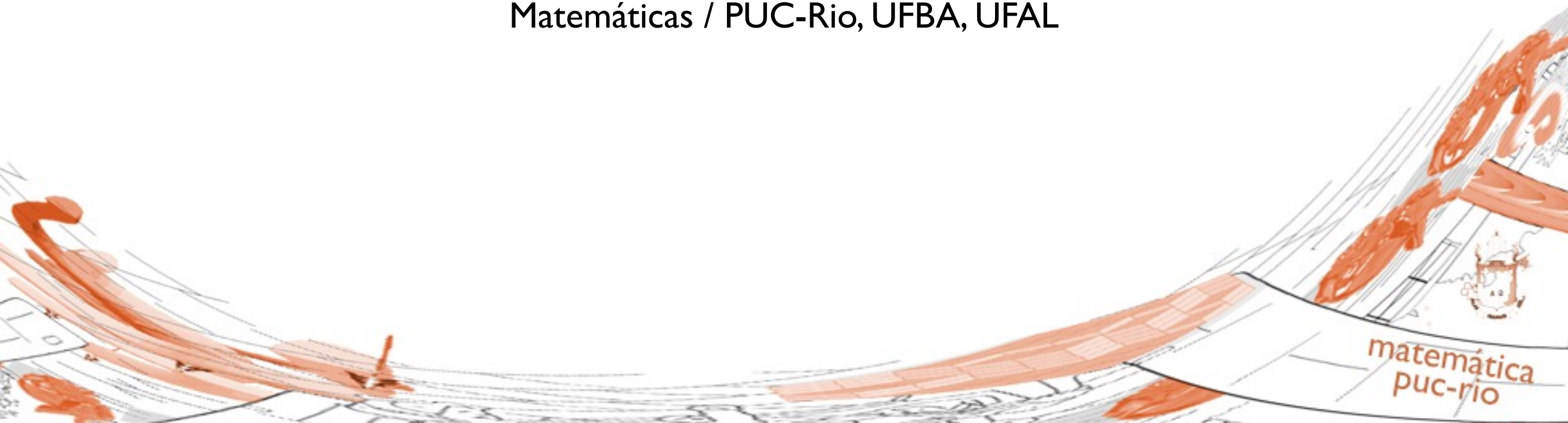


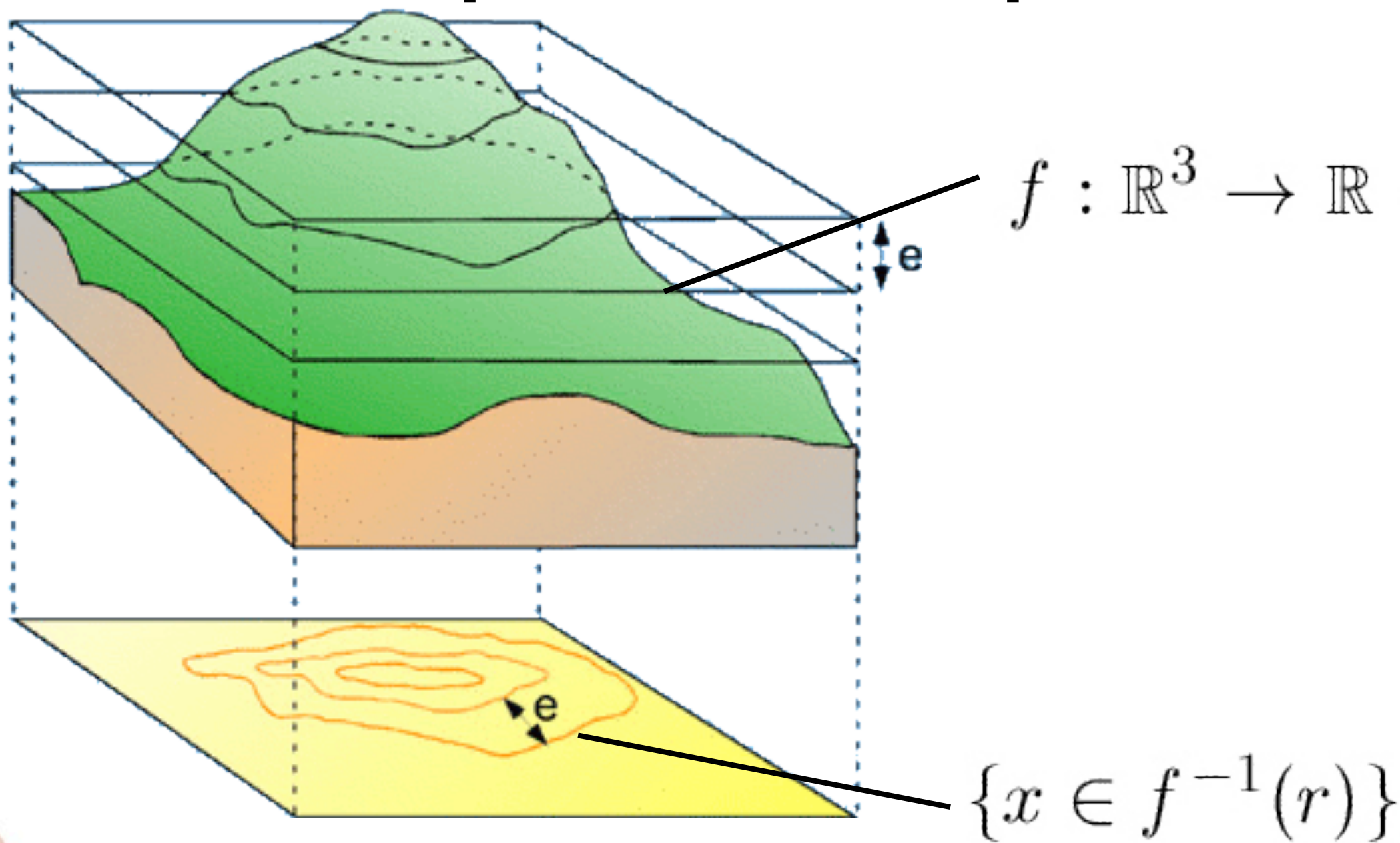
# Curvas e Superfícies Implícitas: Noções de Geometria Diferencial e Discreta

Maria Andrade, Allyson Cabral  
Vinícius Mello, Adelailson Peixoto, *Thomas Lewiner*

