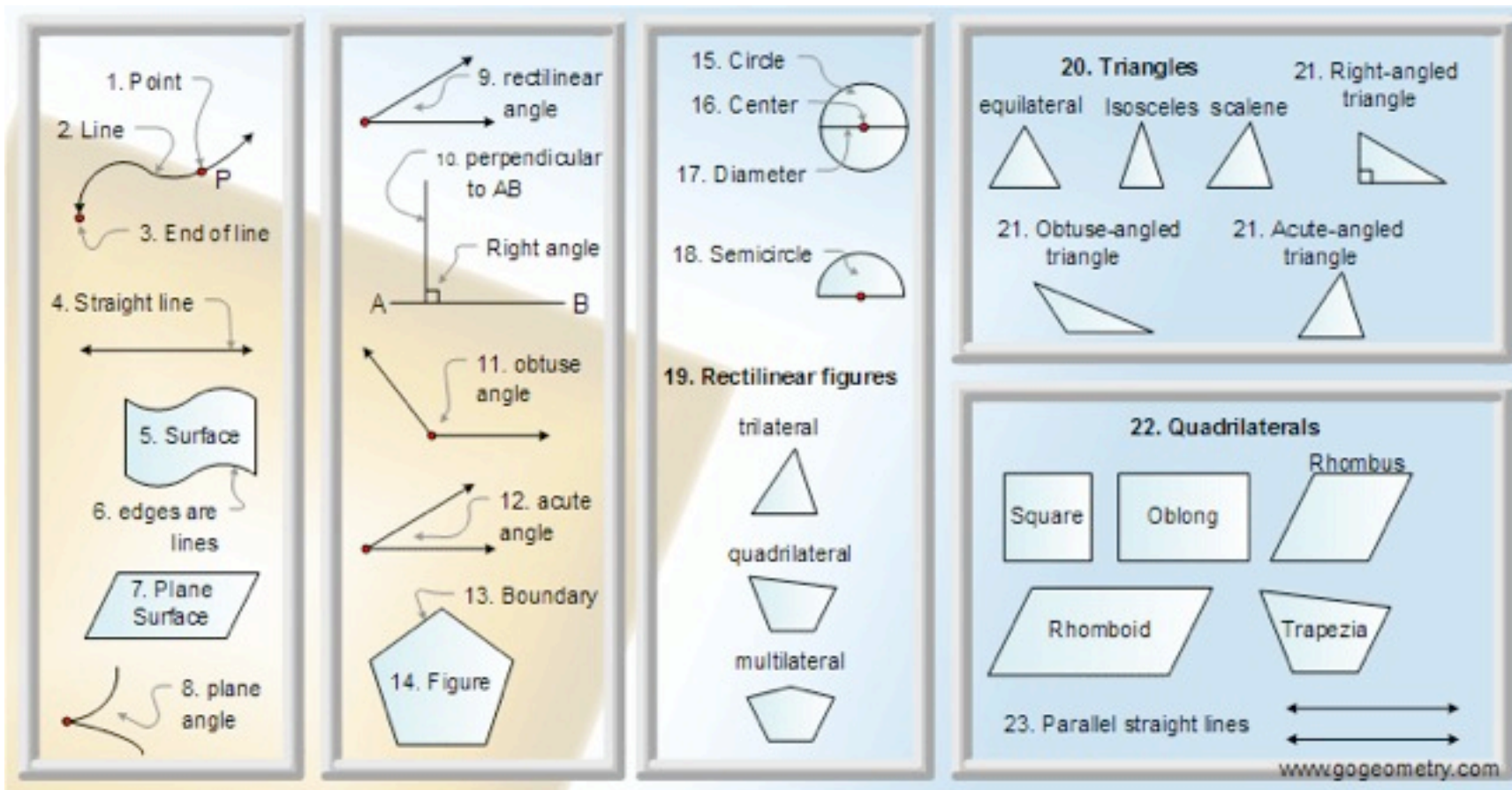


Thales de Mileto



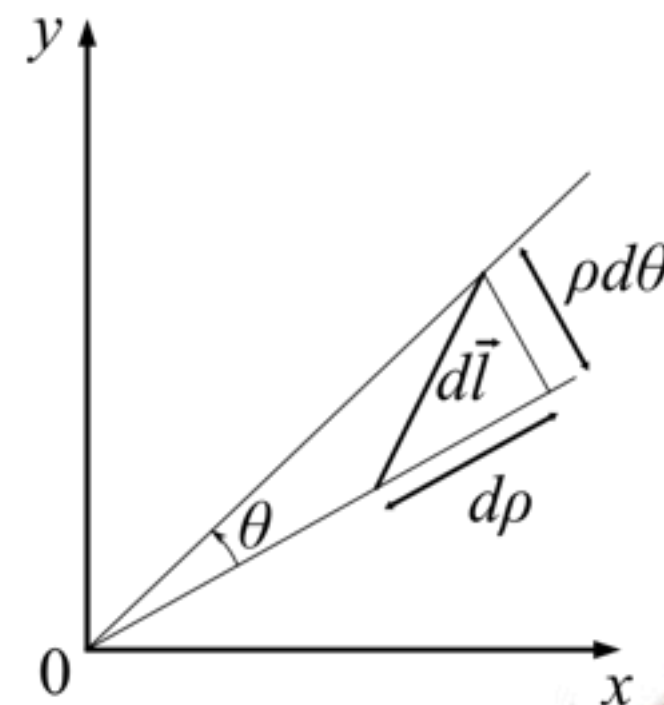
ângulo / proporção

Euclides



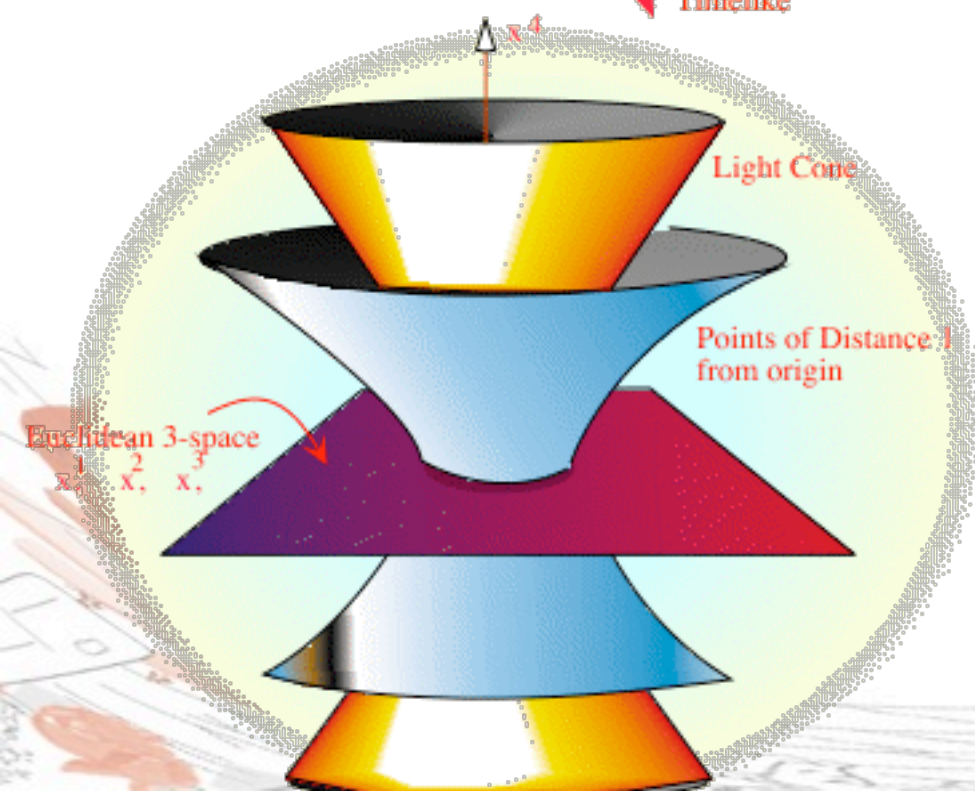
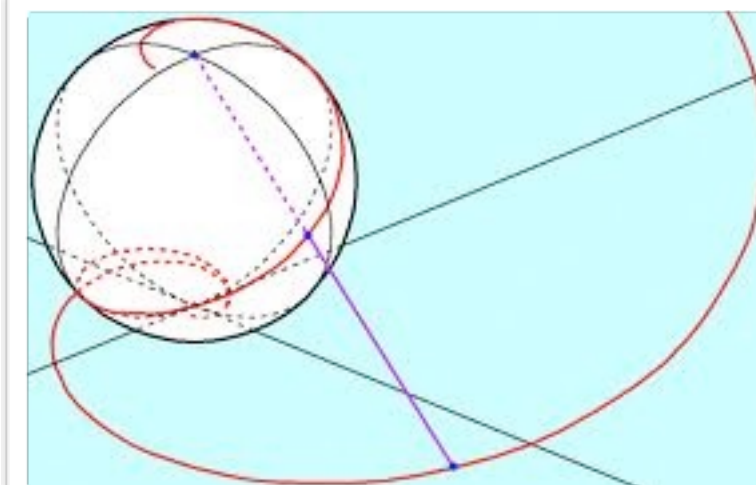
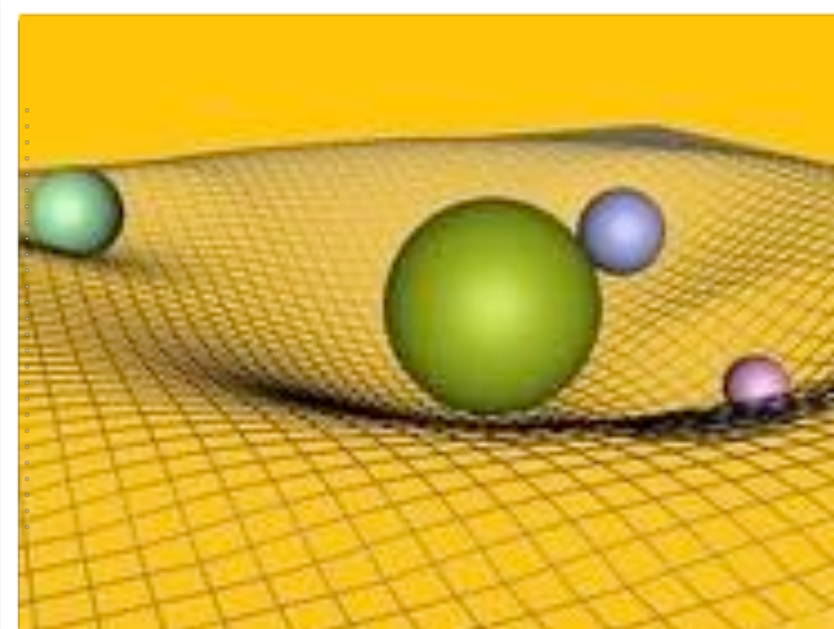
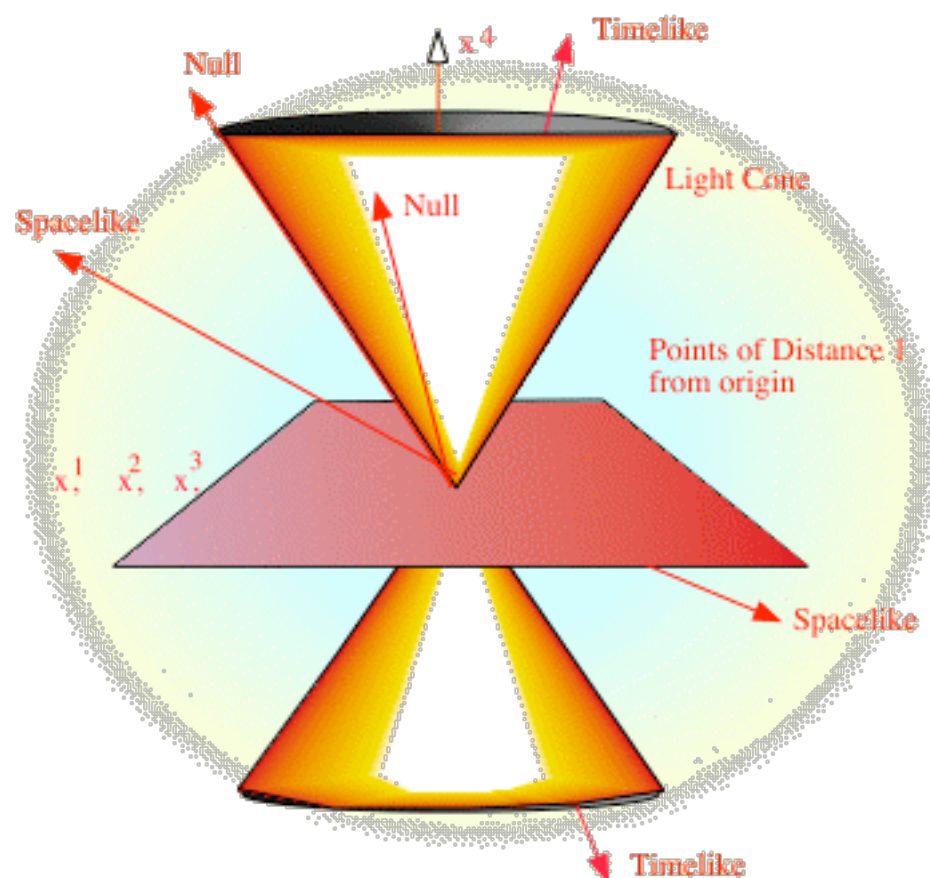
demonstrações

Newton / Leibniz

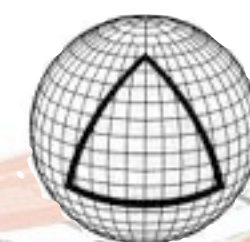


cálculo diferencial
hipótese do infinitesimal

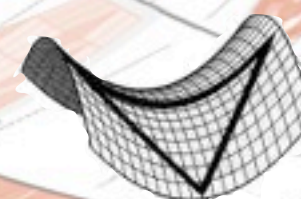
Riemann / Einstein



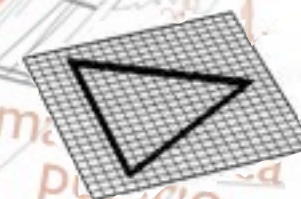
métrica (peso) depende da posição



Positive Curvature

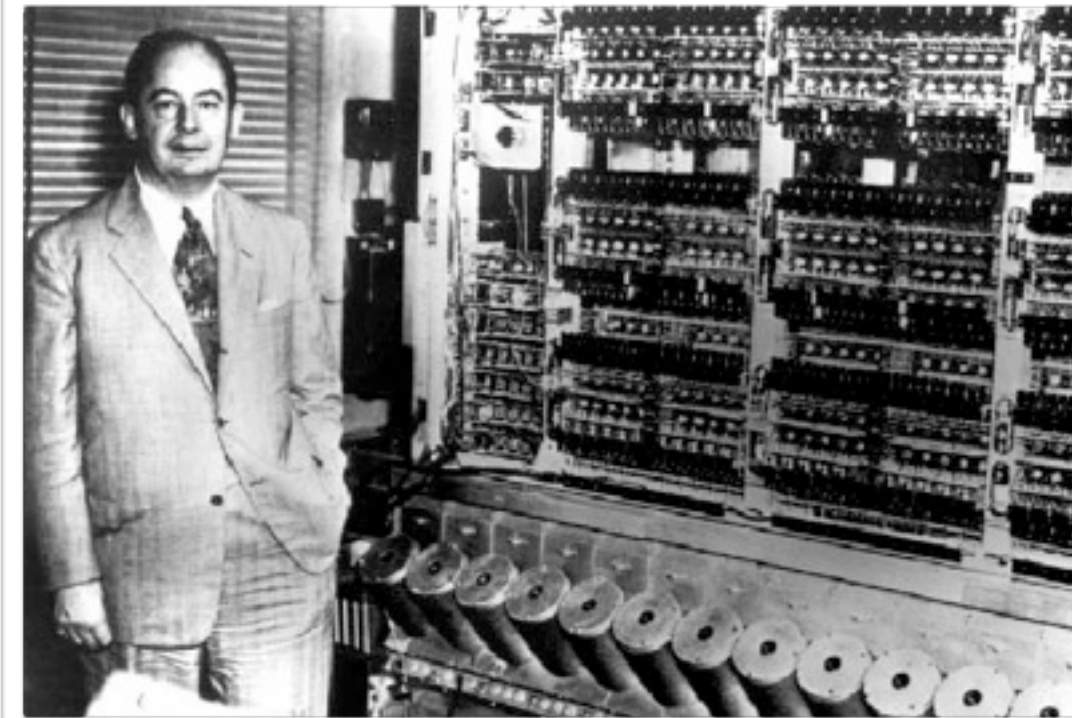
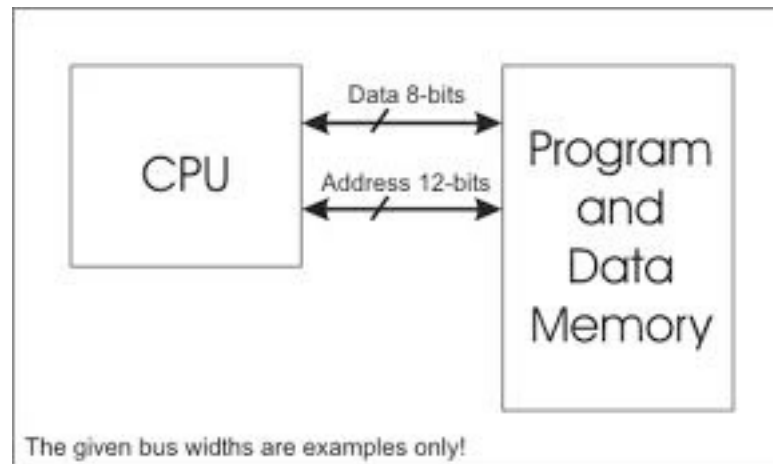


Negative Curvature



Flat Curvature

Máquinas



von Neumann (1949)

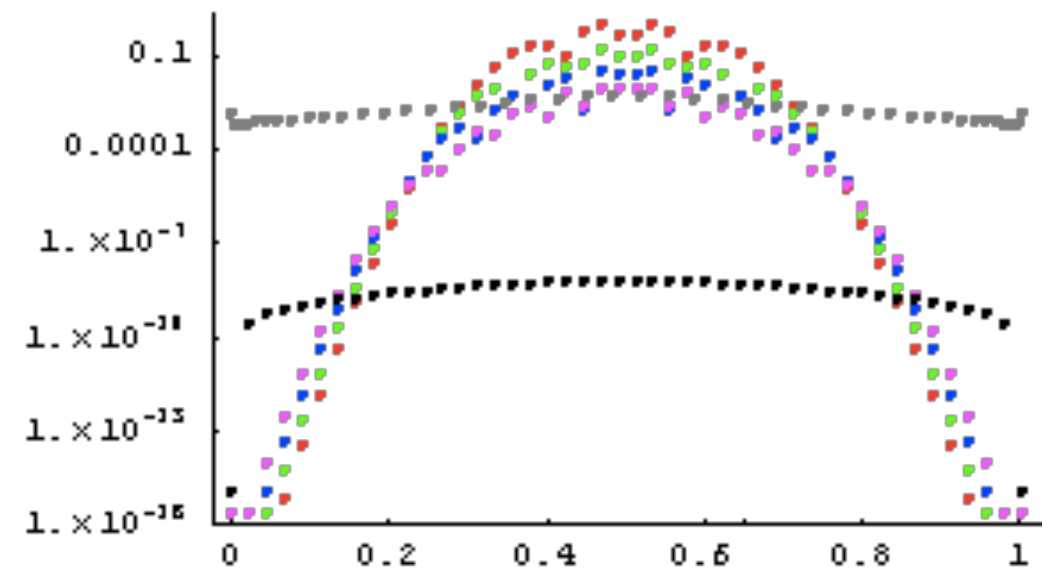
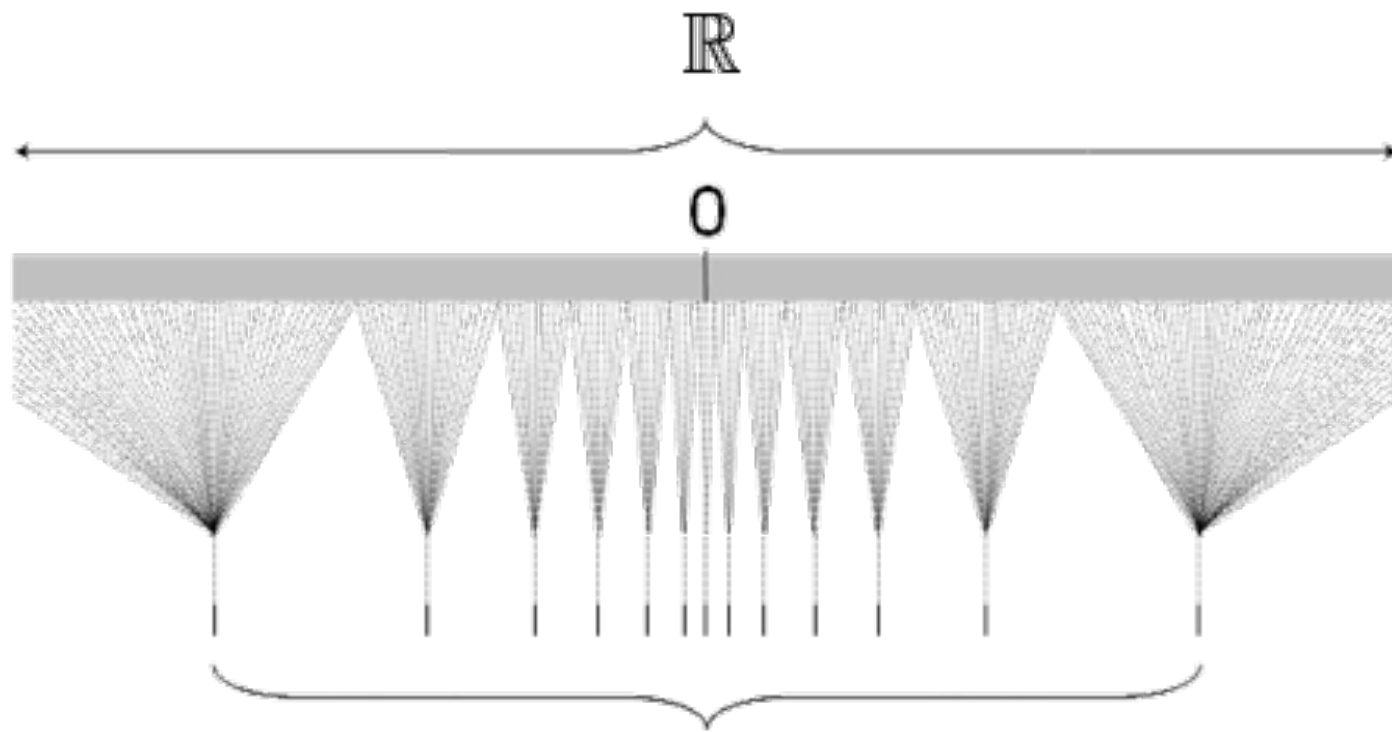


Pascal (1642)

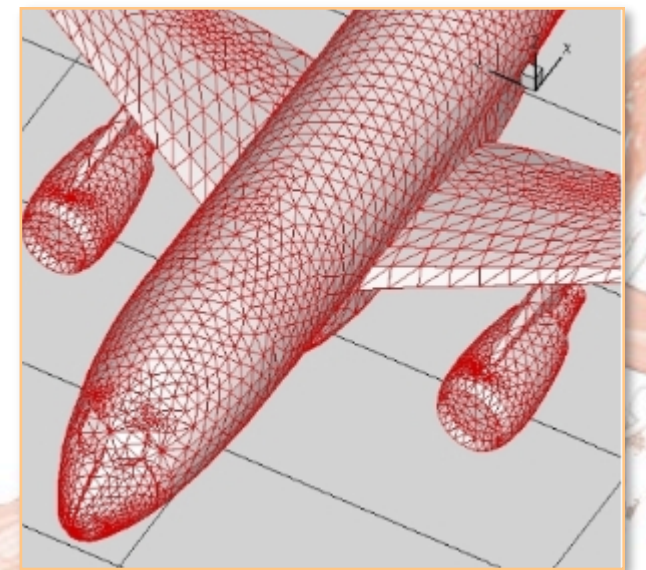


Leibniz (~1650)

Medir com o computador



domínio e imagem discretos!



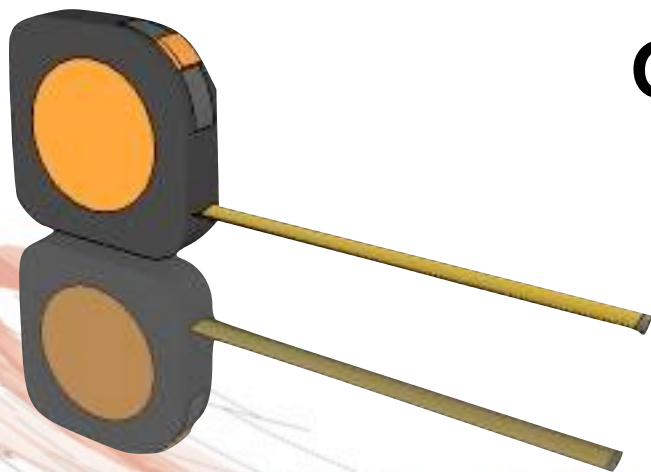
Prática da geometria



medir um objeto: descritiva / integral

definir operações: diferencial

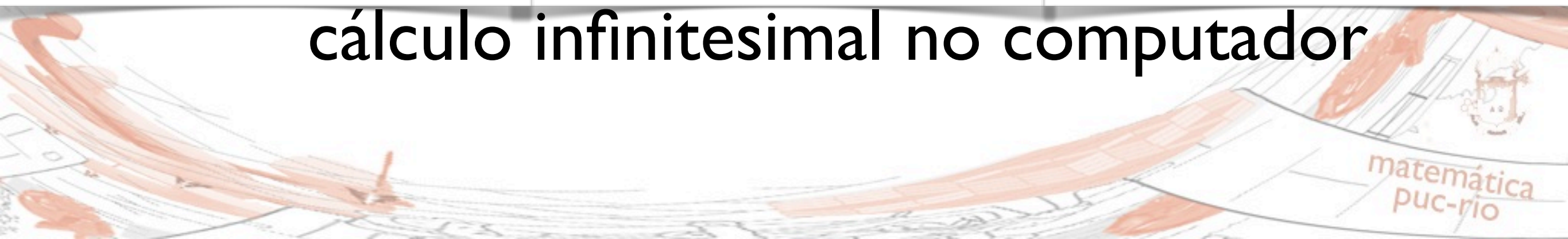
calcular invariantes: discreta



Problemática do curso



cálculo infinitesimal no computador



Formato do curso

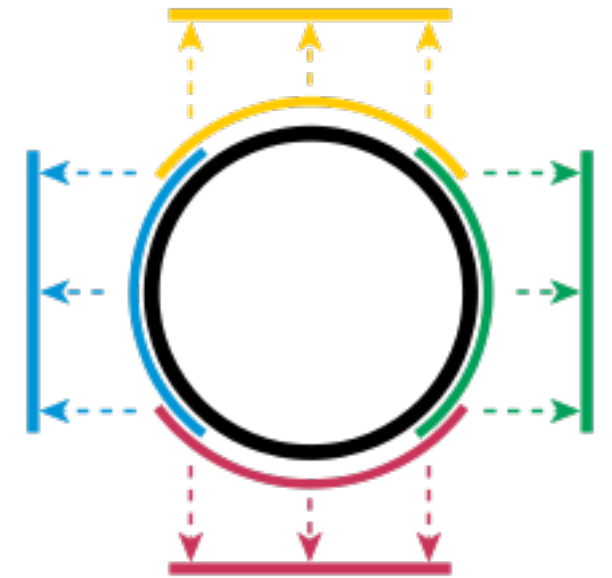
parte formal:
geometria diferencial



parte em construção:
geometria discreta



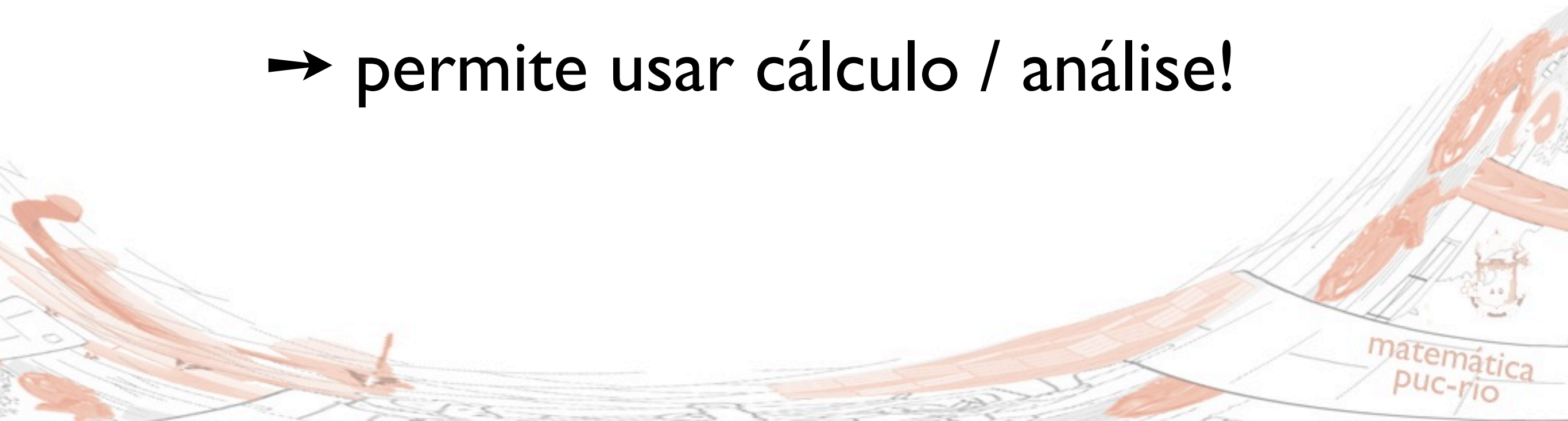
Noção de variedade



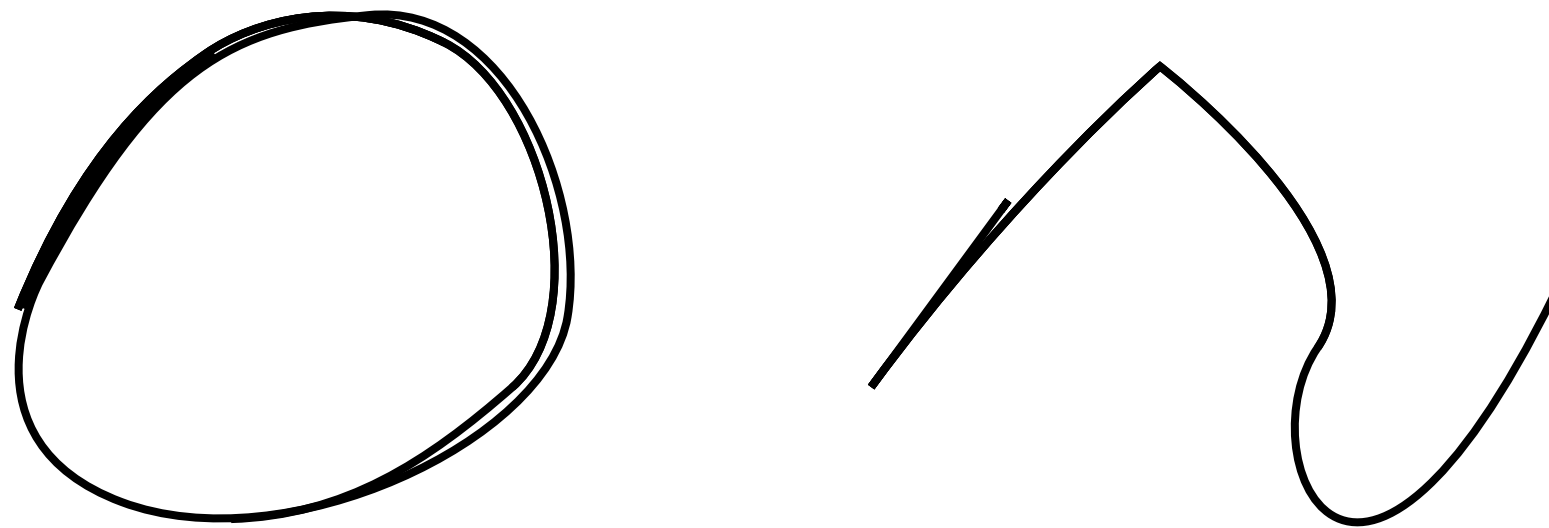
localmente equivalente à reta:

não tem auto-interseção
fronteira...

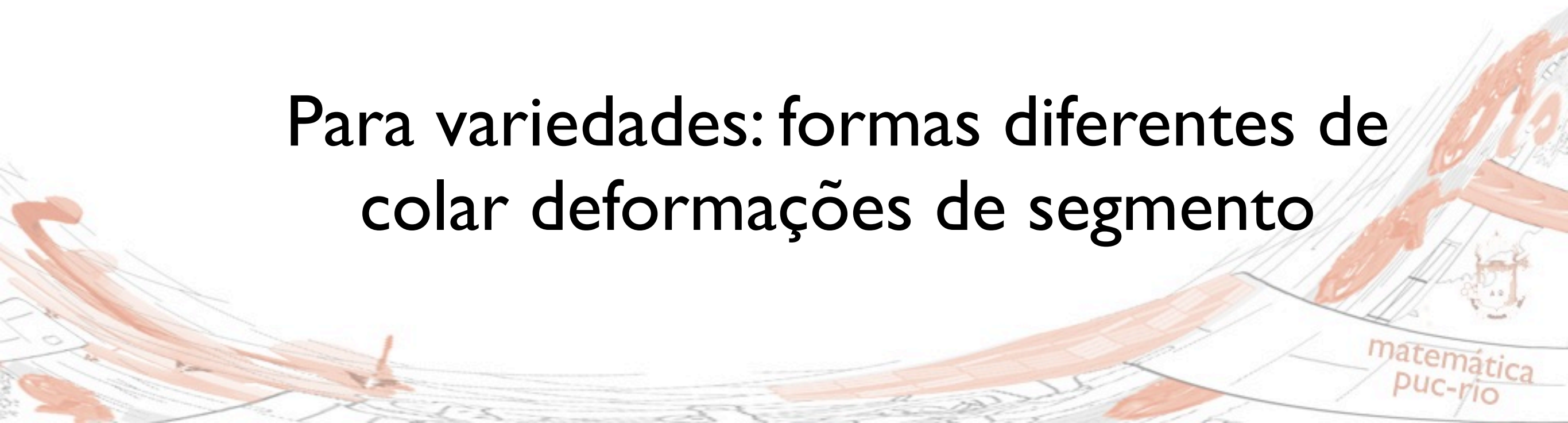
→ permite usar cálculo / análise!