Matemáticas / PUC-Rio, UFBA, UFAL

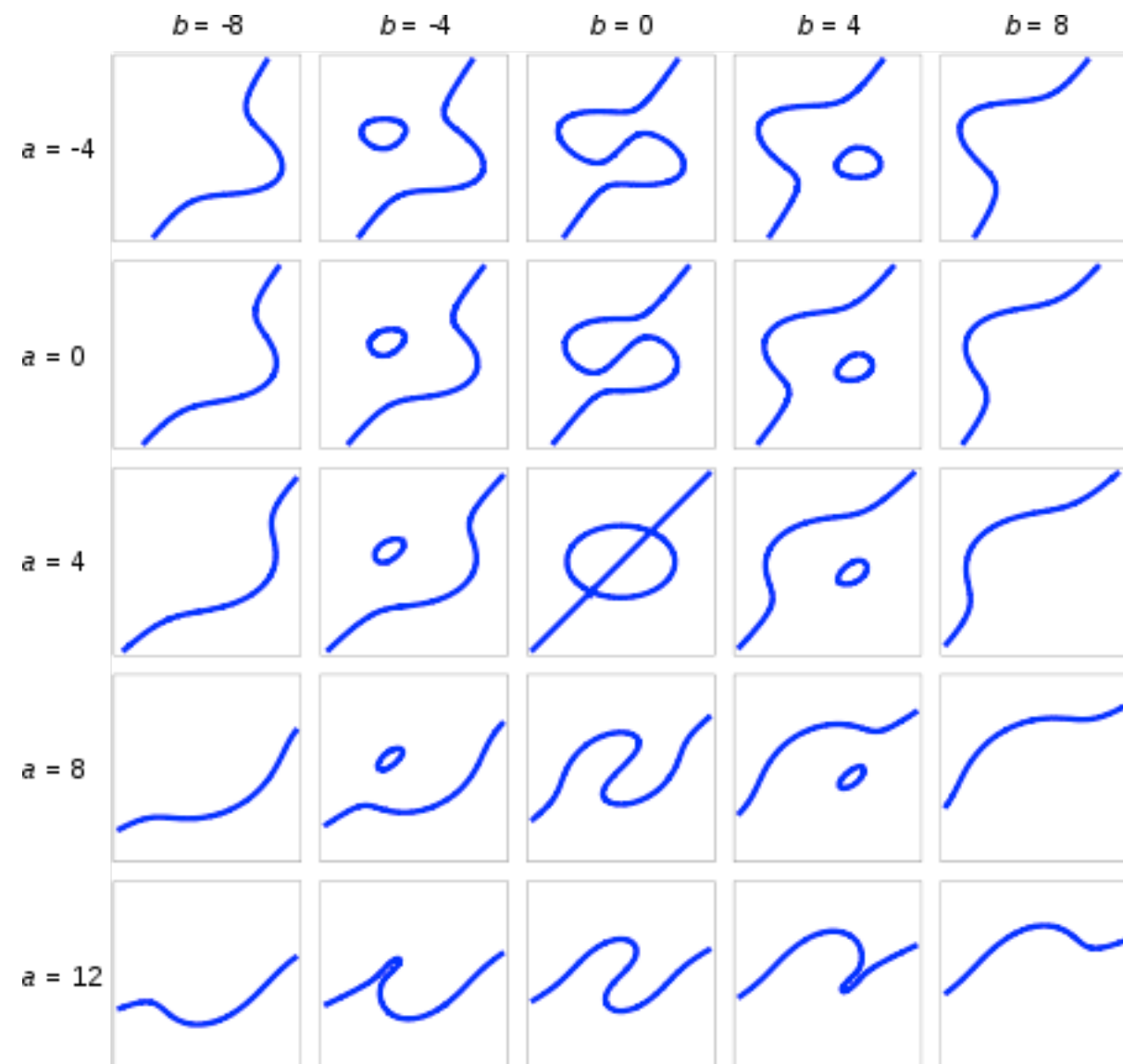


# Superfície implícita



# Interpolação

dim dif	1	2	3
1	1	2	3
2	2	4	9
3	3	9	27



curvas cúbicas

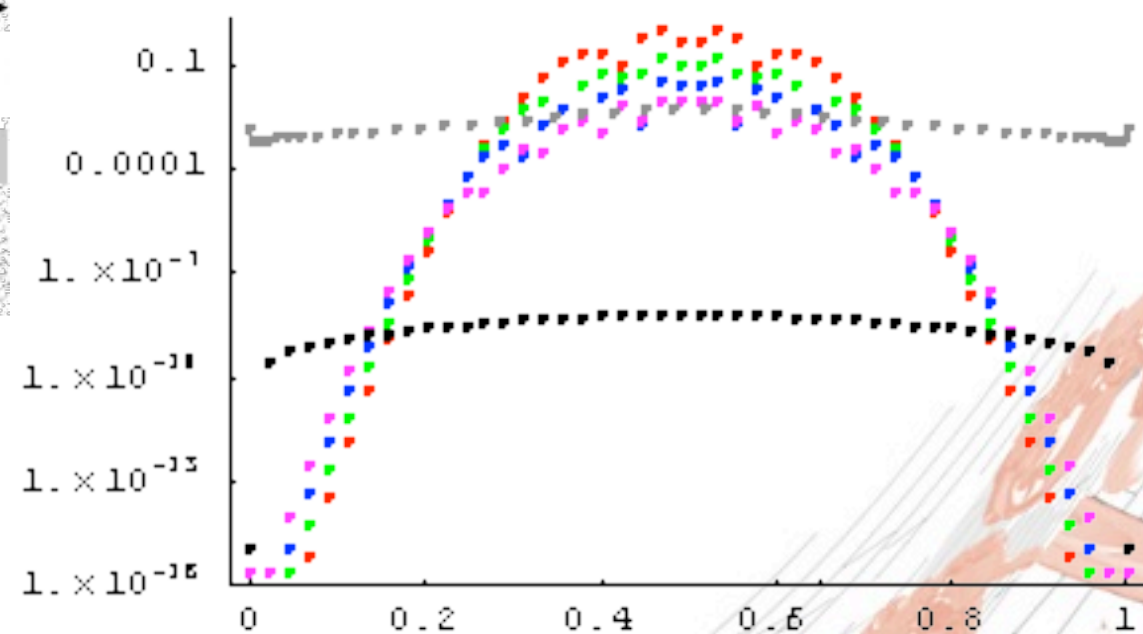
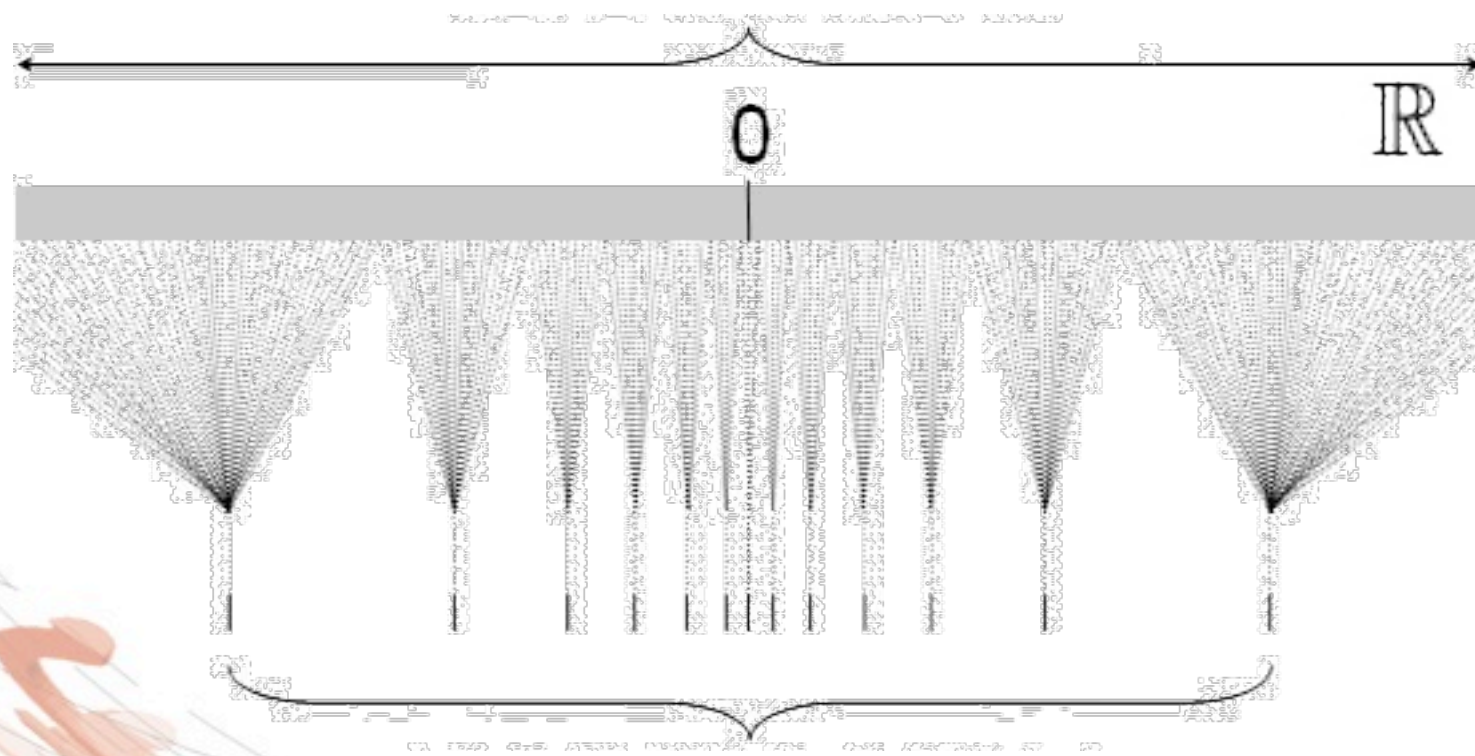


# Cálculo no computador

Números reais?

Funções reais?

Derivadas?



discreto

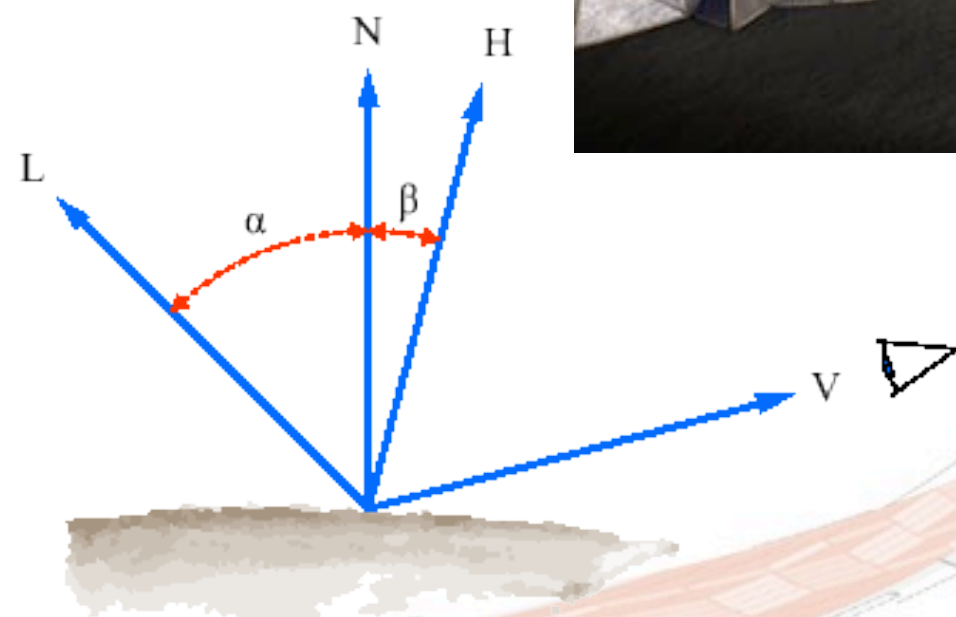


# Geometria no computador

Conjuntos?

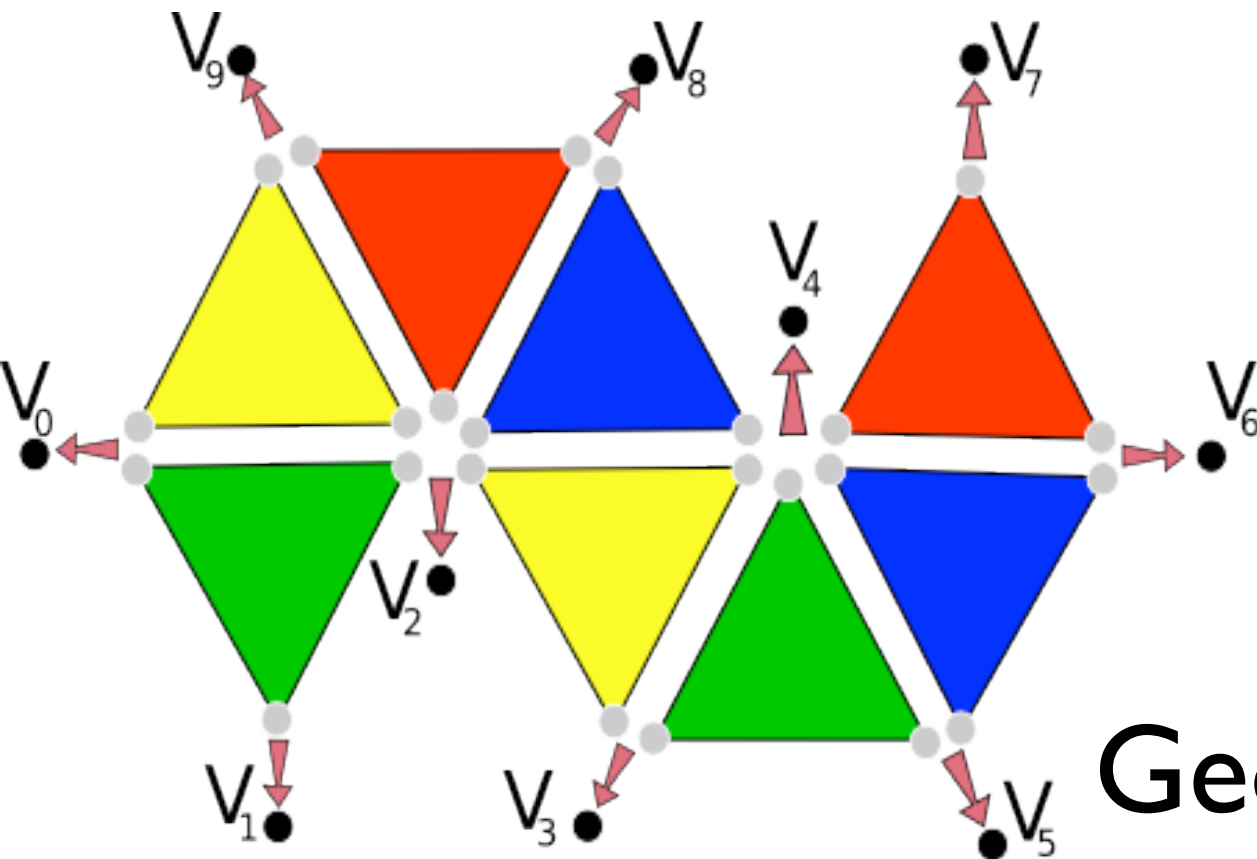
Superfícies?