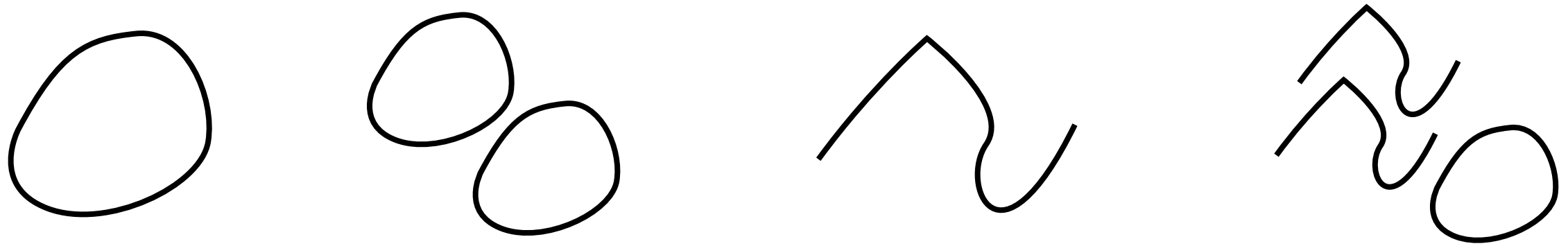
Topologia descritiva



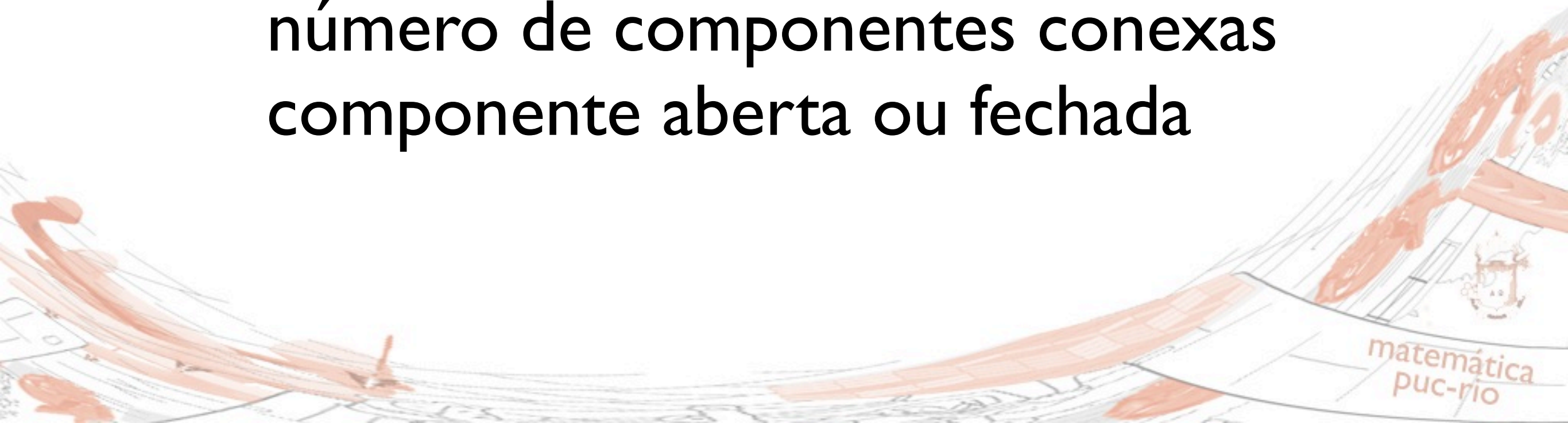
Para variedades: formas diferentes de
colar deformações de segmento



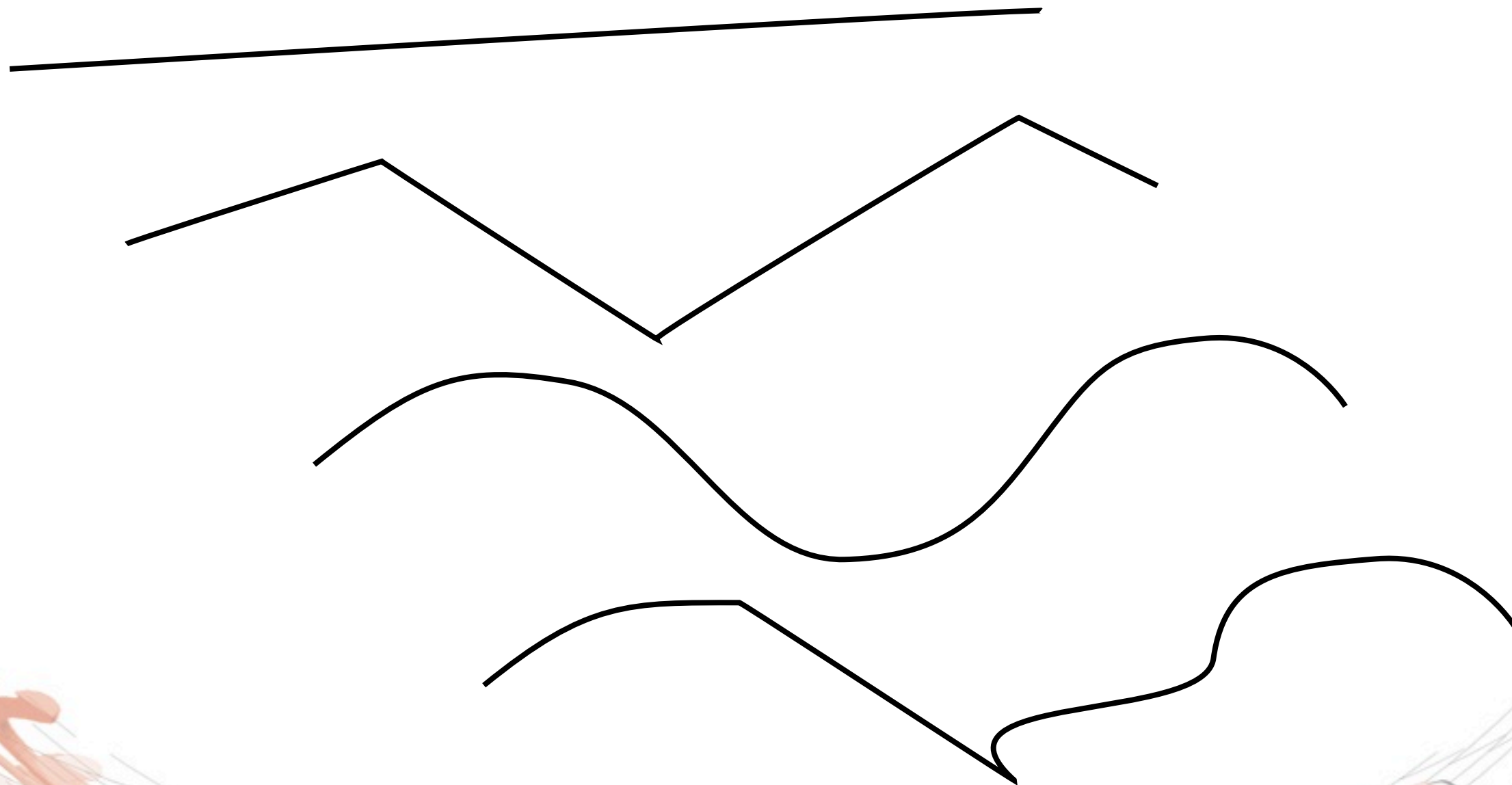
1-variedades: curvas



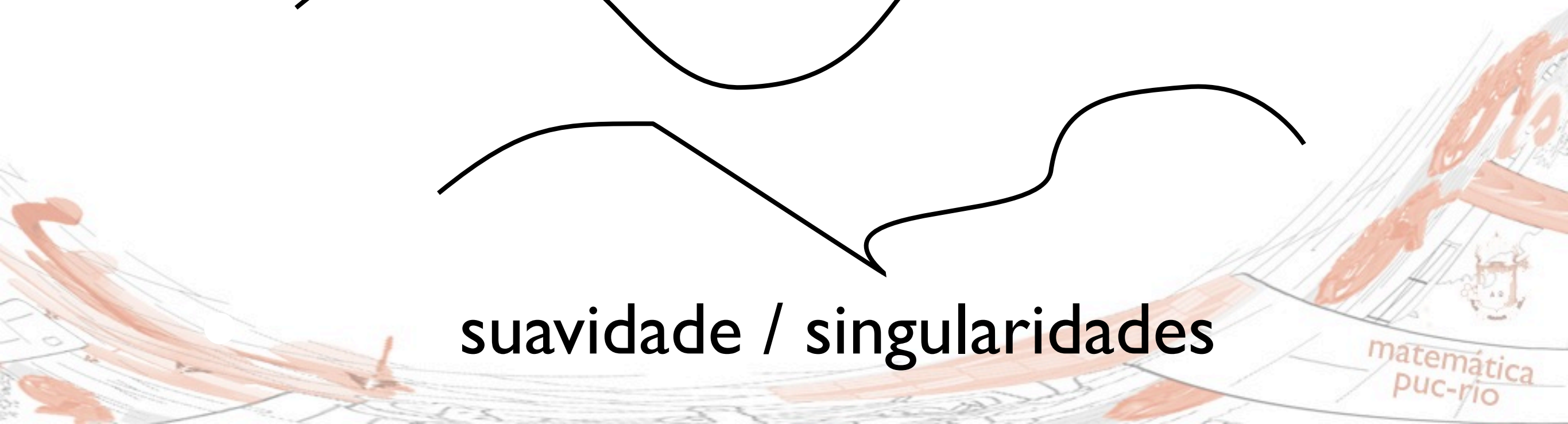
número de componentes conexas
componente aberta ou fechada



Geometria descritiva

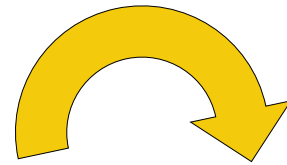


suavidade / singularidades

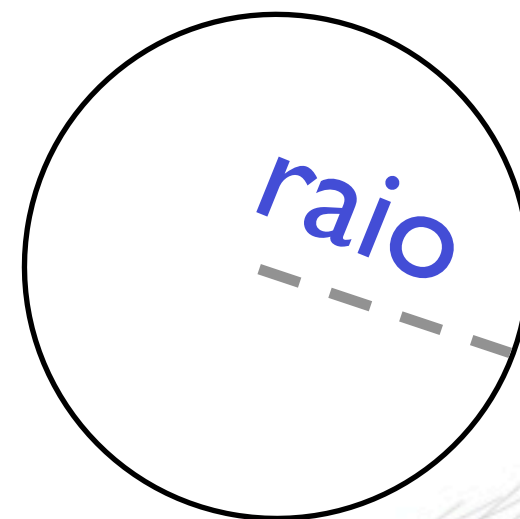
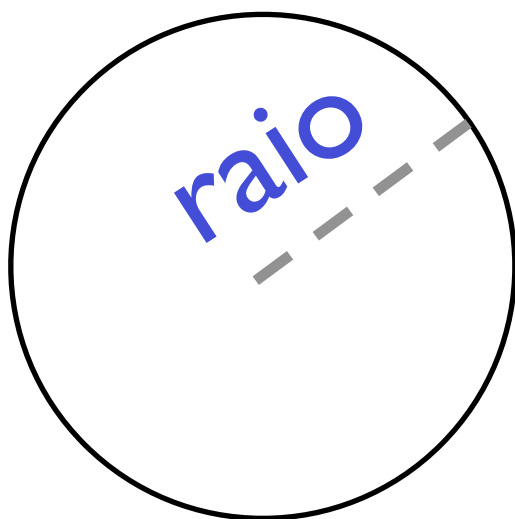
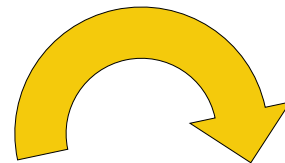


Geometria quantitativa

comprimento



comprimento

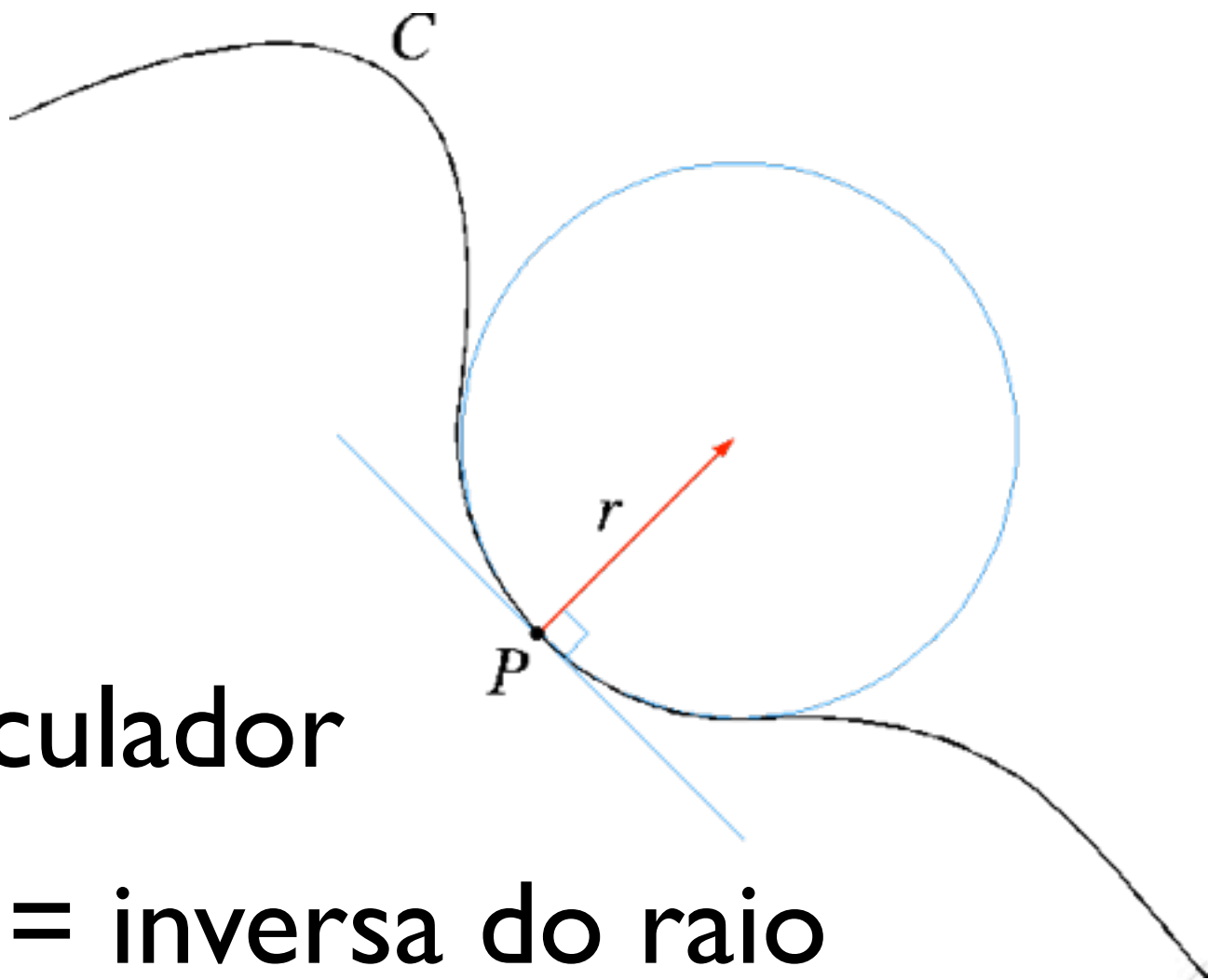


medidas invariantes

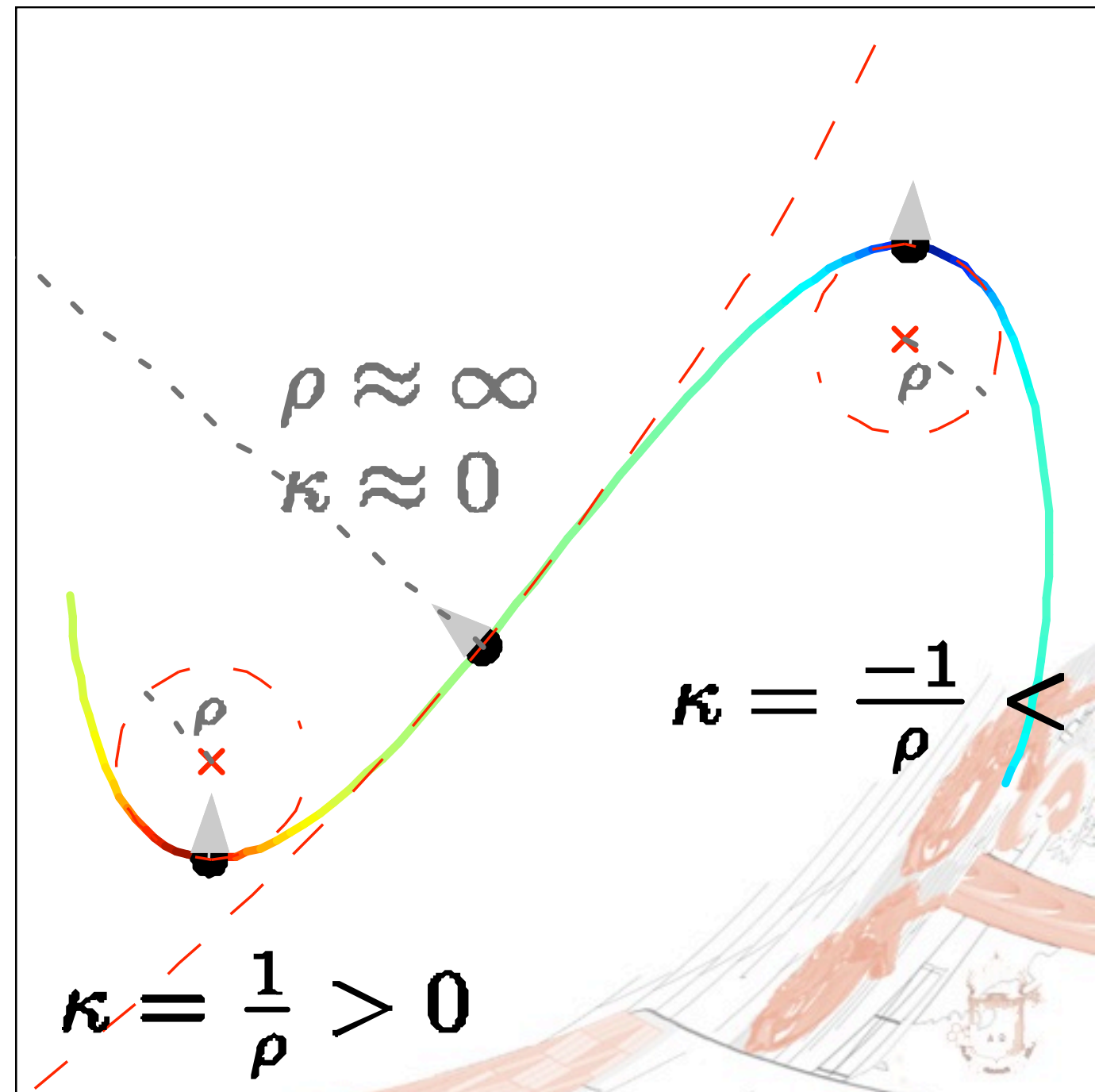
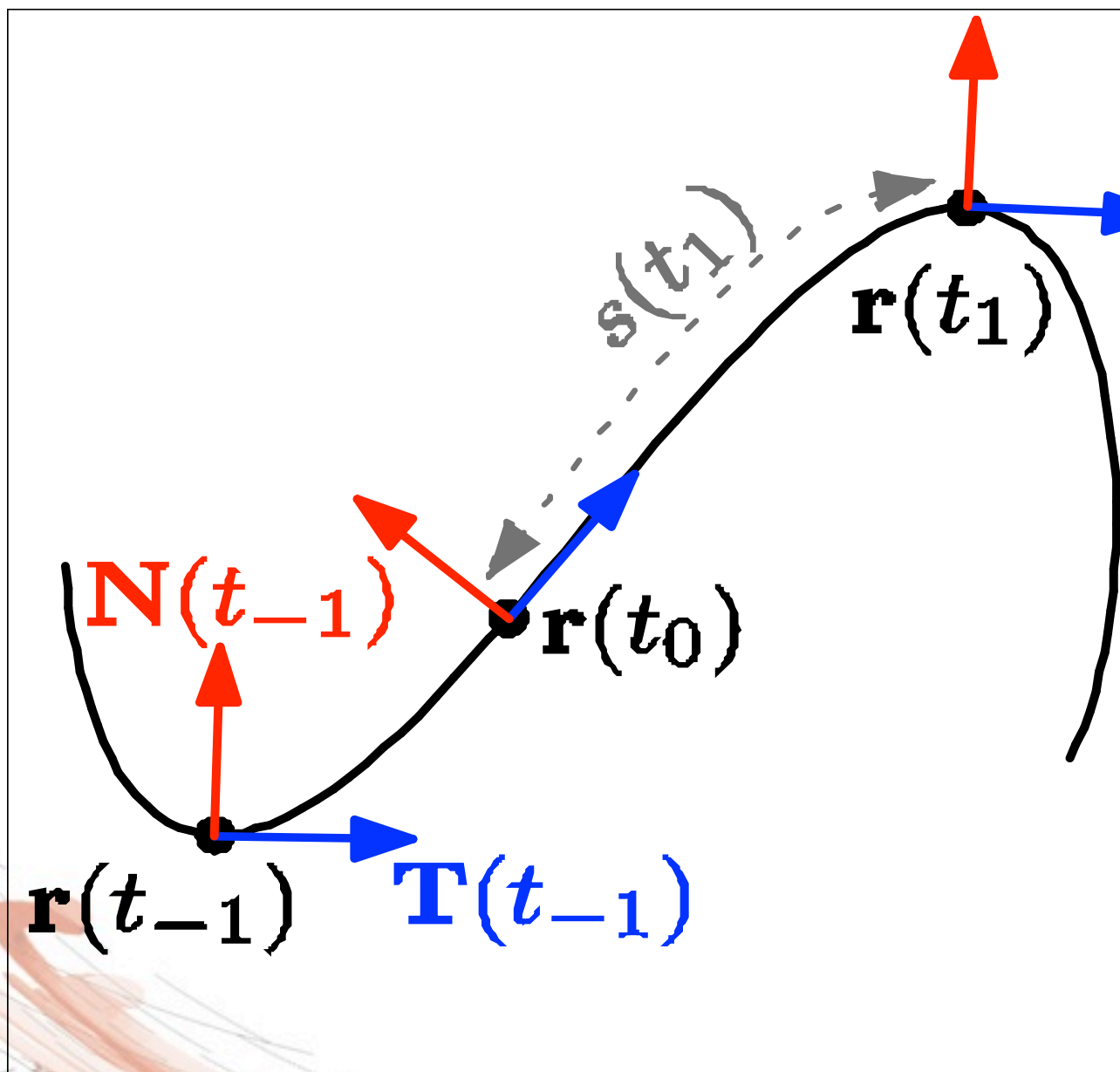
Curvatura