Normais? 



# Superfícies explícitas discretas

Modelo de interpolação



Geometria:

$$V0 = (-3, 0)$$

$$V1 = (-2, -1)$$

$$V2 = (-1, 0)$$

$$V3 = (0, -1)$$

Conectividade

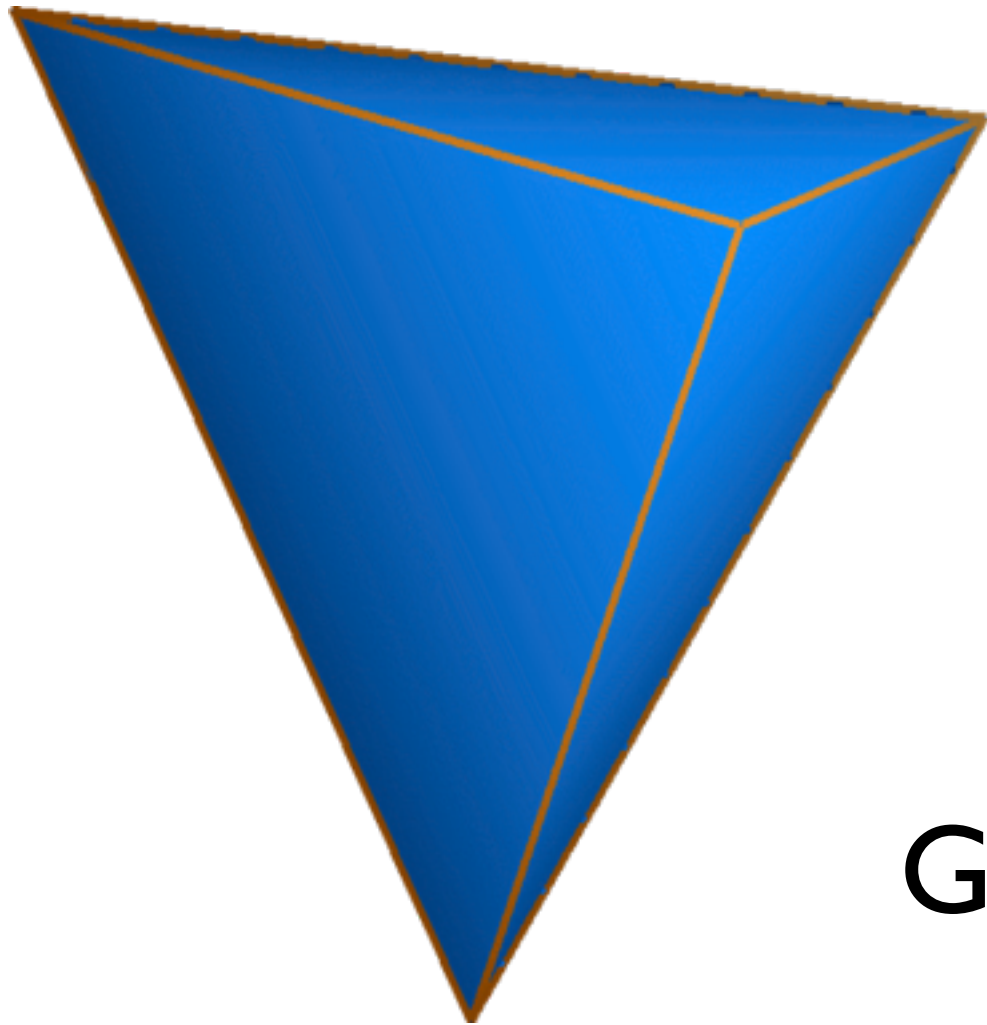
$$T0 = V0V1V2$$

$$T1 = V0V2V9$$

$$T2 = V2V8V9$$

$$T3 = V2V4V8$$

# Exemplo: Tetraedro



Geometria:

$$V0 = (0,0,0)$$

$$V1 = (1,0,0)$$

$$V2 = (0,1,0)$$

$$V3 = (0,0,1)$$

Conectividade

$$T0 = V0V1V2$$

$$T1 = V0V2V3$$

$$T2 = V0V3V1$$

$$T3 = V1V3V2$$

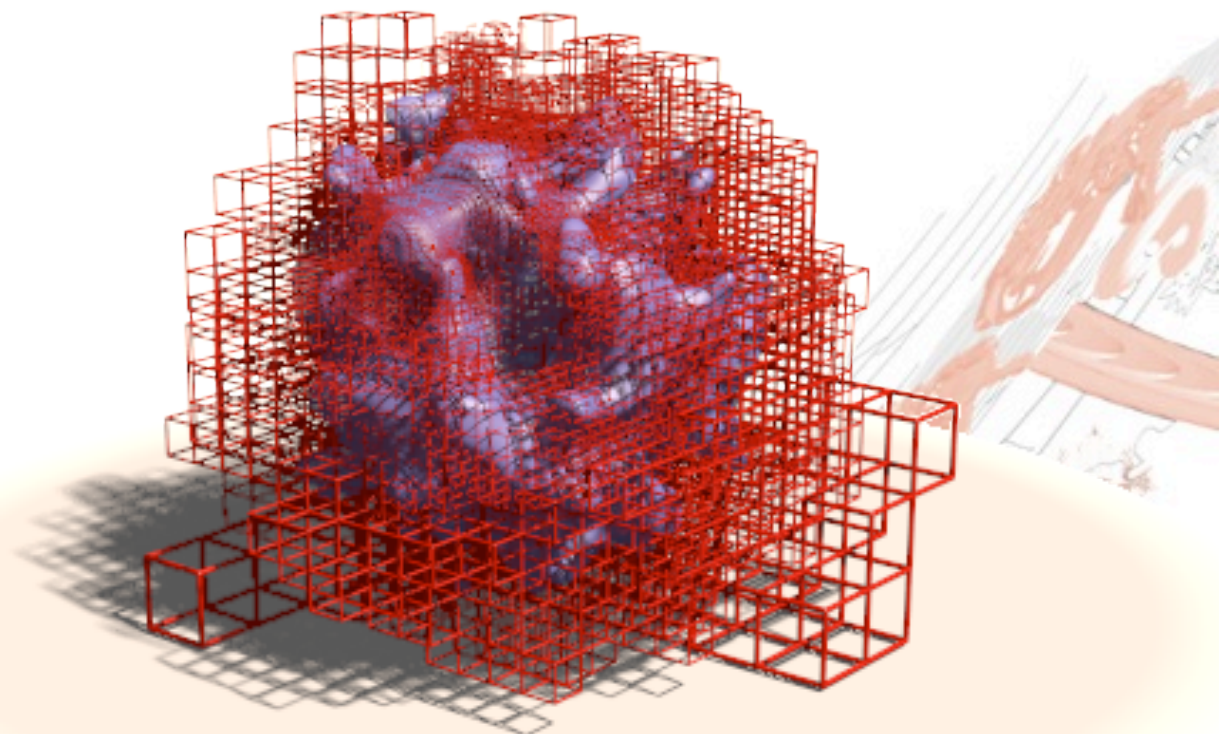
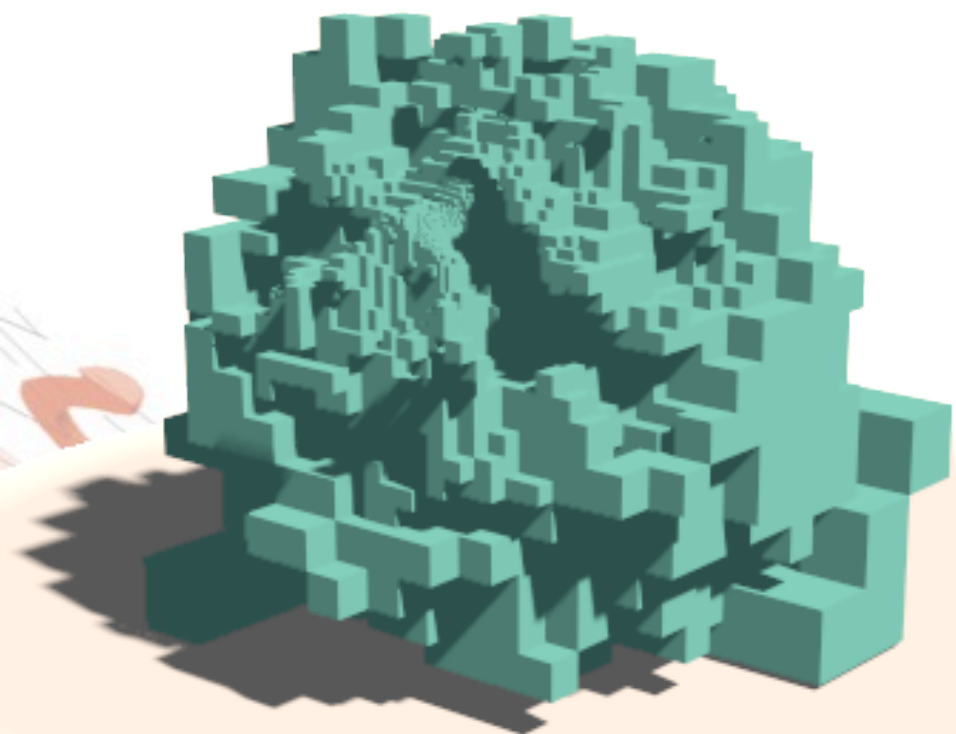