Circulo osculador

Curvatura = inversa do raio



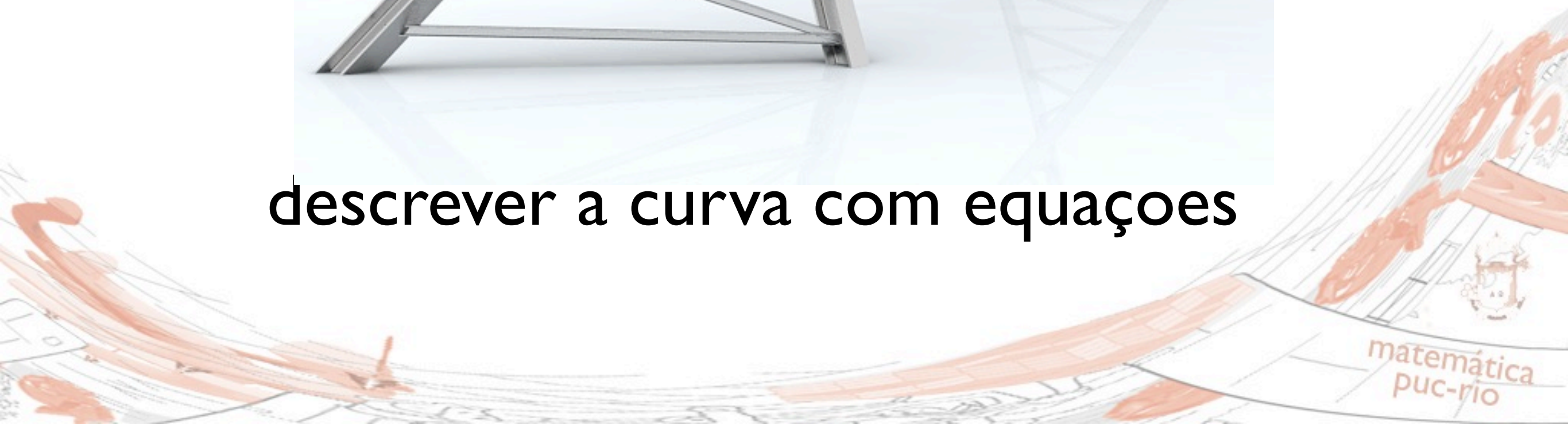
Geometria descritiva de curvas



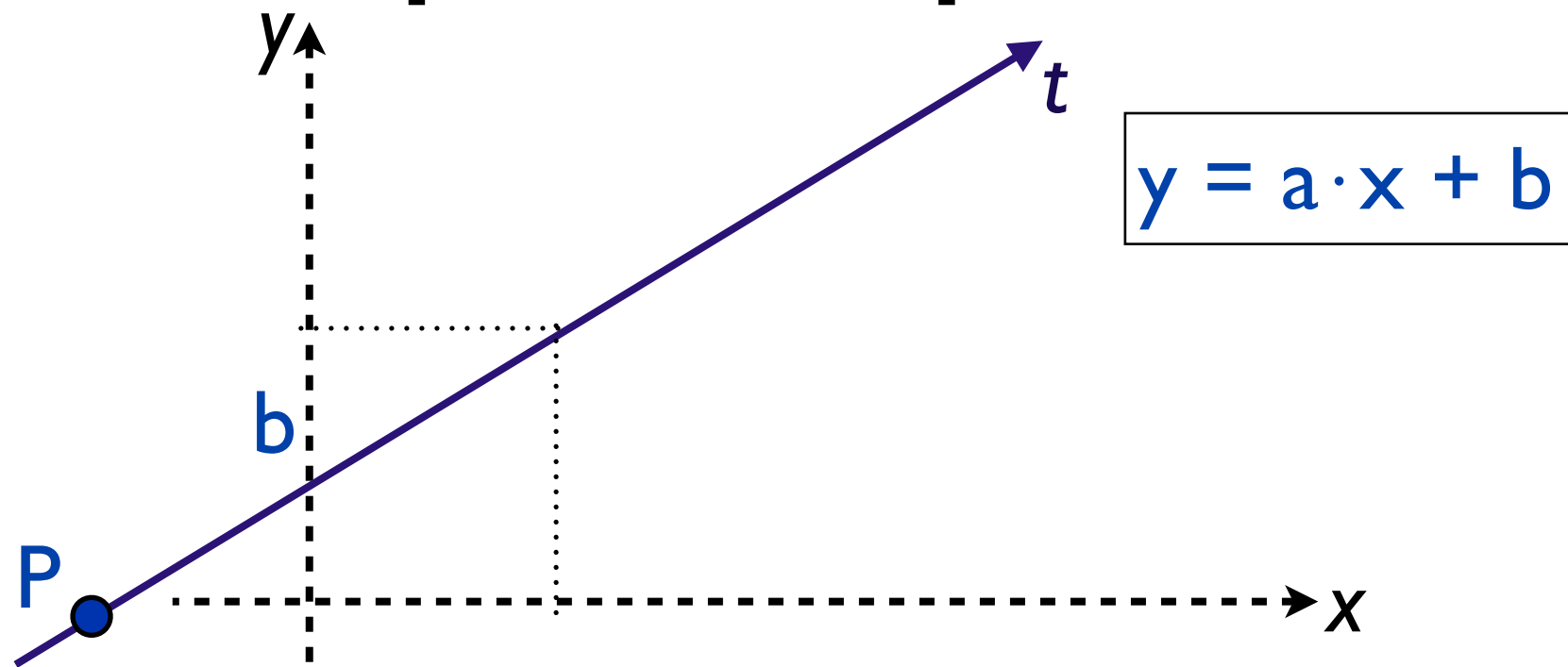
Cálculo de medidas



descrever a curva com equações



Exemplo simplíssimo: reta



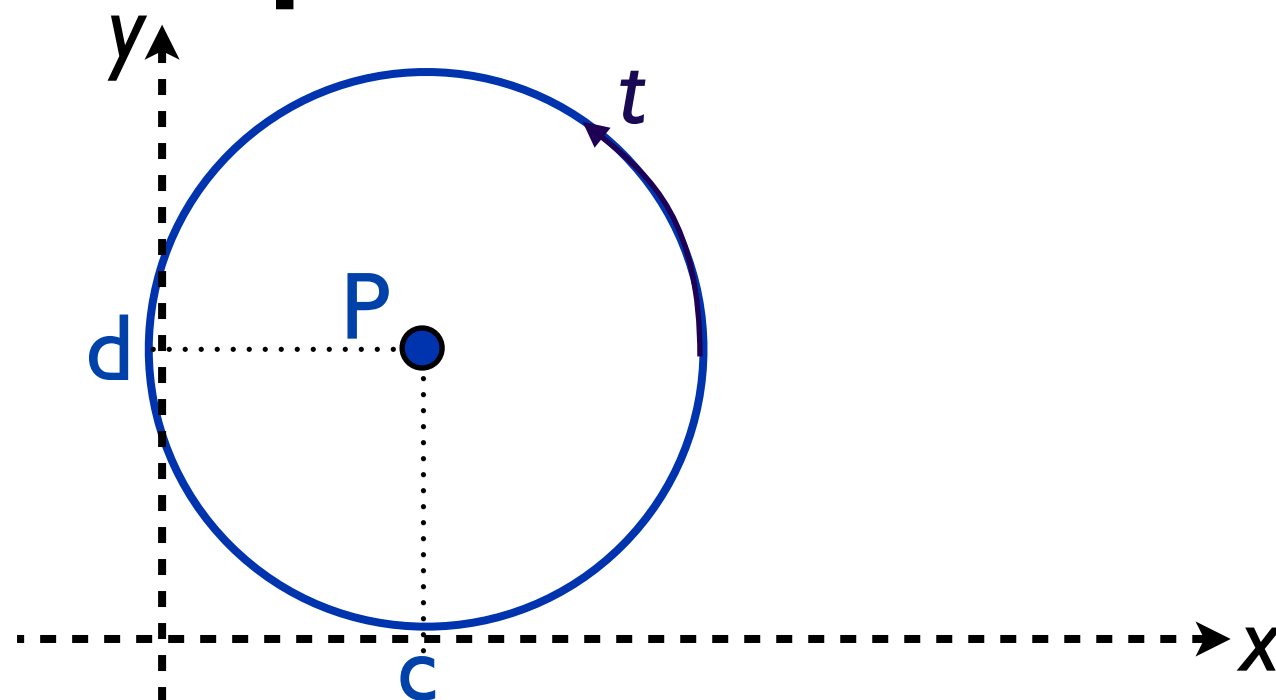
descritivo: ponto / tangente

gráfico: $y = a \cdot x + b$

implícito: $c \cdot x + d \cdot y + e = 0$

parametrização: $\{ x = t, y = a \cdot t + b \}$

Exemplo básico: círculo



descritivo: centro / curvatura

implícito: $(x-c)^2 + (y-d)^2 = r^2$

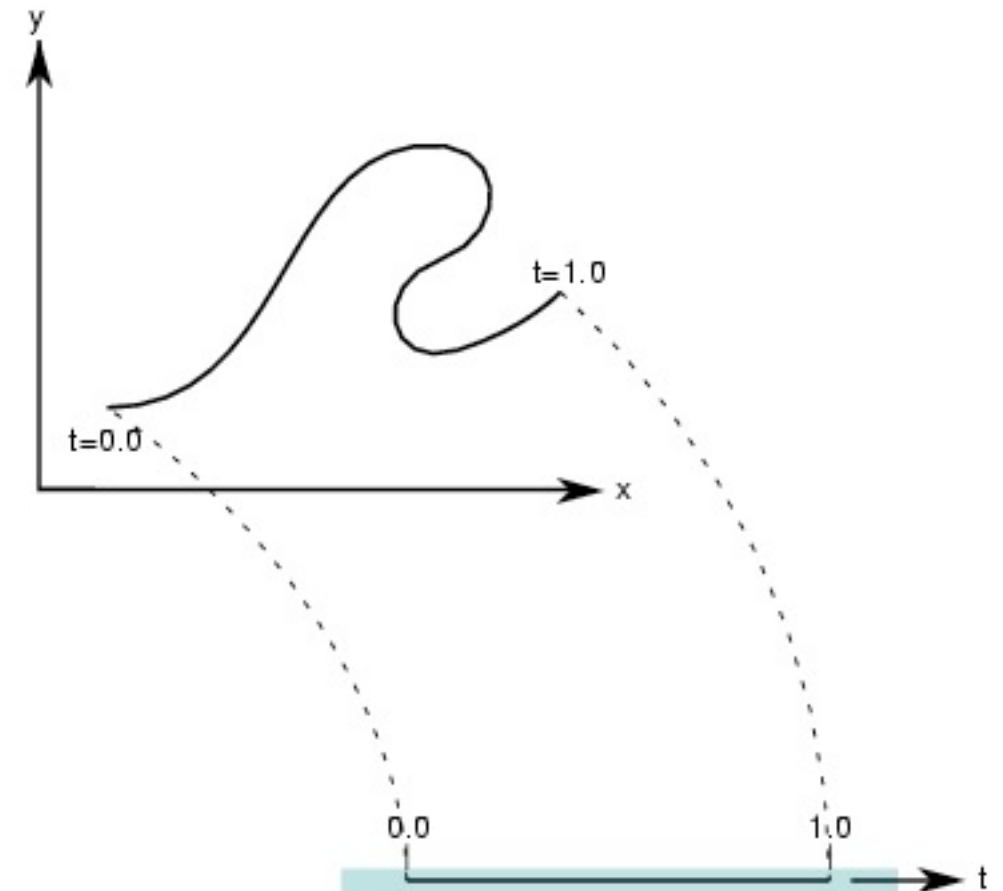
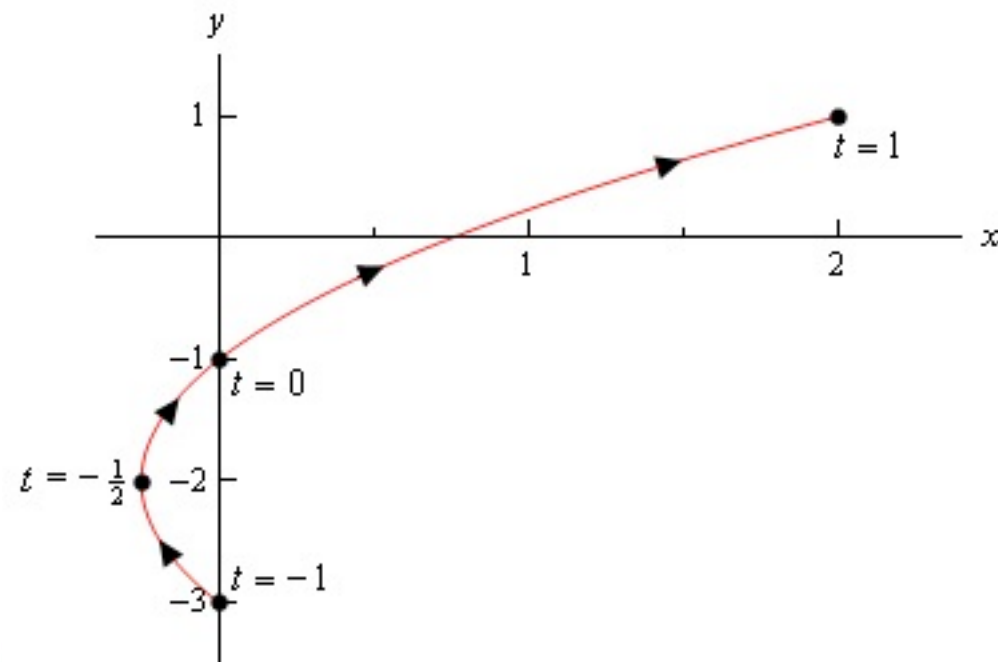
parametrização: $\{ x = r \cdot \cos(t) + c, \\ y = r \cdot \sin(t) + d \}$