# Superfícies implícitas discretas = isosuperfícies

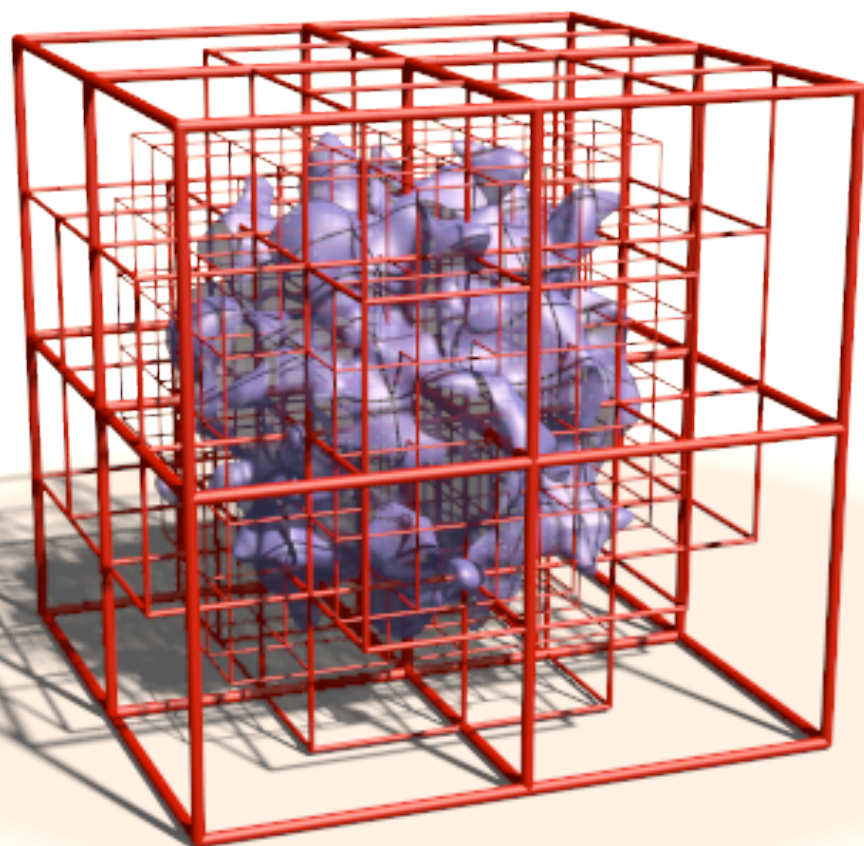
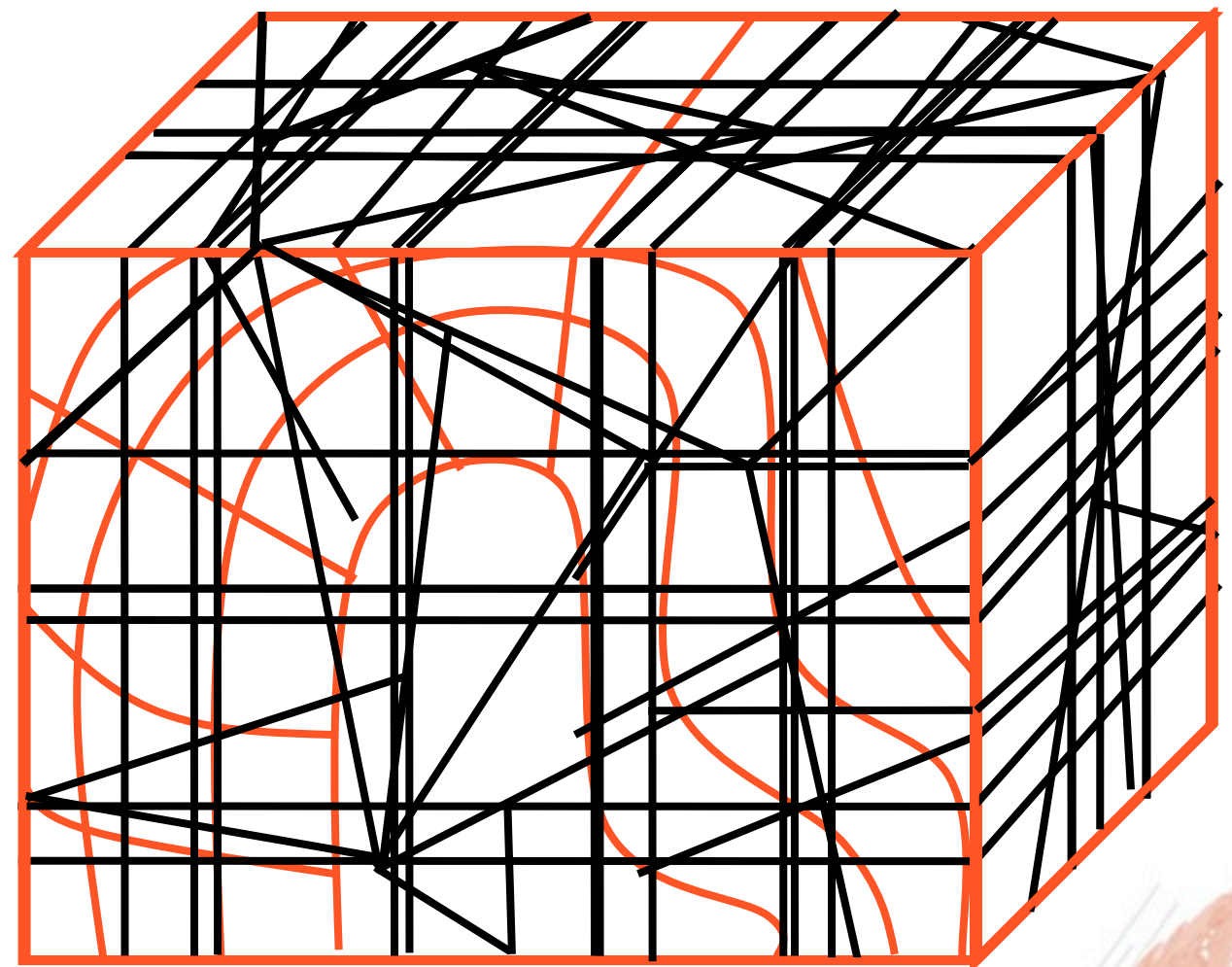
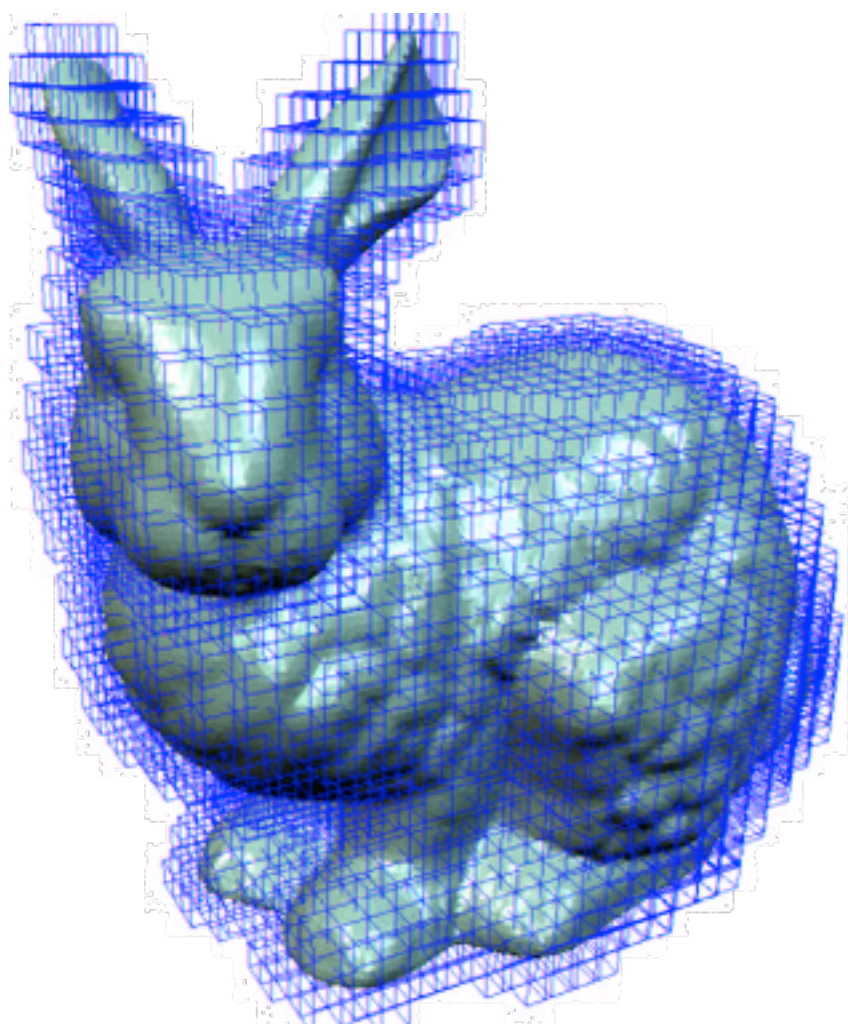
Discretização em reticulados (grid)