Parametrização

$$\alpha : I \rightarrow \mathbb{R}^2$$

$$\alpha(t) = (x(t), y(t)) \in \mathbb{R}^2$$

$$\mathbf{C} = \{(x(t), y(t)), t \in I\}$$



Parametrização regular

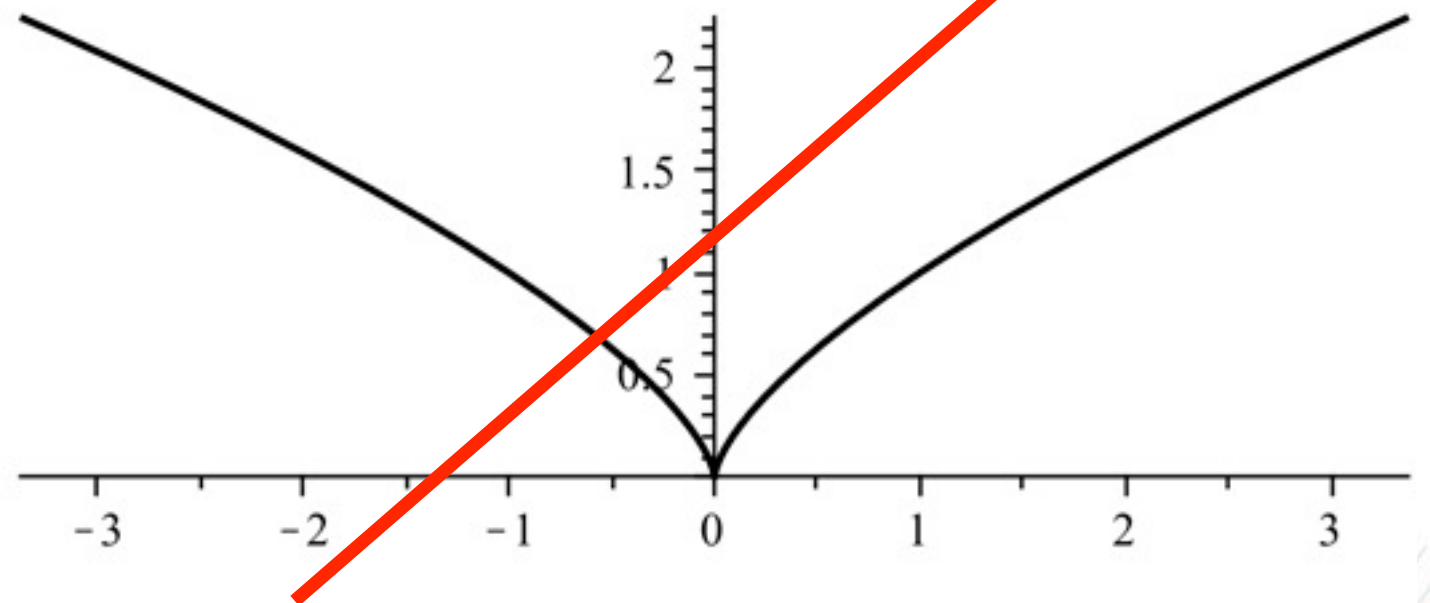
$$\alpha : I \rightarrow \mathbb{R}^2$$

$$\alpha'(t) = (x'(t), y'(t))$$

$$\forall t, \alpha'(t) \neq (0, 0)$$

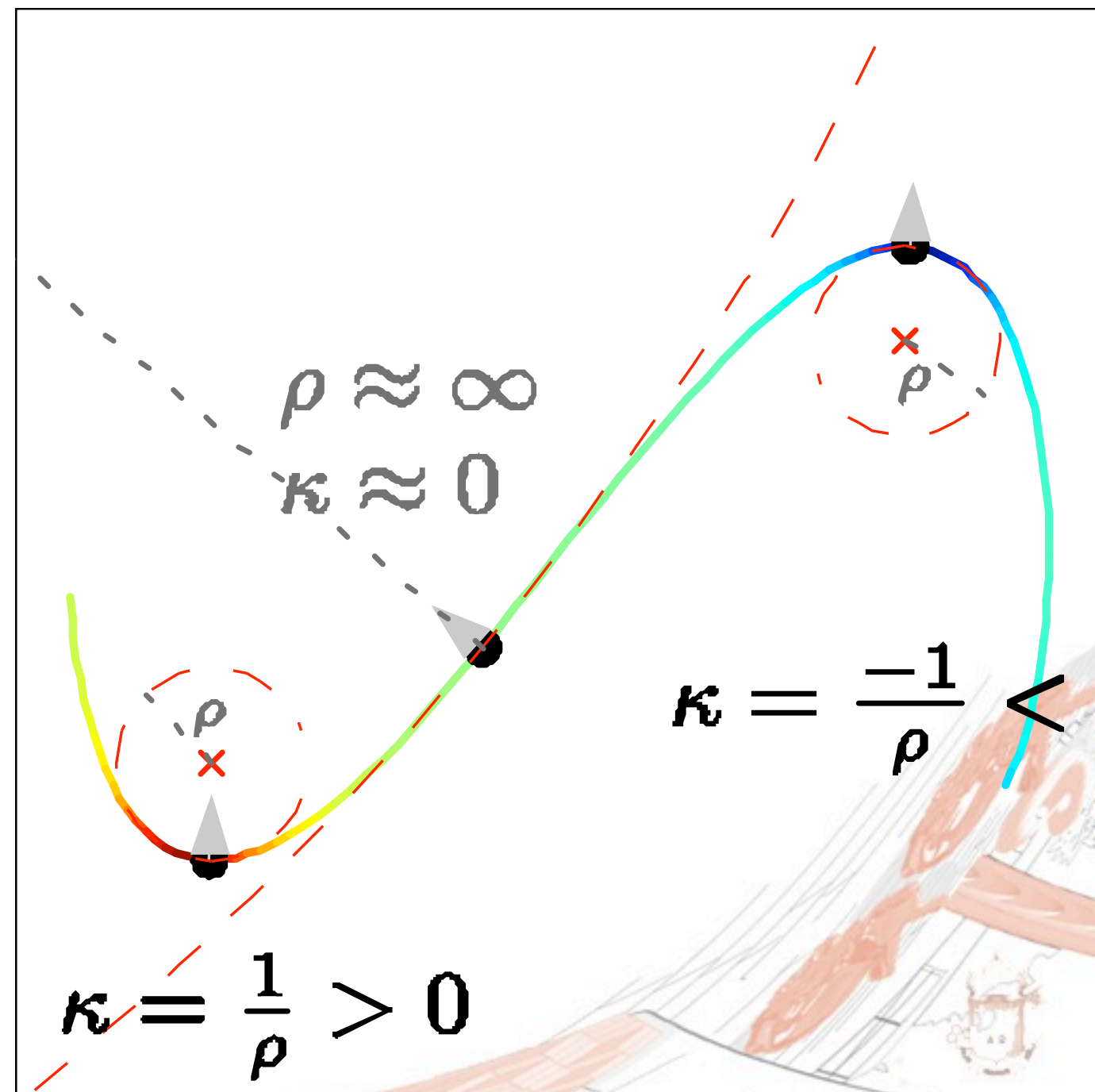
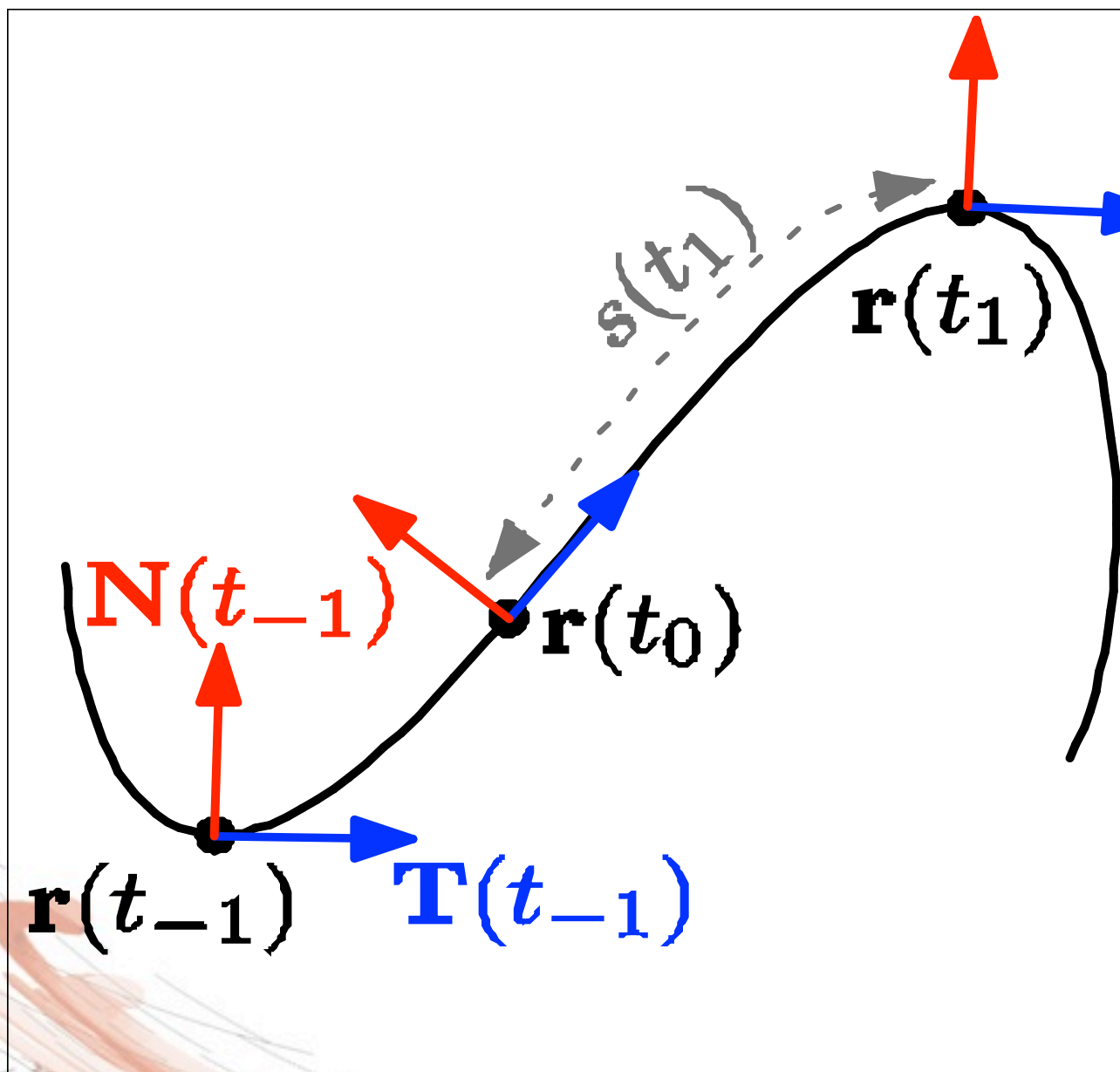
$$\alpha(t) = (t^3, t^2)$$

$$\alpha'(0) = (0, 0)$$

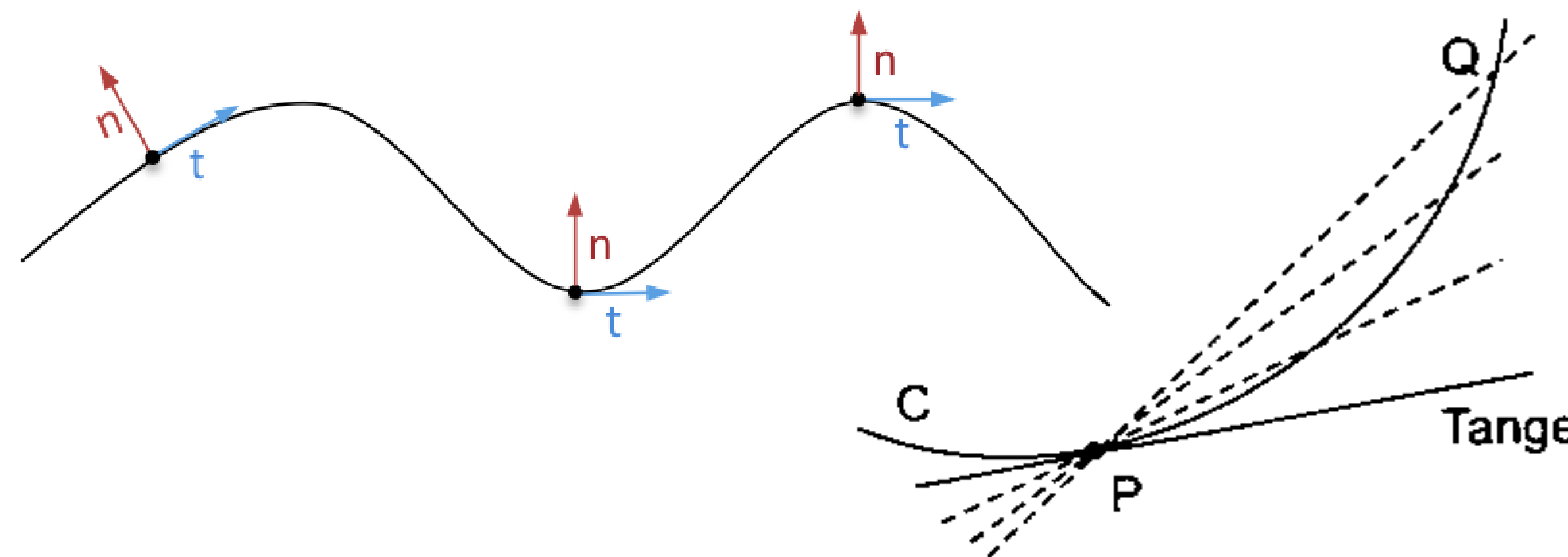


curva regular =
admite uma parametrização regular

Quantidades a calcular



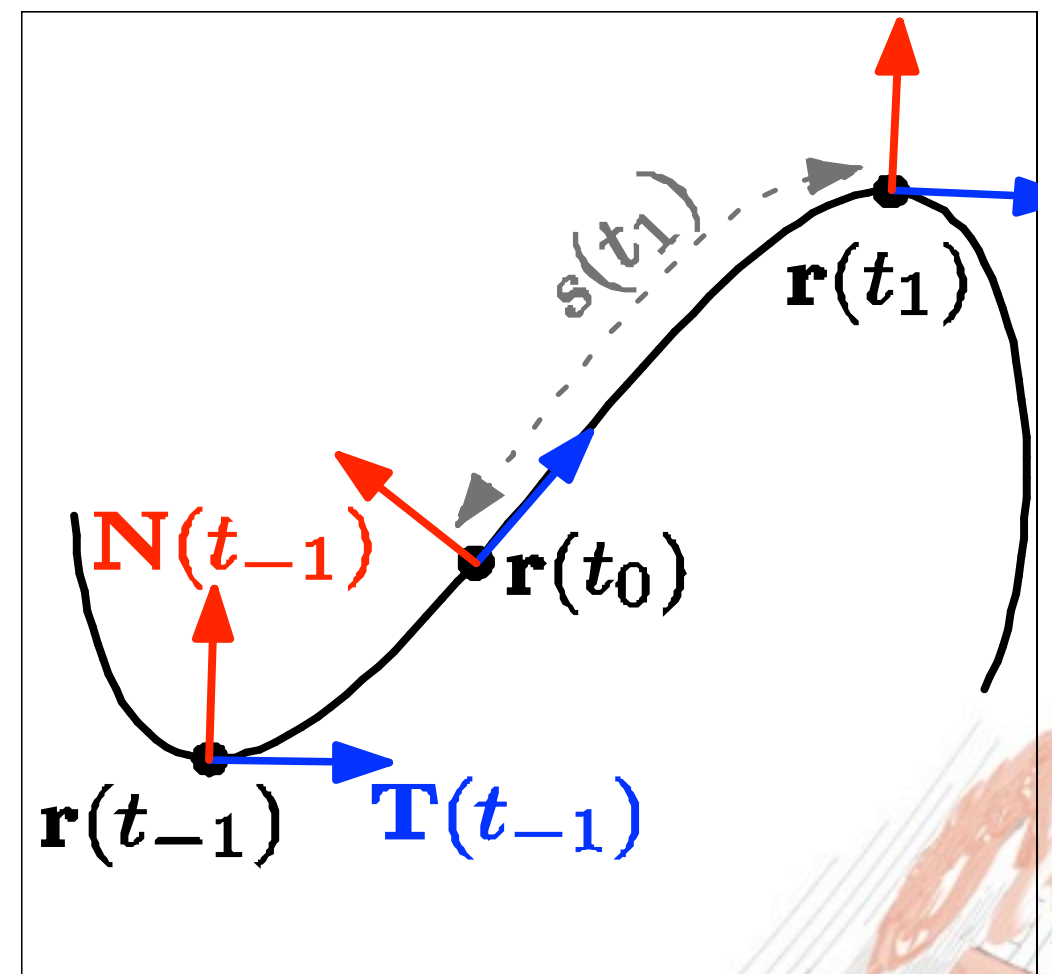
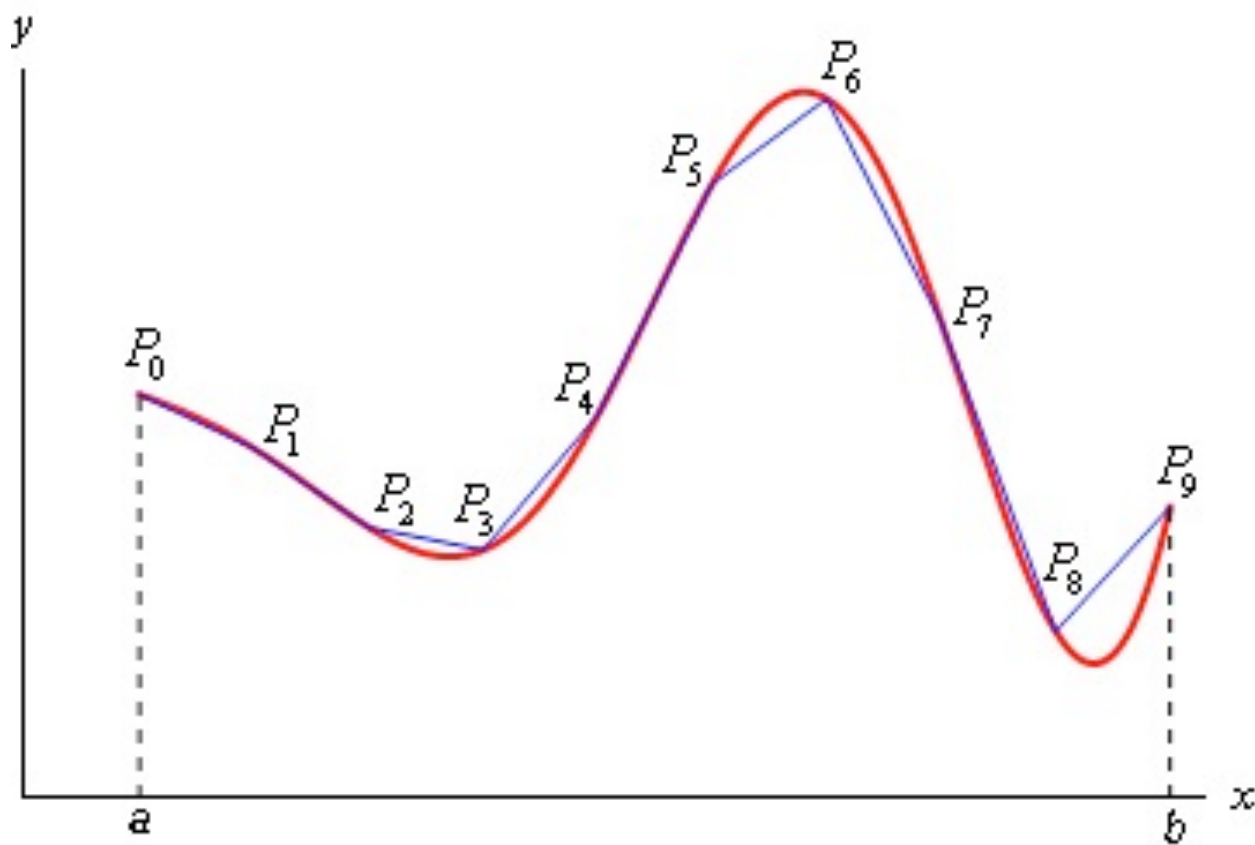
Vetor tangente / normal



$$\mathbf{t} = \alpha' = (x', y')$$

$$\mathbf{n} = \alpha'^{\perp} = (-y', x')$$

Comprimento de arco

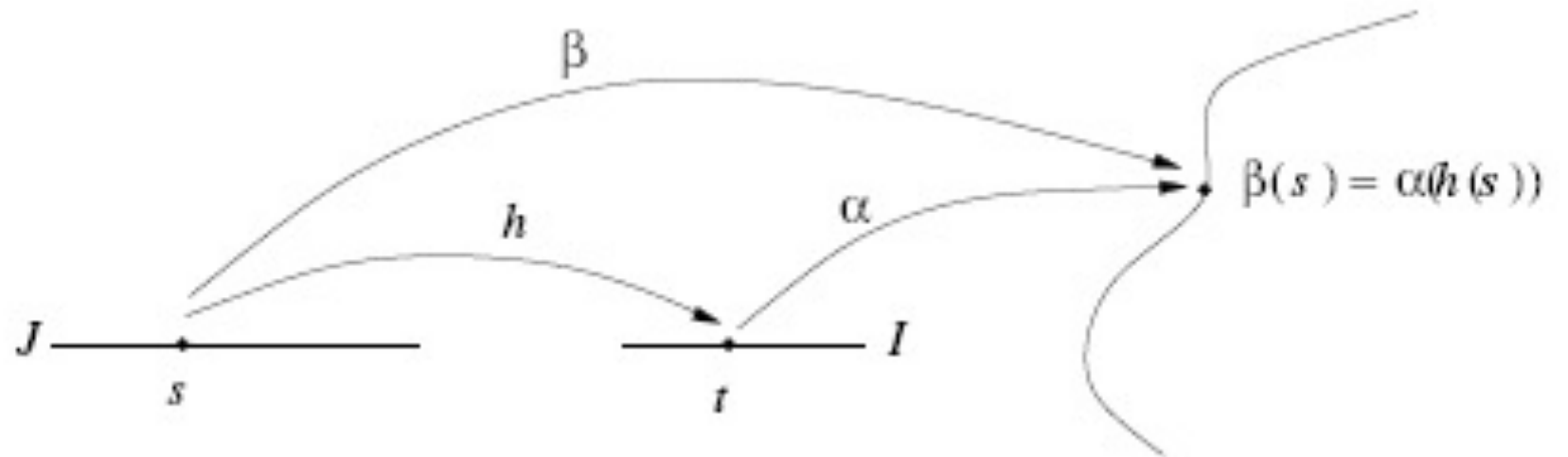


$$s(t) = \int_{t=a}^t \|\alpha'(u)\| du$$

Re-parametrização

$$\alpha : I \rightarrow \mathbb{R}^2$$

$$h : J \rightarrow I$$



$$\beta = \alpha \circ h : J \rightarrow \mathbb{R}^2$$

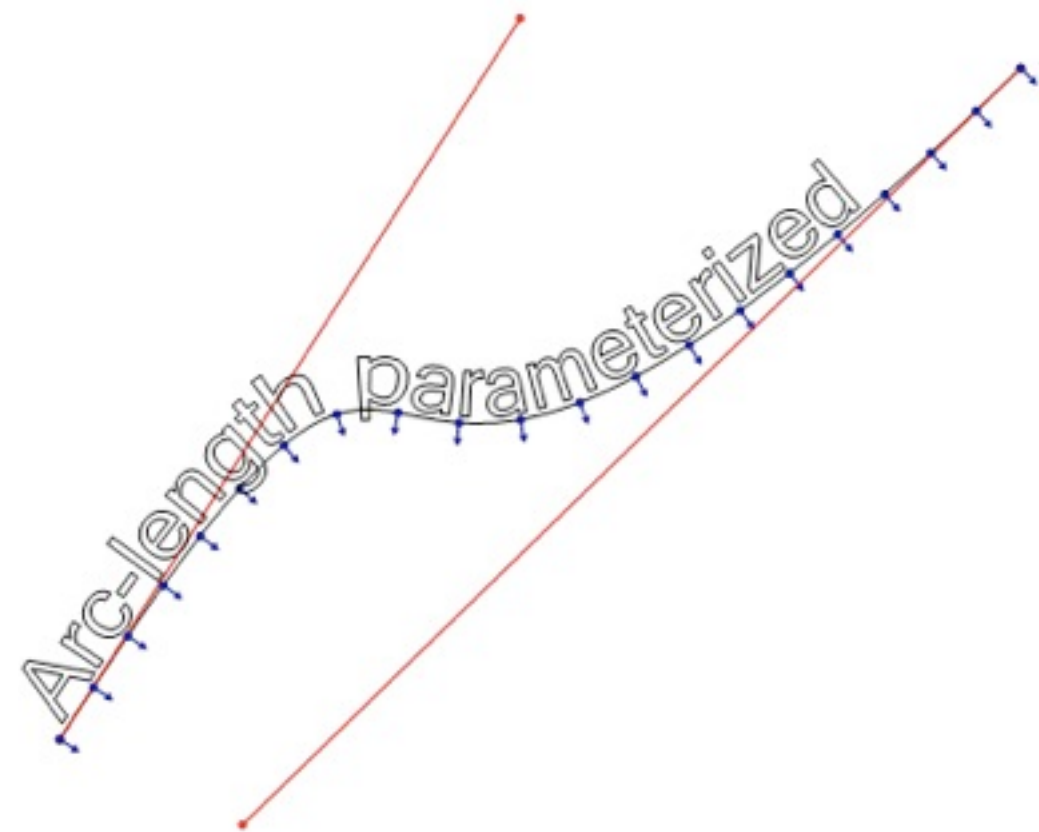
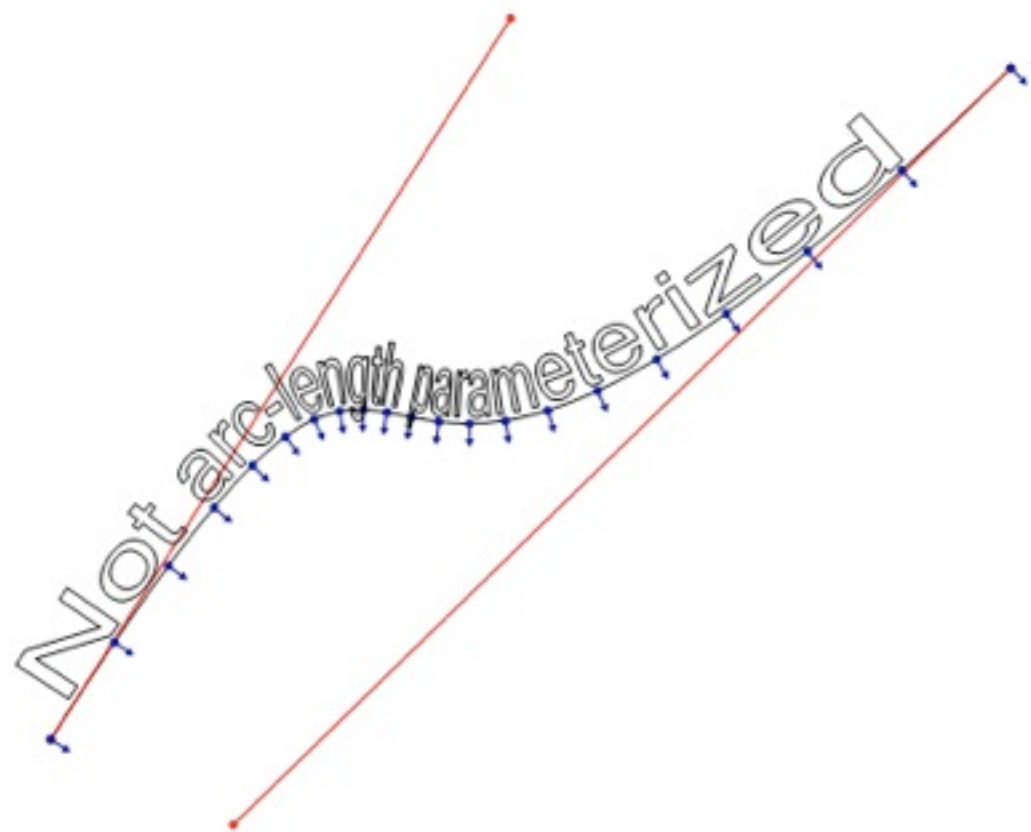
$$\beta(u) = \alpha(h(u))$$

Some text warped to a curve.

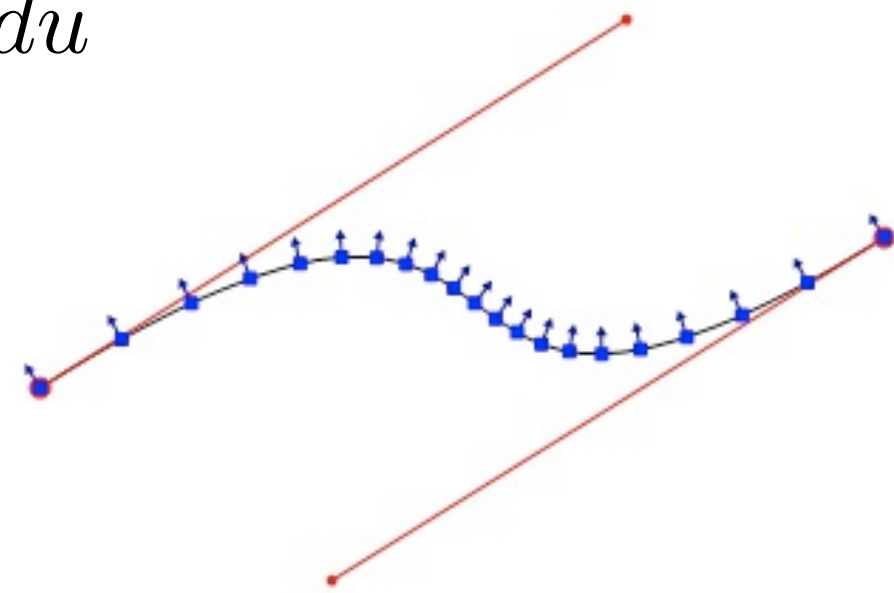
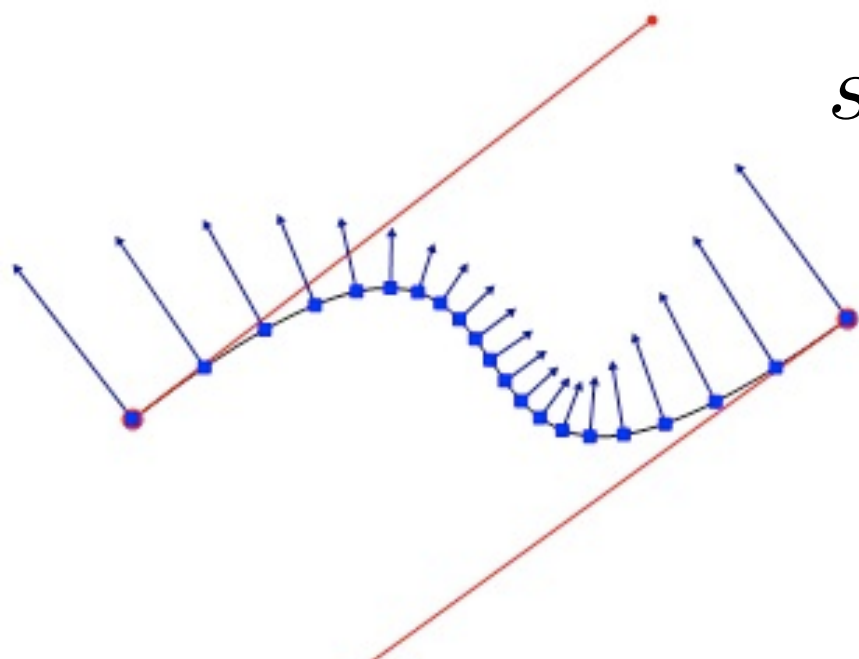
An example of some squashed text. Looks like the font has been over dieting

ex: $t = h(u) = 2u$

Re-parametrização por comprimento de arco



$$s(t) = \int_{t=a}^t \|\alpha'(u)\| du$$



Re-parametrização por comprimento de arco

$$\alpha : I \rightarrow \mathbb{R}^2$$

$$h : J \rightarrow I$$

$$\beta = \alpha \circ h : J \rightarrow \mathbb{R}^2$$

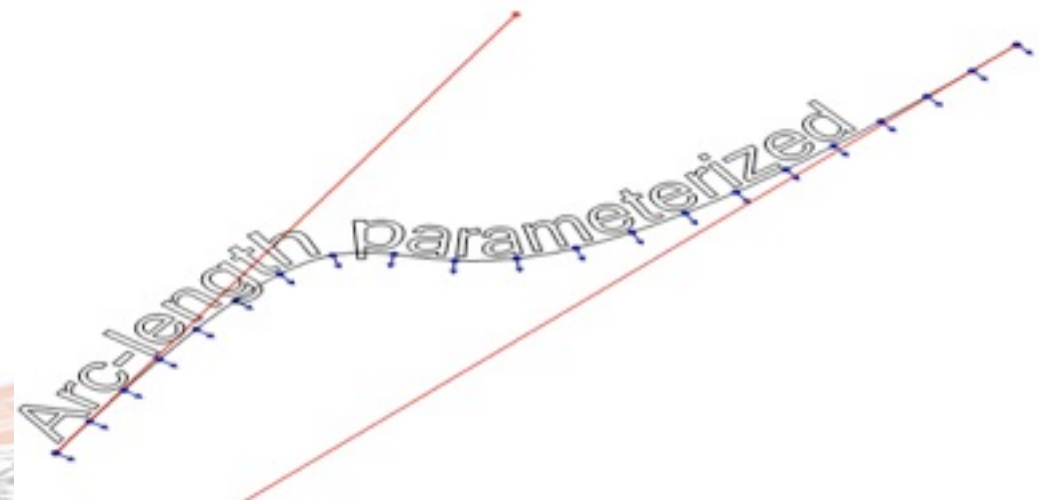
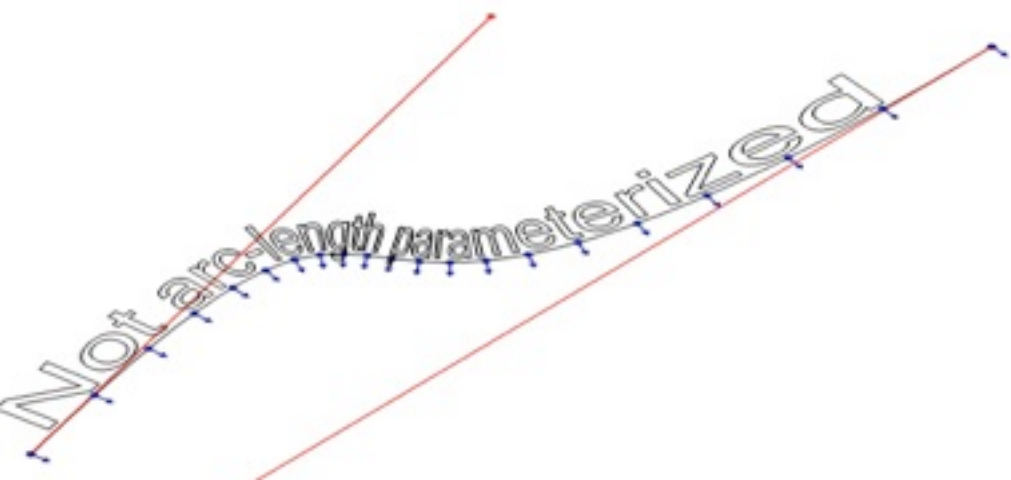
$$\beta(u) = \alpha(h(u))$$

$$\beta'(u) = h'(u) \cdot \alpha'(h(u))$$

$$h = s^{-1}$$

$$h'(u) = \frac{1}{s'(h(u))} = \frac{1}{\|\alpha'(h(u))\|}$$

$$\begin{aligned}\|\beta'(u)\| &= |h'(u)| \cdot \|\alpha'(h(u))\| \\ &= \frac{\|\alpha'(h(u))\|}{\|\alpha'(h(u))\|} = 1\end{aligned}$$

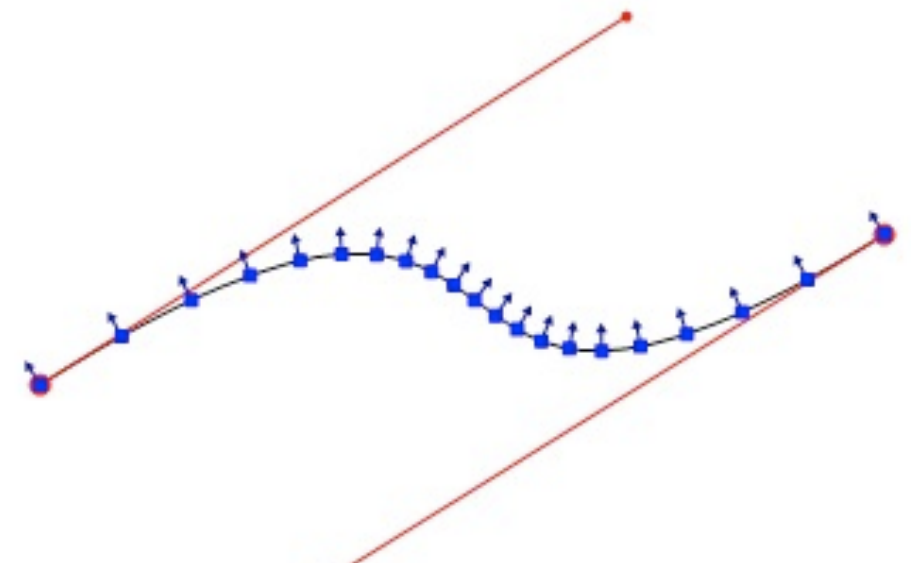
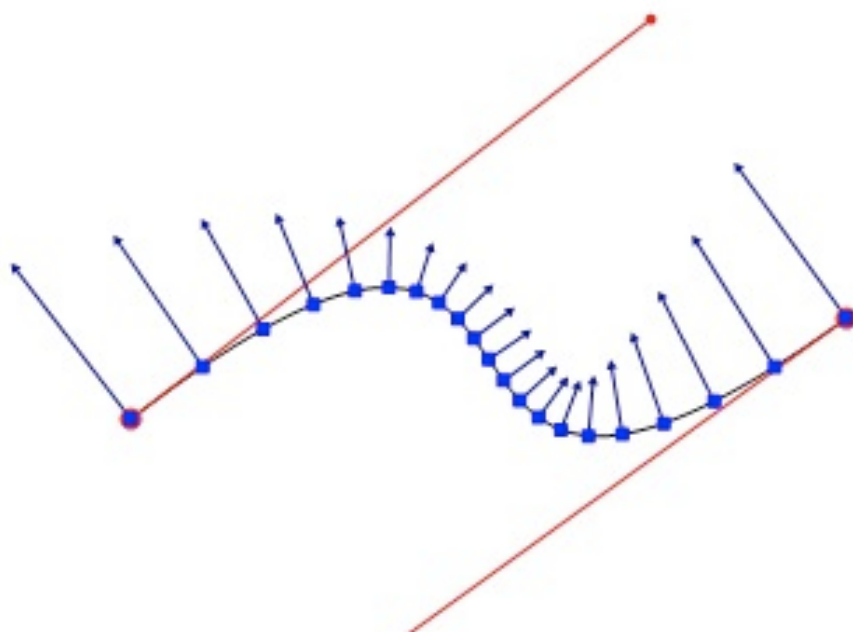