Localização da isosuperfície

Modelos de interpolação

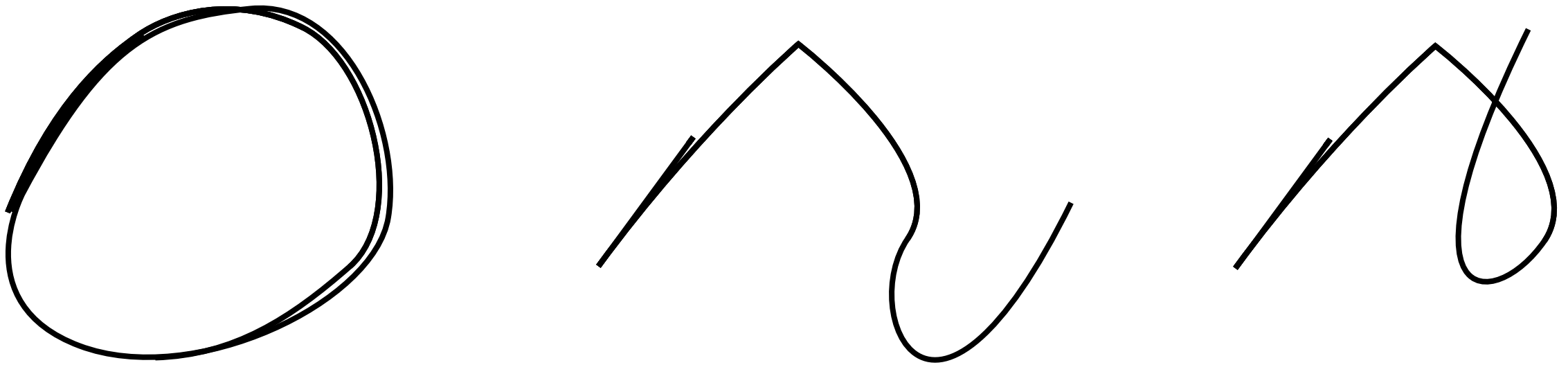


# Grid

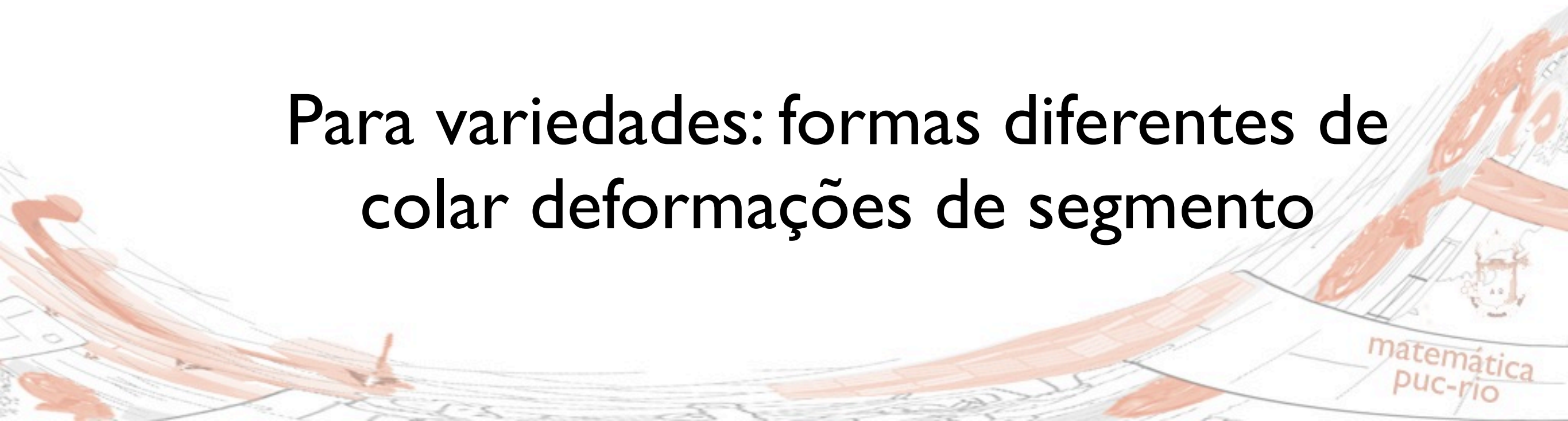




# Definir a topologia

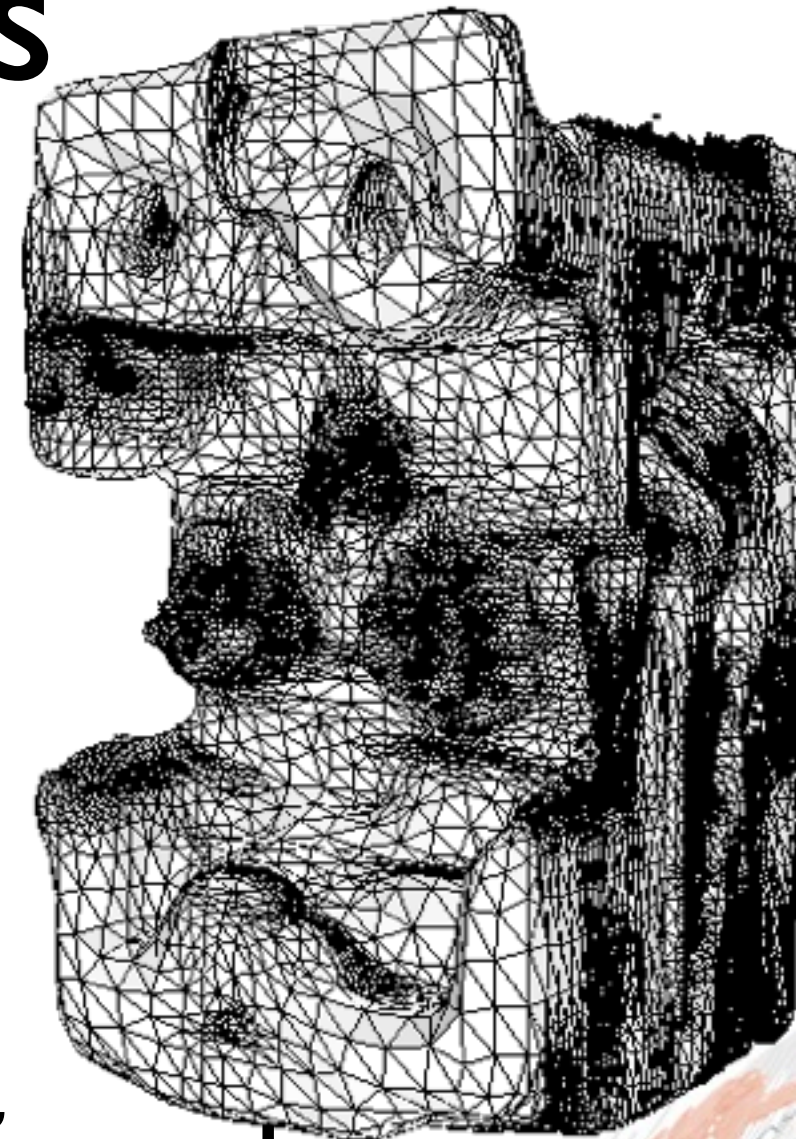
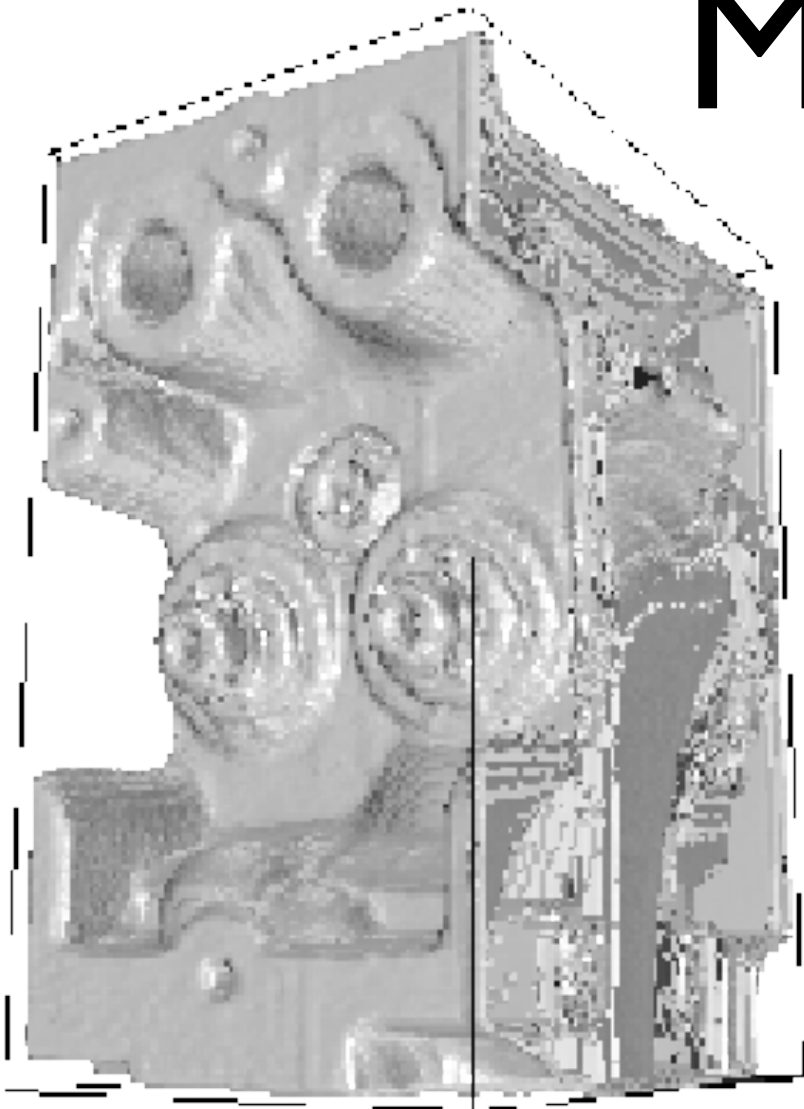