Invariância do comprimento

transformações geométricas

parametrização

Jacobiano compensa: $s(t) = \int_{t=a}^t ||\alpha'(u)|| du$



Curvatura: variação da tangente

C parametrizada pelo comprimento de arco

$$\|\alpha'(s)\| = 1$$

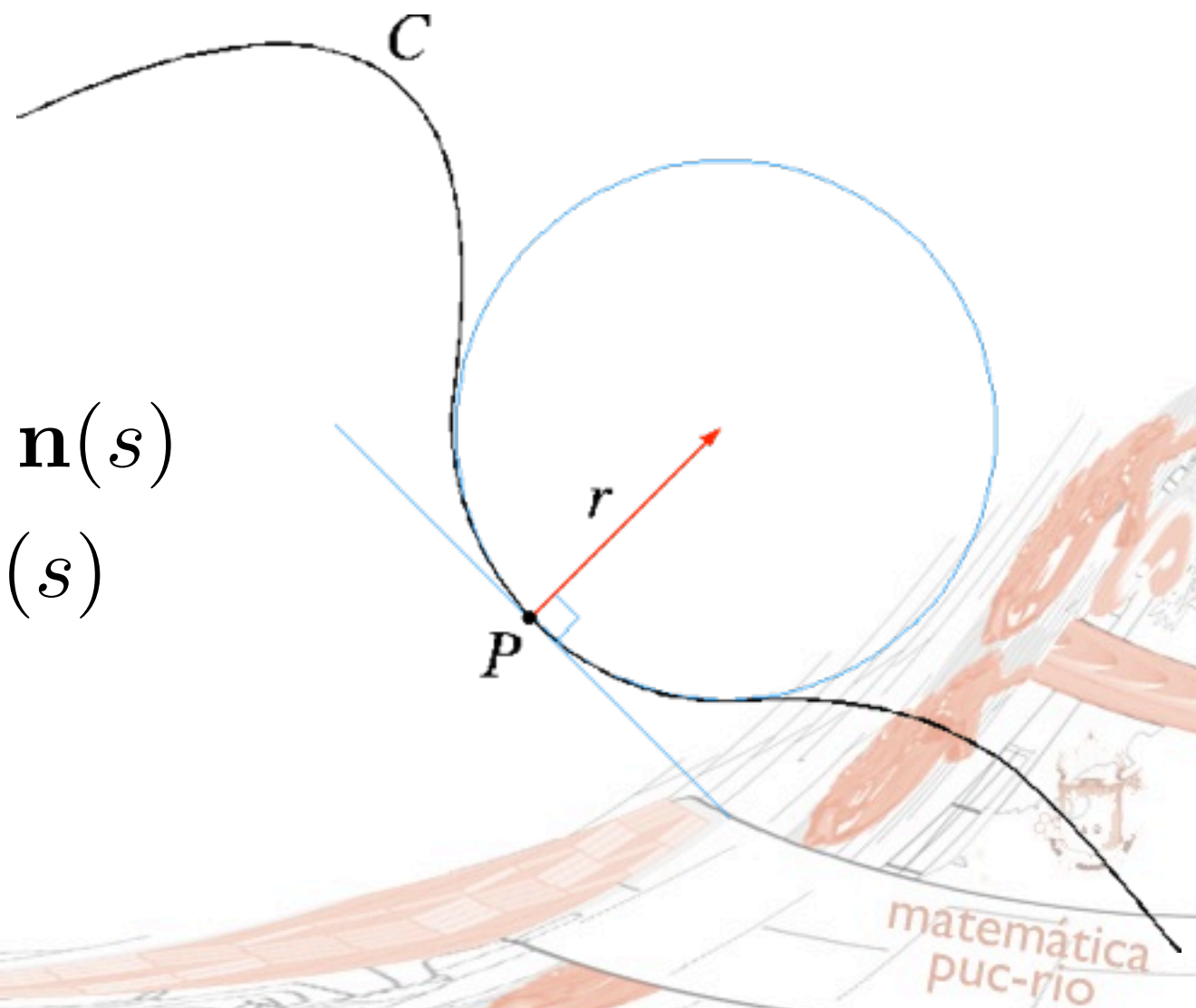
$$\|\mathbf{t}(s)\| = 1$$

$$\Rightarrow \langle \mathbf{t}(s), \mathbf{t}(s) \rangle = 1 \quad \forall s$$

$$\Rightarrow 2\langle \mathbf{t}'(s), \mathbf{t}(s) \rangle = 0 \quad \forall s$$

$$\Rightarrow \mathbf{t}'(s) \perp \mathbf{t}(s) \quad \Rightarrow \quad \mathbf{t}'(s) \parallel \mathbf{n}(s)$$

$$\Rightarrow \forall s, \exists \kappa(s), \mathbf{t}'(s) = \kappa(s) \cdot \mathbf{n}(s)$$



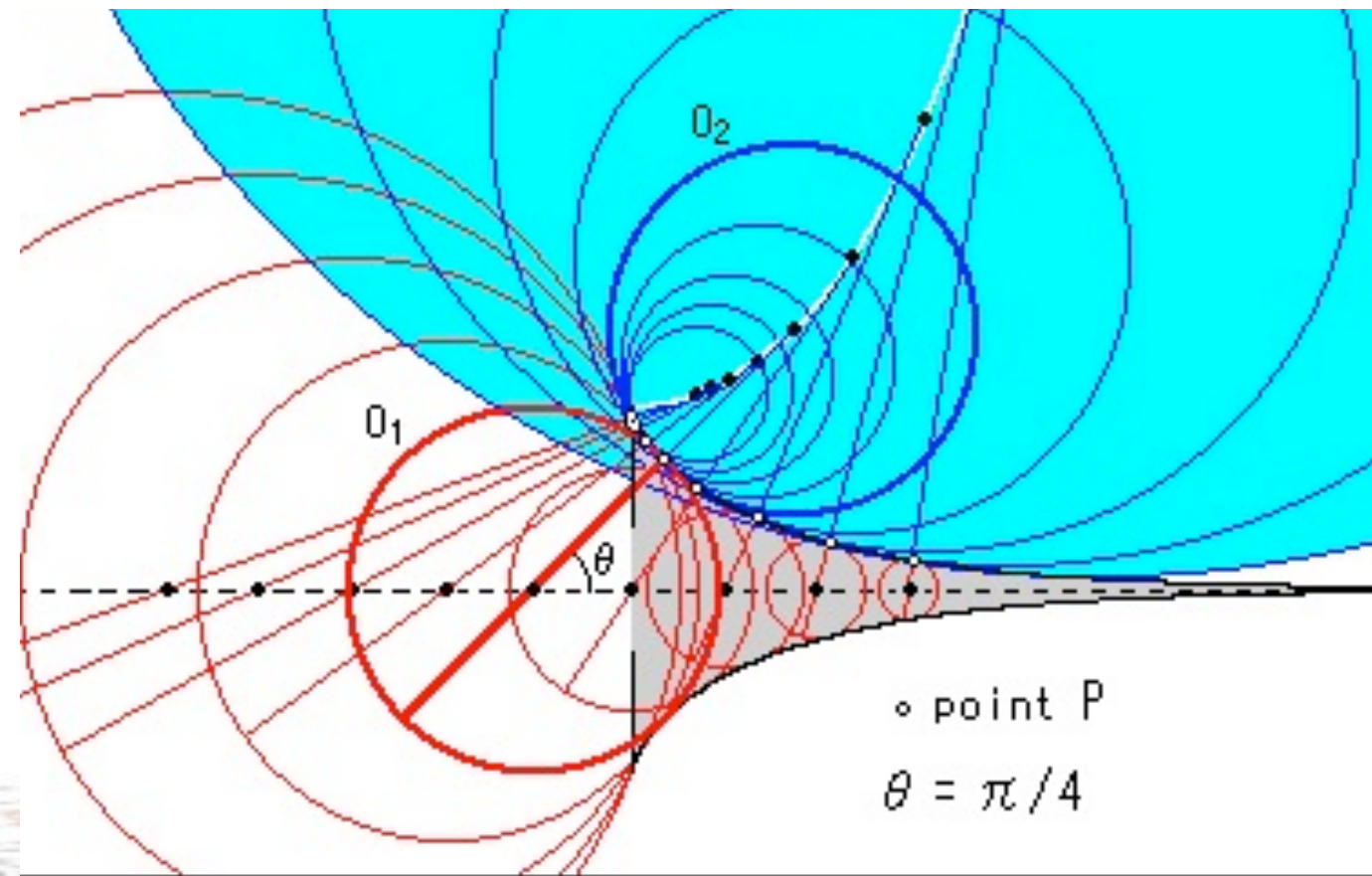
Curvatura: limite de círculos

Tangente

limite das cordas (2 pontos)

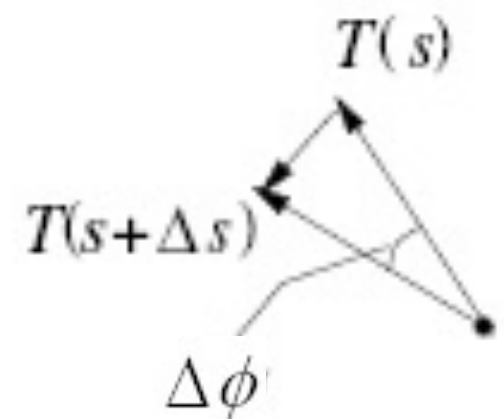
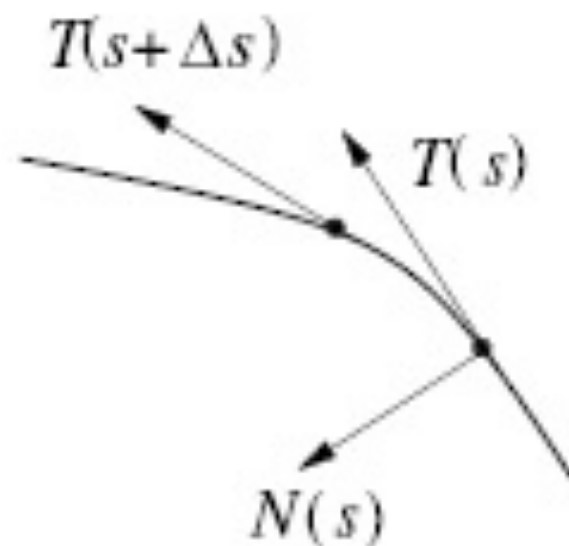
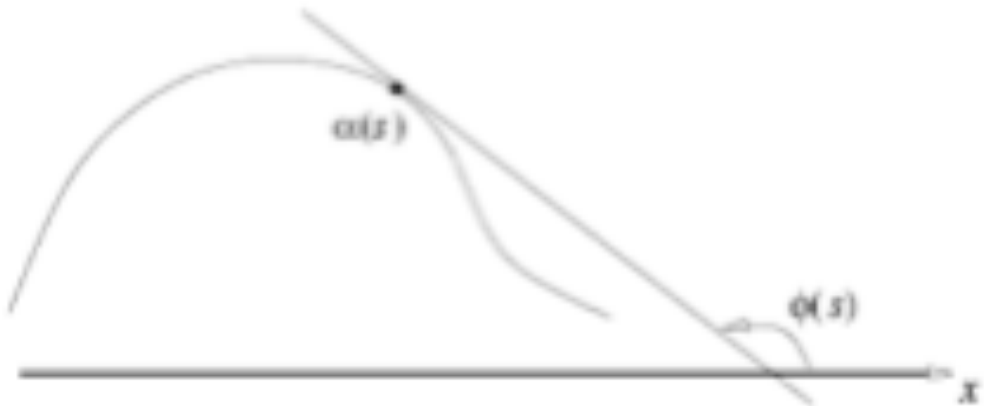
Curvatura

limite dos círculos (3 pontos)

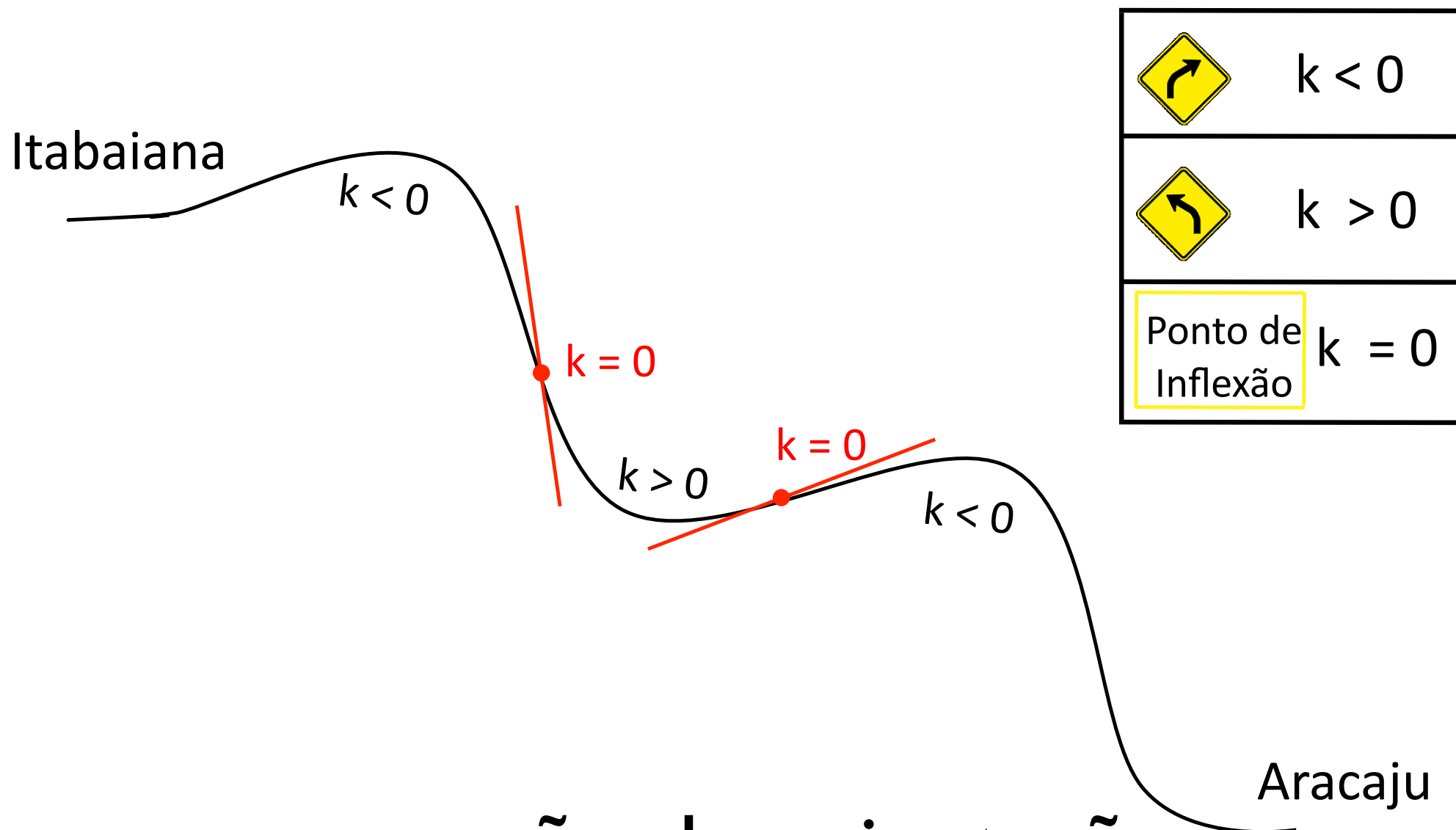


Curvatura: variação de ângulo

$$\begin{aligned}
 \kappa(s) &= \langle \mathbf{t}'(s), \mathbf{n}(s) \rangle \\
 &= \left\langle \lim_{ds \rightarrow 0} \frac{\mathbf{t}(s + ds) - \mathbf{t}(s)}{ds}, \mathbf{n}(s) \right\rangle \\
 &\approx \left\langle \lim_{ds \rightarrow 0} \frac{d\phi \cdot \mathbf{n}}{ds}, \mathbf{n} \right\rangle \\
 &= \lim_{ds \rightarrow 0} \frac{d\phi}{ds} \\
 &= \phi'(s)
 \end{aligned}$$



Sinal da curvatura



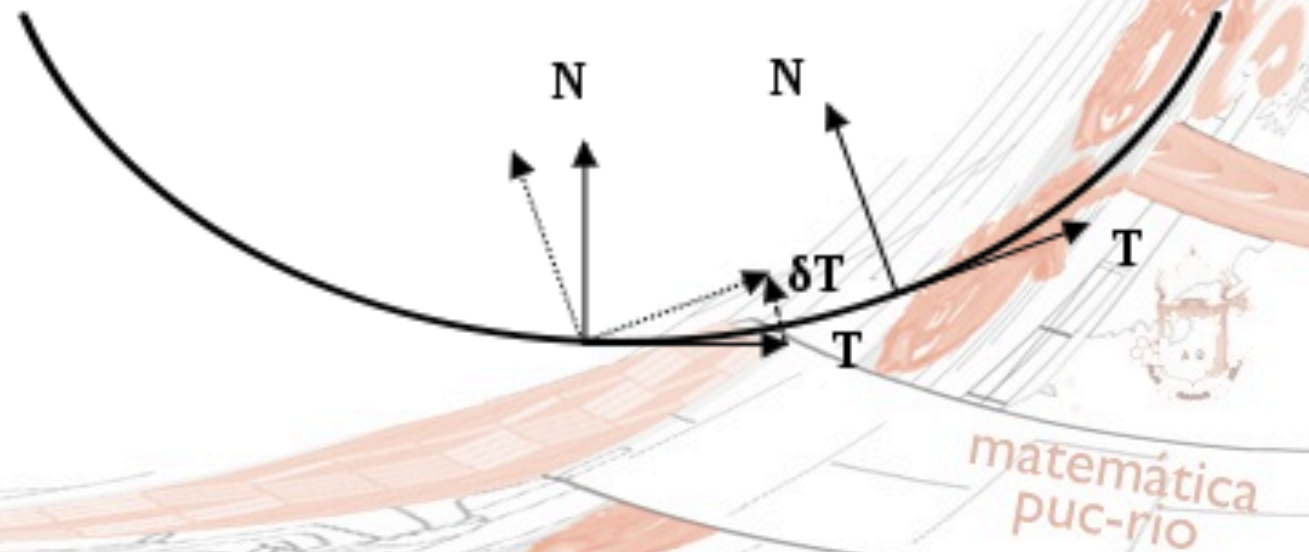
convenção de orientação

Curvatura sem comprimento de arco

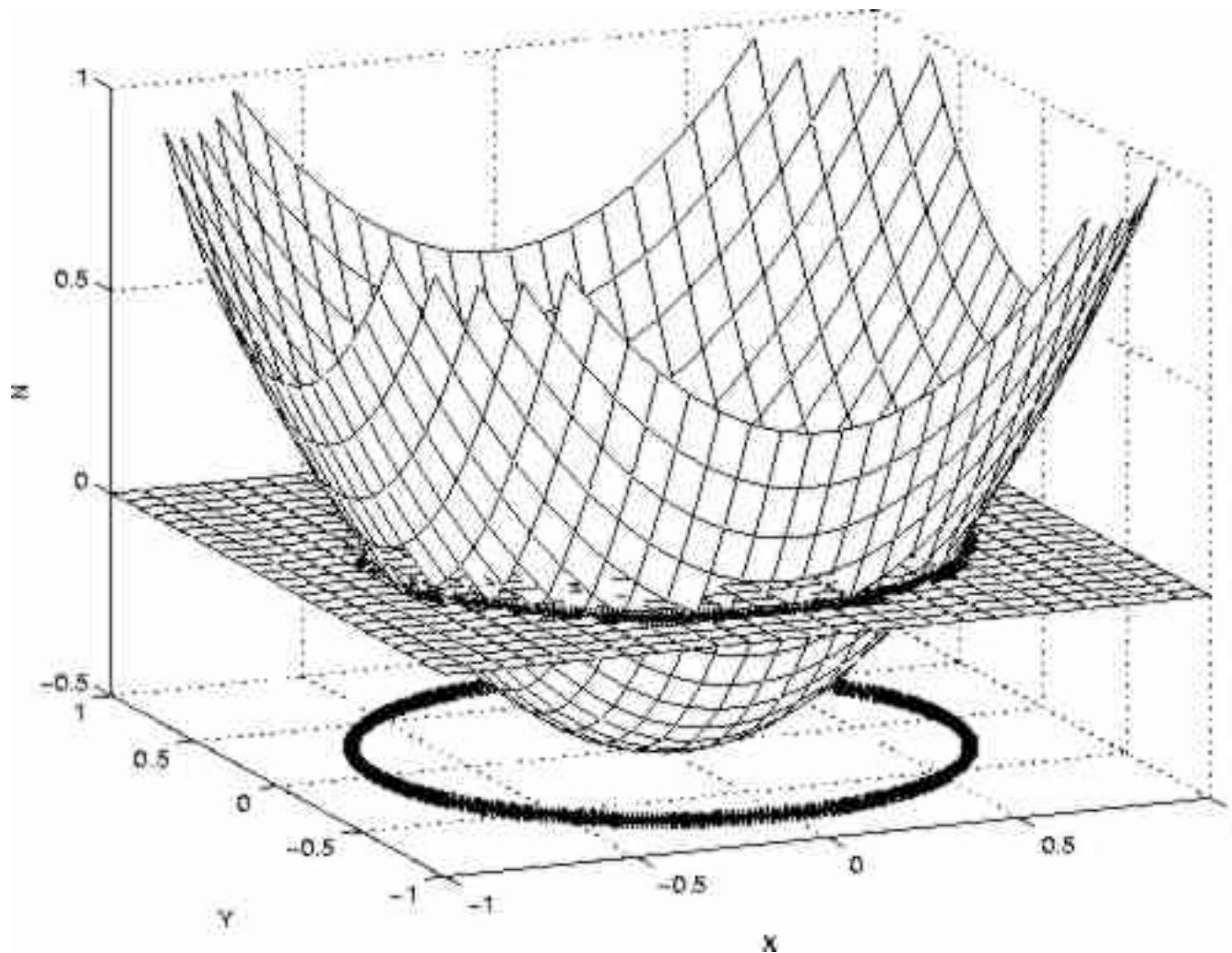
$$\begin{aligned}\kappa(t) &= \frac{\alpha'(t) \times \alpha''(t)}{\|\alpha'\|^3} \\ &= \frac{x'(t) \cdot y''(t) - x''(t) \cdot y'(t)}{(x'(t)^2 + y'(t)^2)^{\frac{3}{2}}}\end{aligned}$$

$$\beta'(u) = h'(u) \cdot \alpha'(h(u))$$

$$h'(u) = \frac{1}{s'(h(u))} = \frac{1}{\|\alpha'(h(u))\|}$$

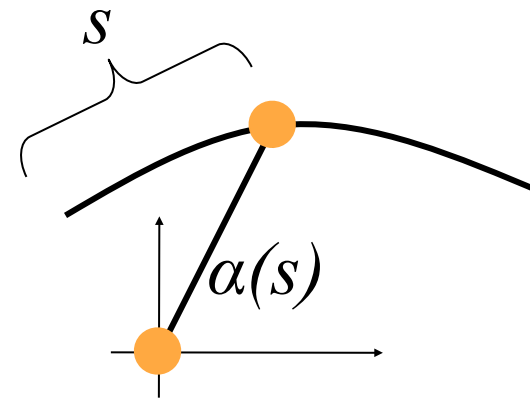


Curvatura de curva implícita?

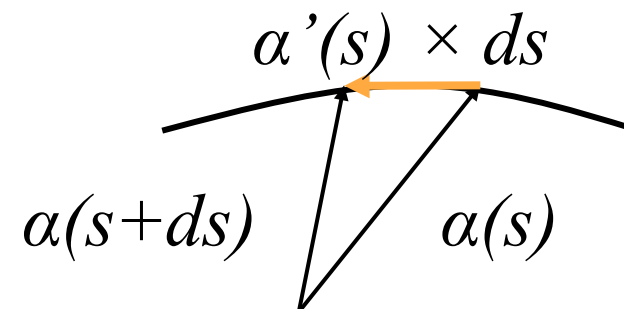


Resumo

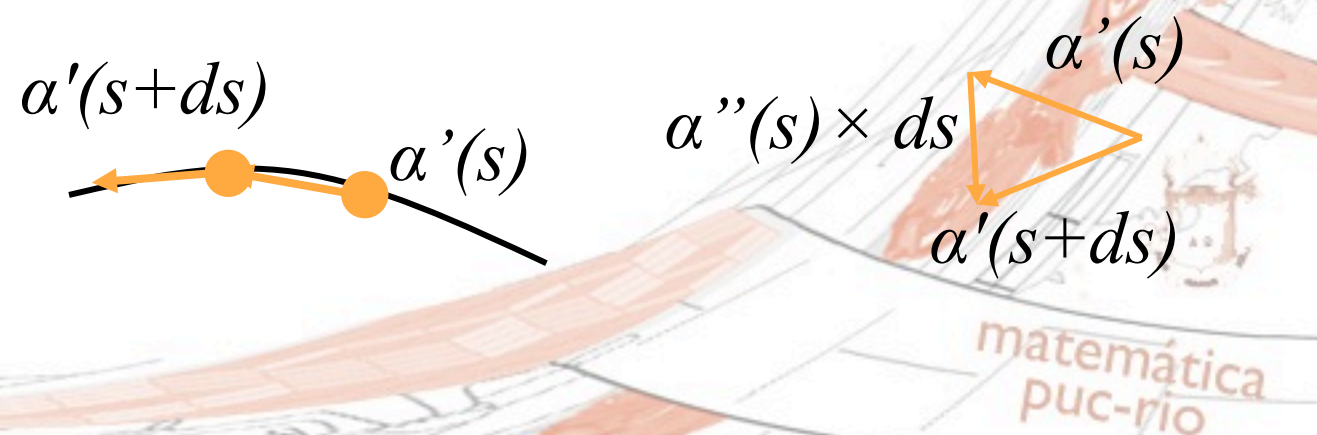
comprimento de arco



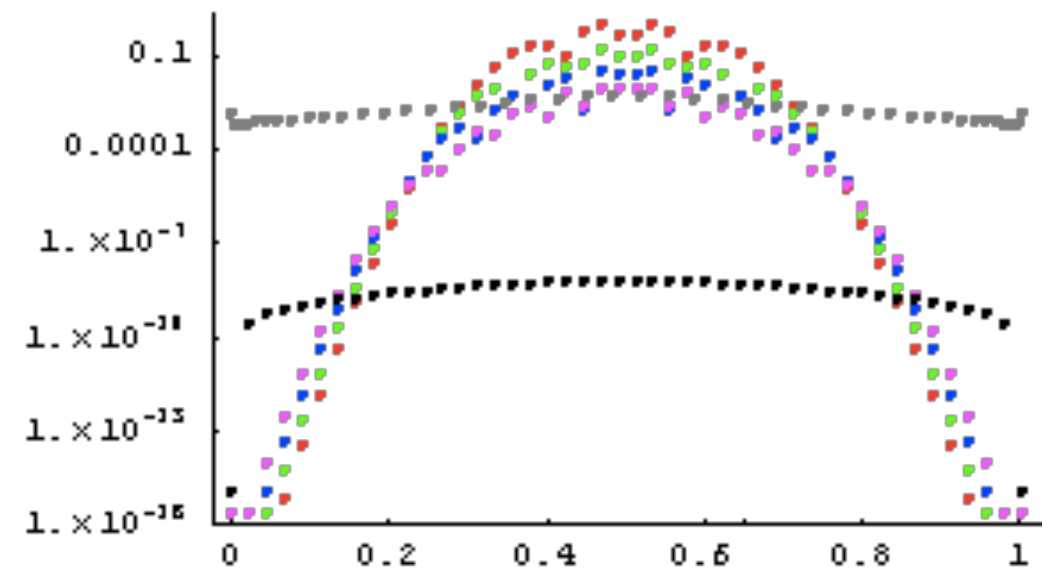
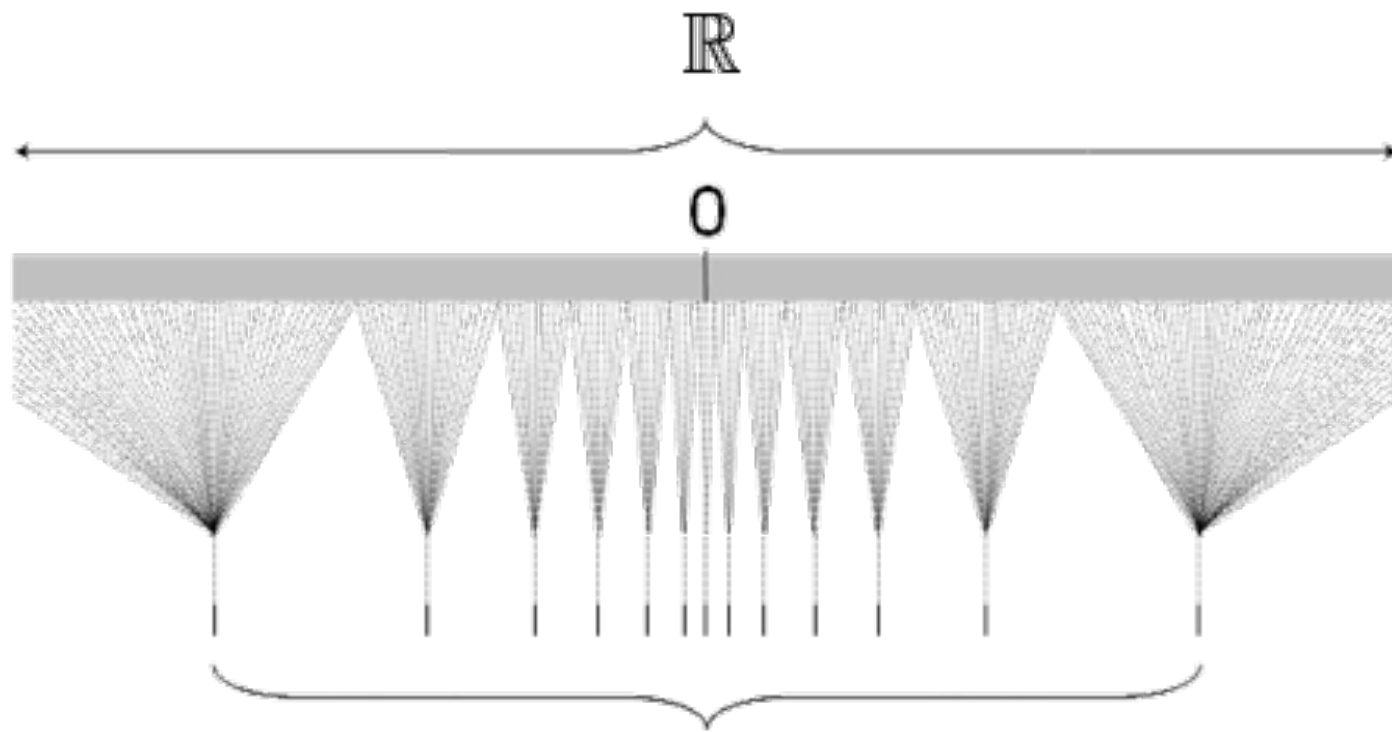
tangente



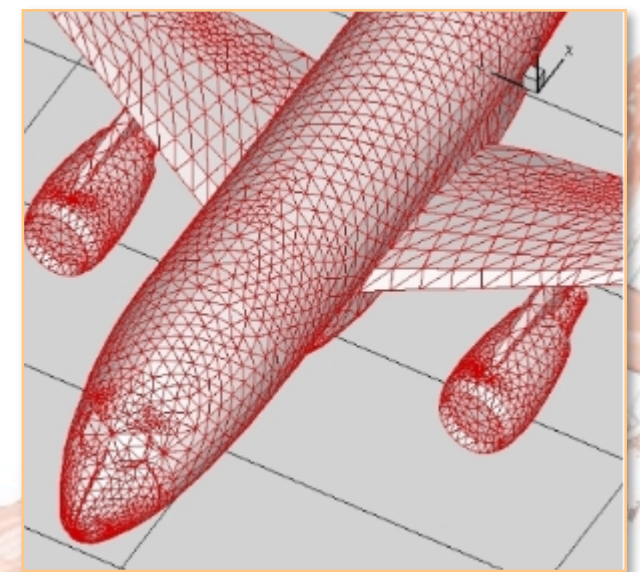
curvatura



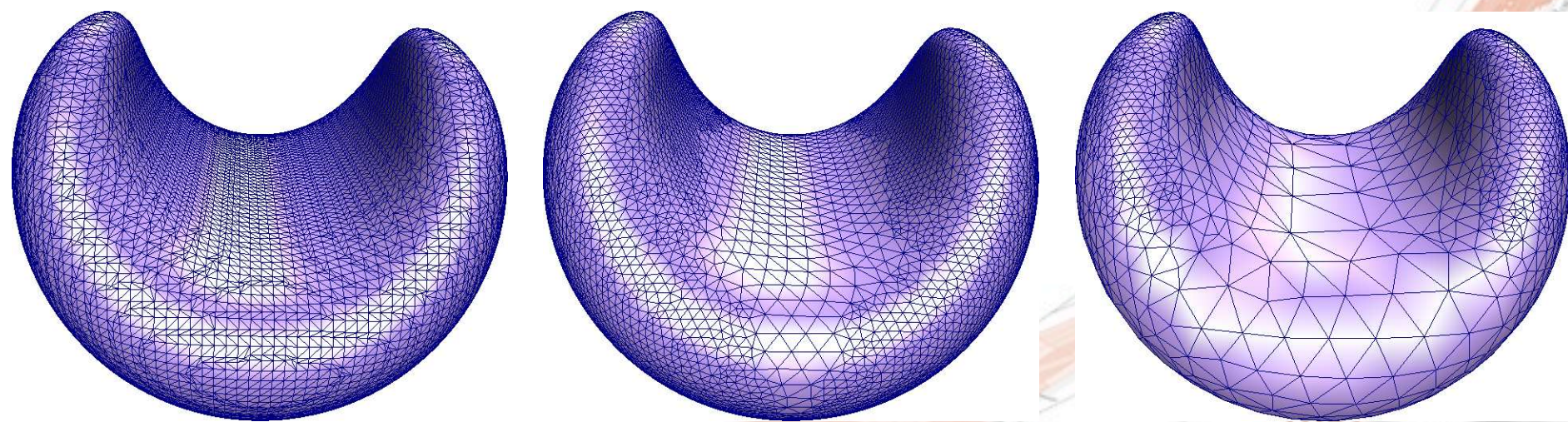
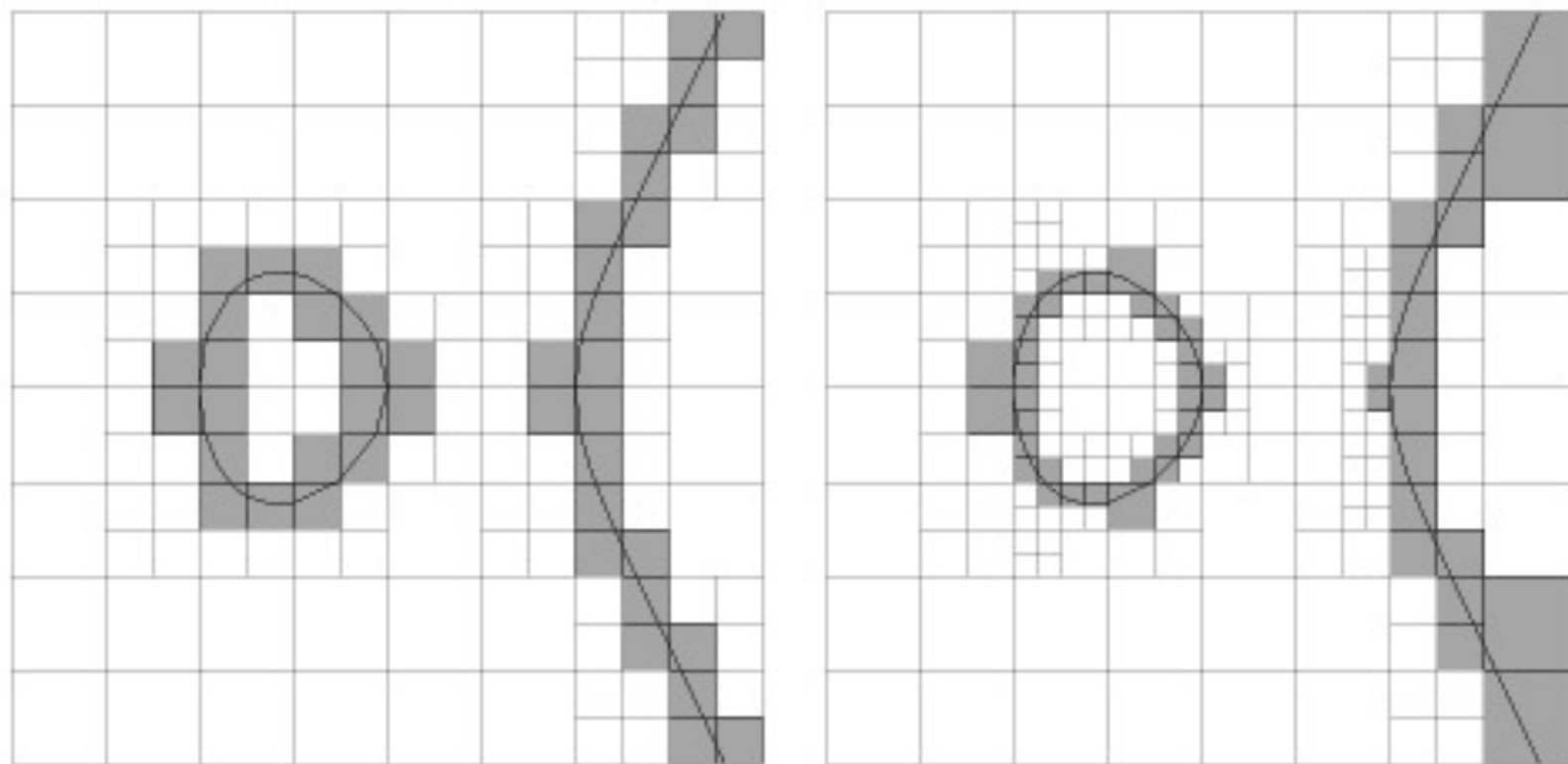
Medir com o computador



infinitesimal??



Necessário no computador!



Obrigado pela atenção

Curvas e Superfícies Implícitas:
Noções de Geometria Diferencial e Discreta

amanhã: geometria diferencial de curvas e superfícies implícitas