Para variedades: formas diferentes de  
colar deformações de segmento





# Marching Cubes



Mais citado da computação e da  
matemática

Representação explícita de  
isosuperfície

$$f(x) \neq 0$$

# Caso I D

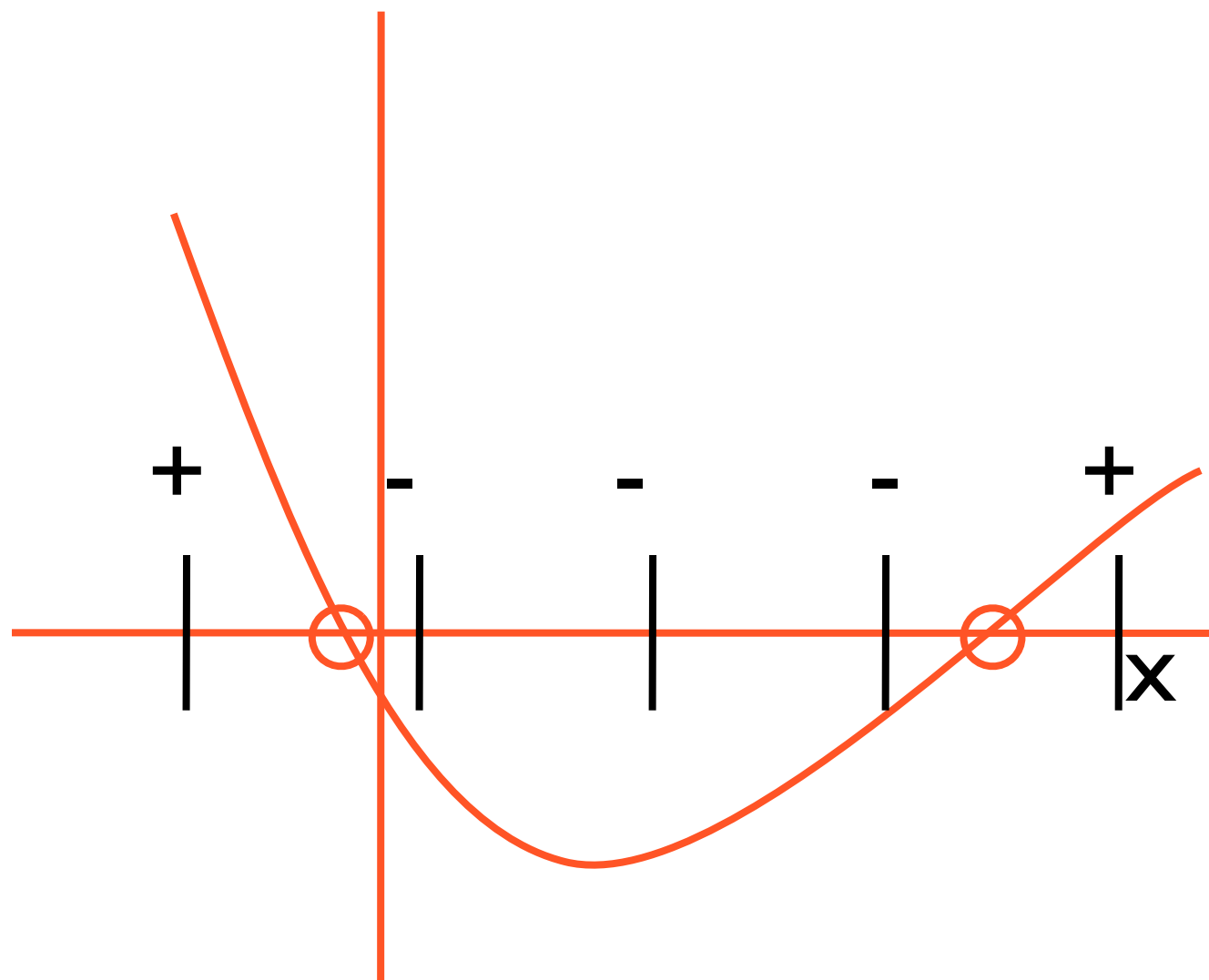
$$f_{\sim}(x) = f(x) - a$$

$a$

$x$



# $f_{\sim}(x)=0$      Caso ID

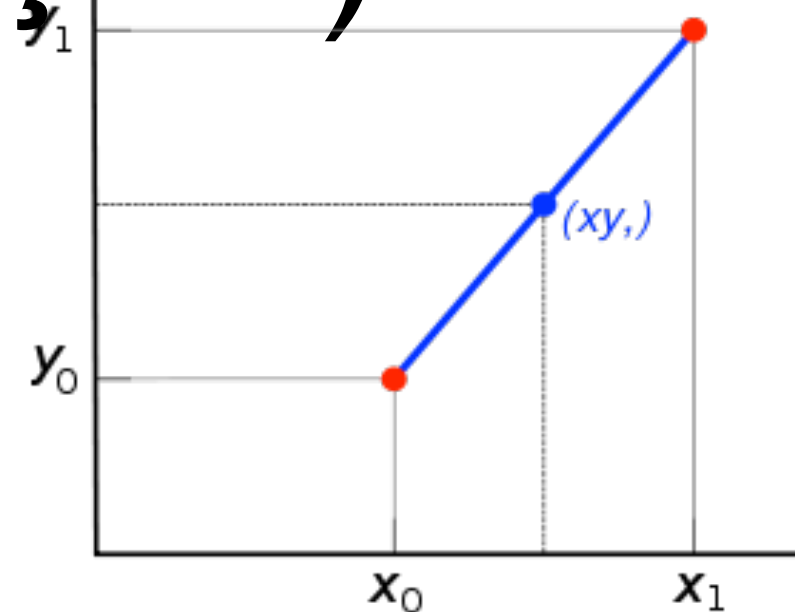
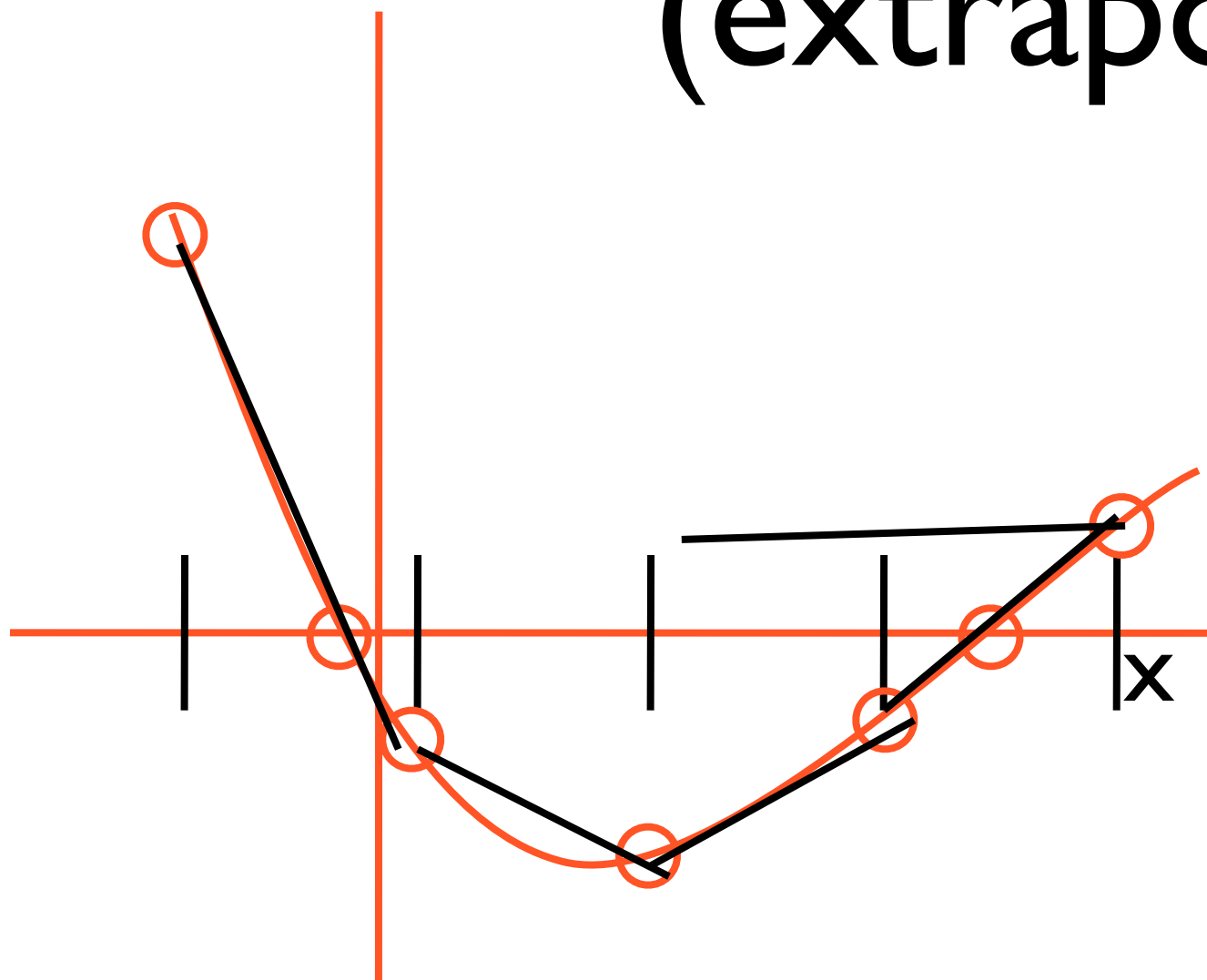


Ambiguidade por falta de dados



# Interpolação (extrapolação?)

$$f_{\sim}(x)=0$$

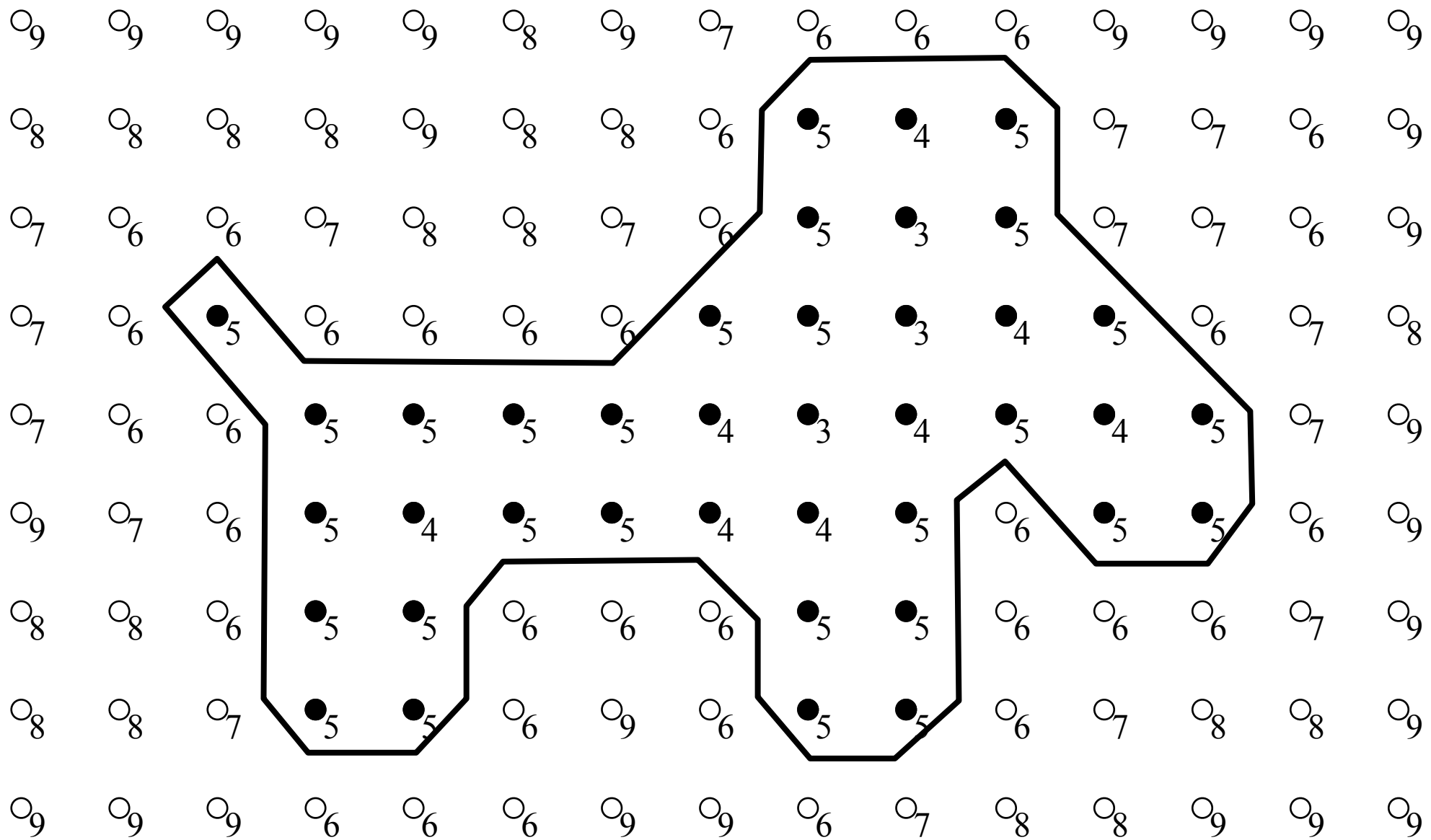


$$\frac{y - y_0}{y_1 - y_0} = \frac{x - x_0}{x_1 - x_0}$$

Localização: mudança de sinal

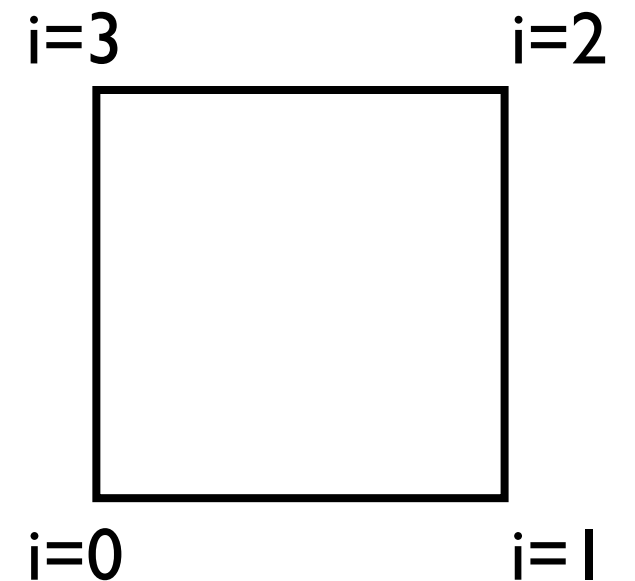
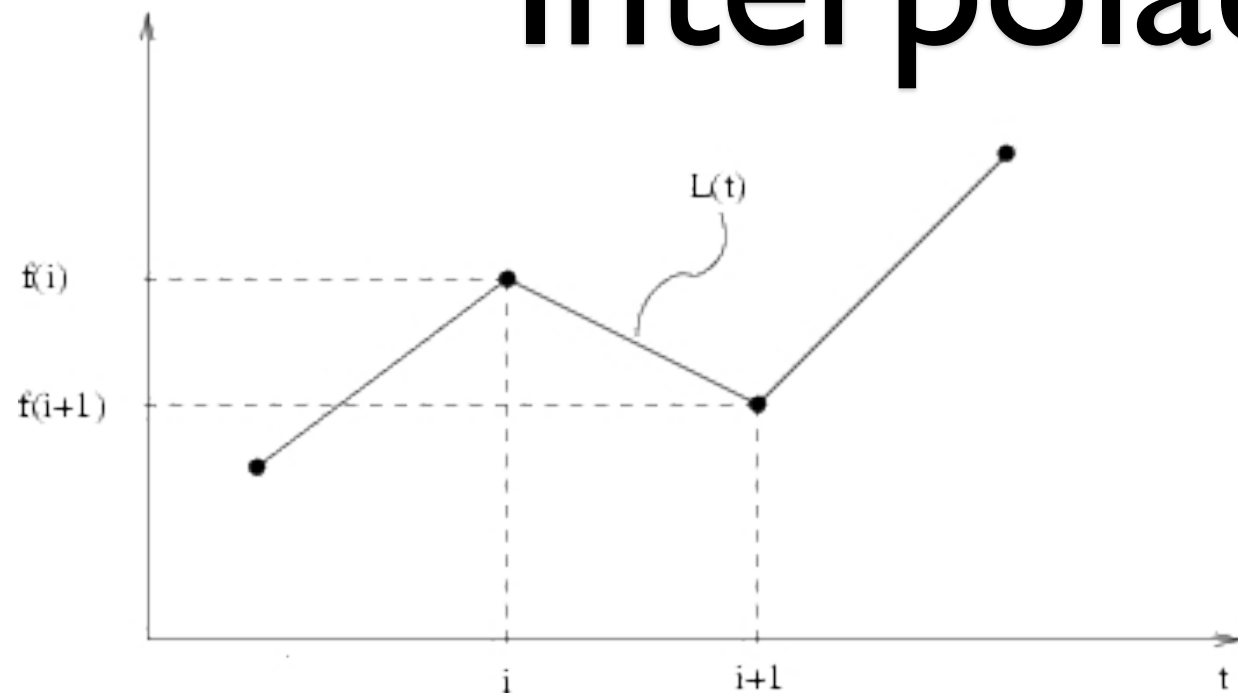
Interpolação: linear

# Caso 2D



# Curvas discretas

# Interpolação bilinear



$$f(s,t) = (1-s) \cdot (1-t) \cdot g[0,0] +$$

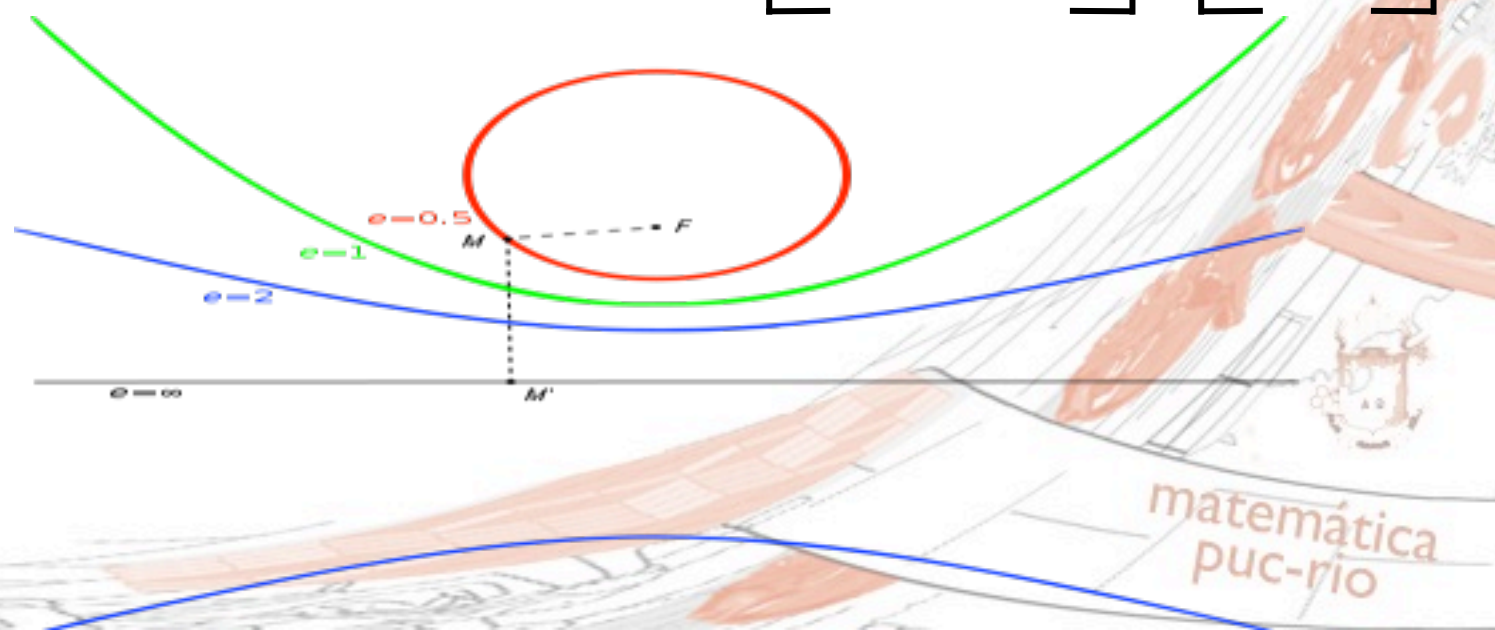
$$s \cdot (1-t) \cdot g[1,0] +$$

$$(1-s) \cdot t \cdot g[0,1] +$$

$$s \cdot t \cdot g[1,1]$$

cônicas!

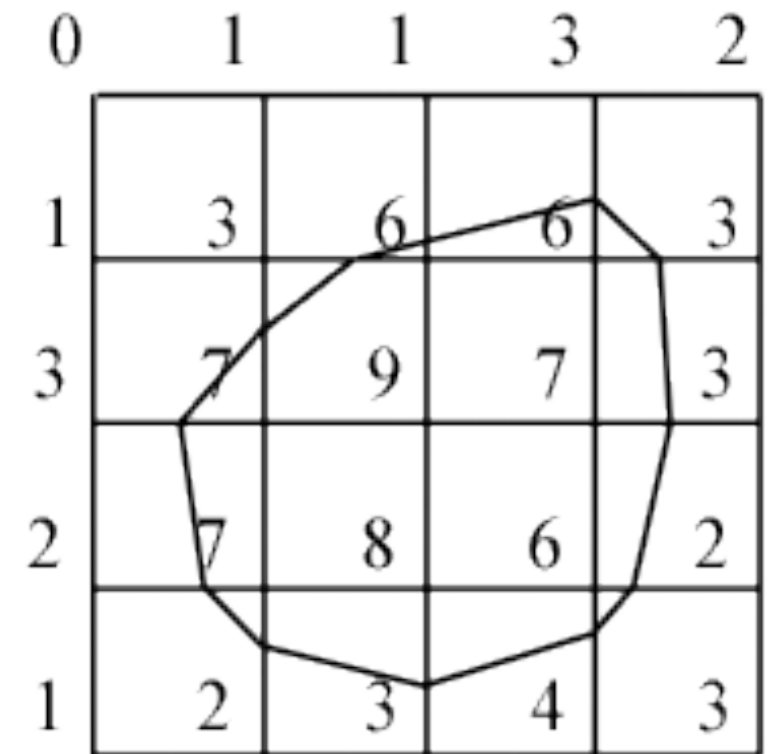
$$f(s,t) = [1-s, s] \begin{bmatrix} v_0 & v_1 \\ v_2 & v_3 \end{bmatrix} \begin{bmatrix} 1-t \\ t \end{bmatrix}$$



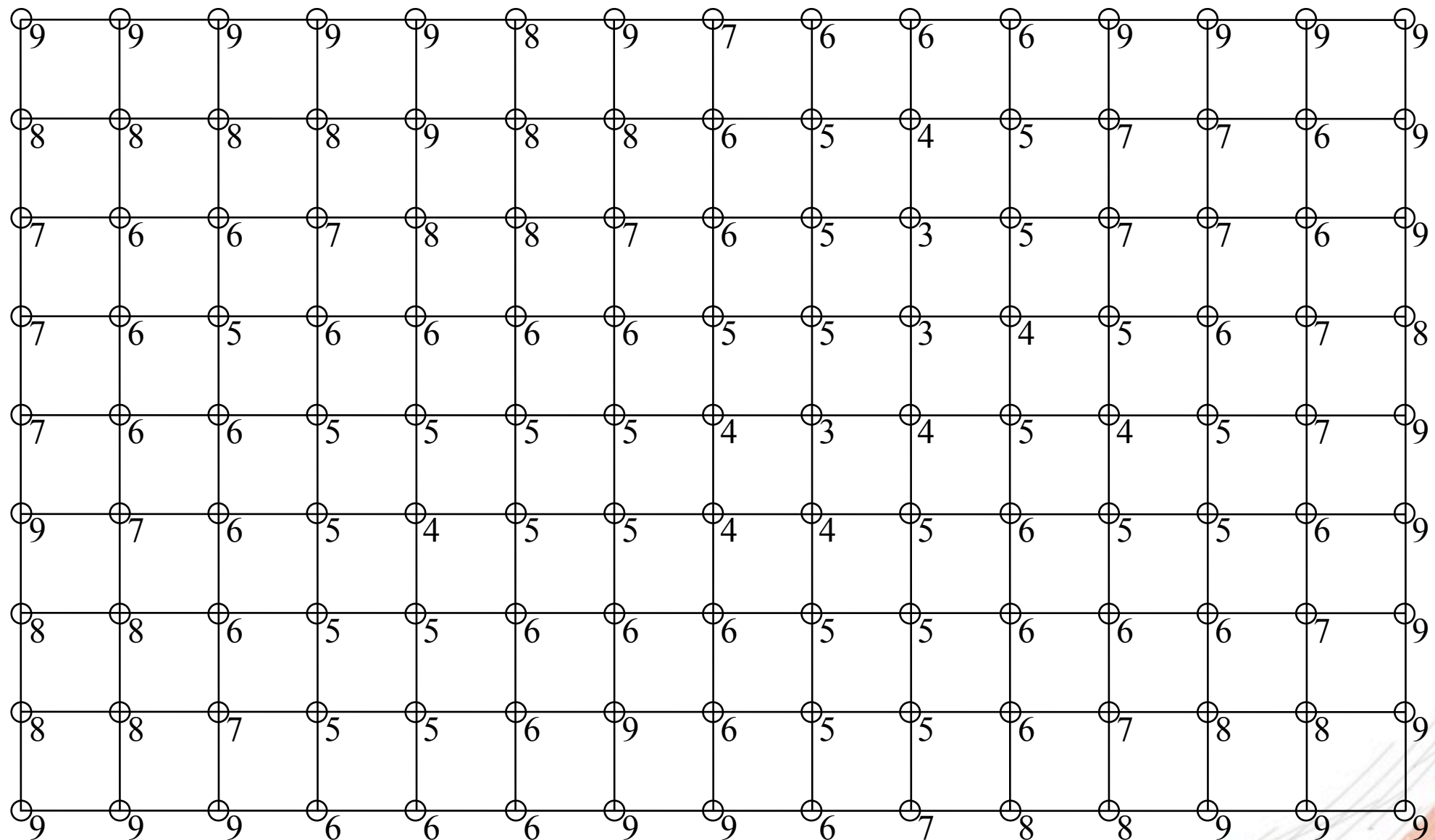


# *Marching Squares*

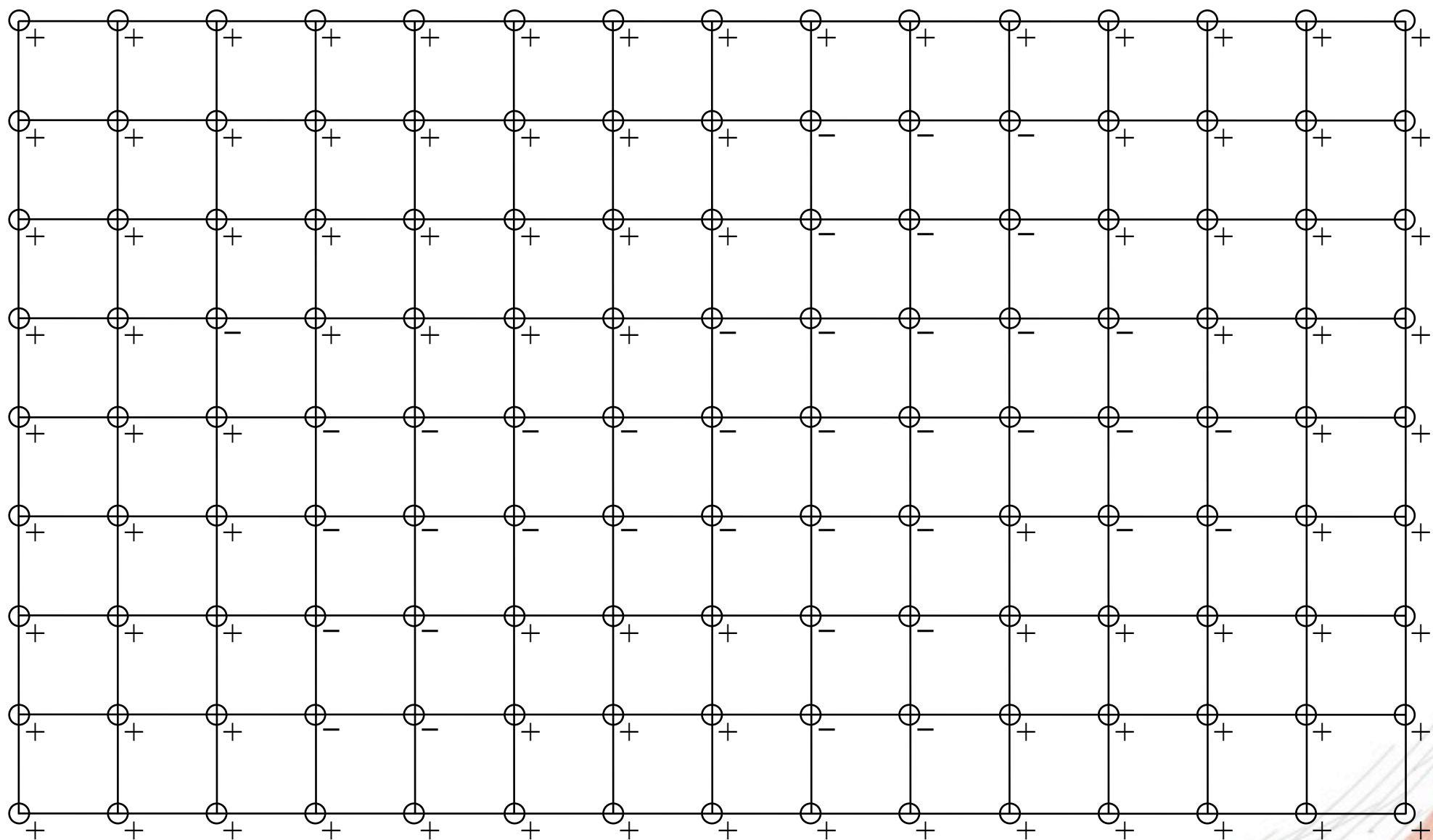
quadrado por quadrado  
tabela de casos para acelerar  
ambigüidades



# Amostras regulares

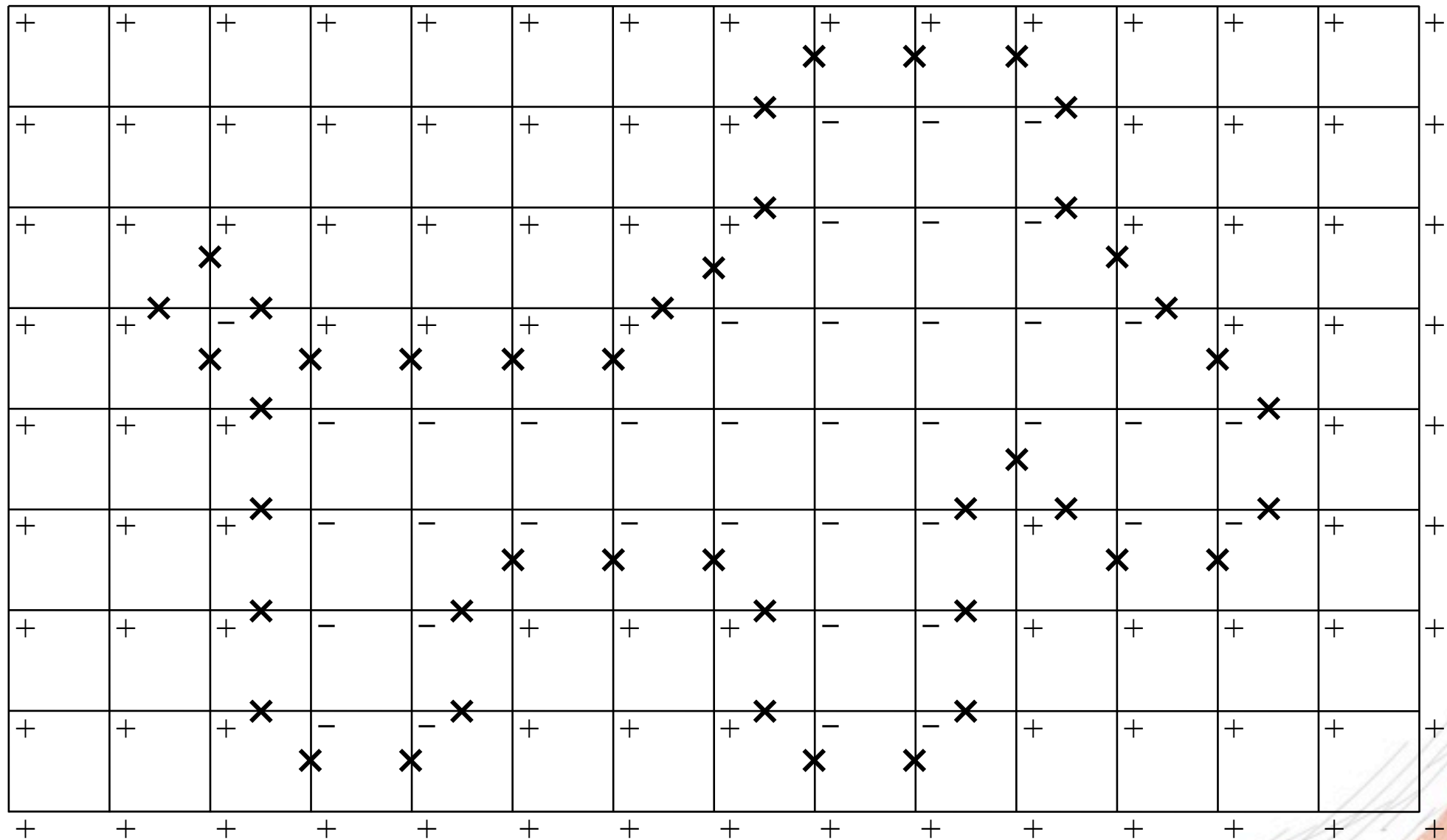


# Limiar

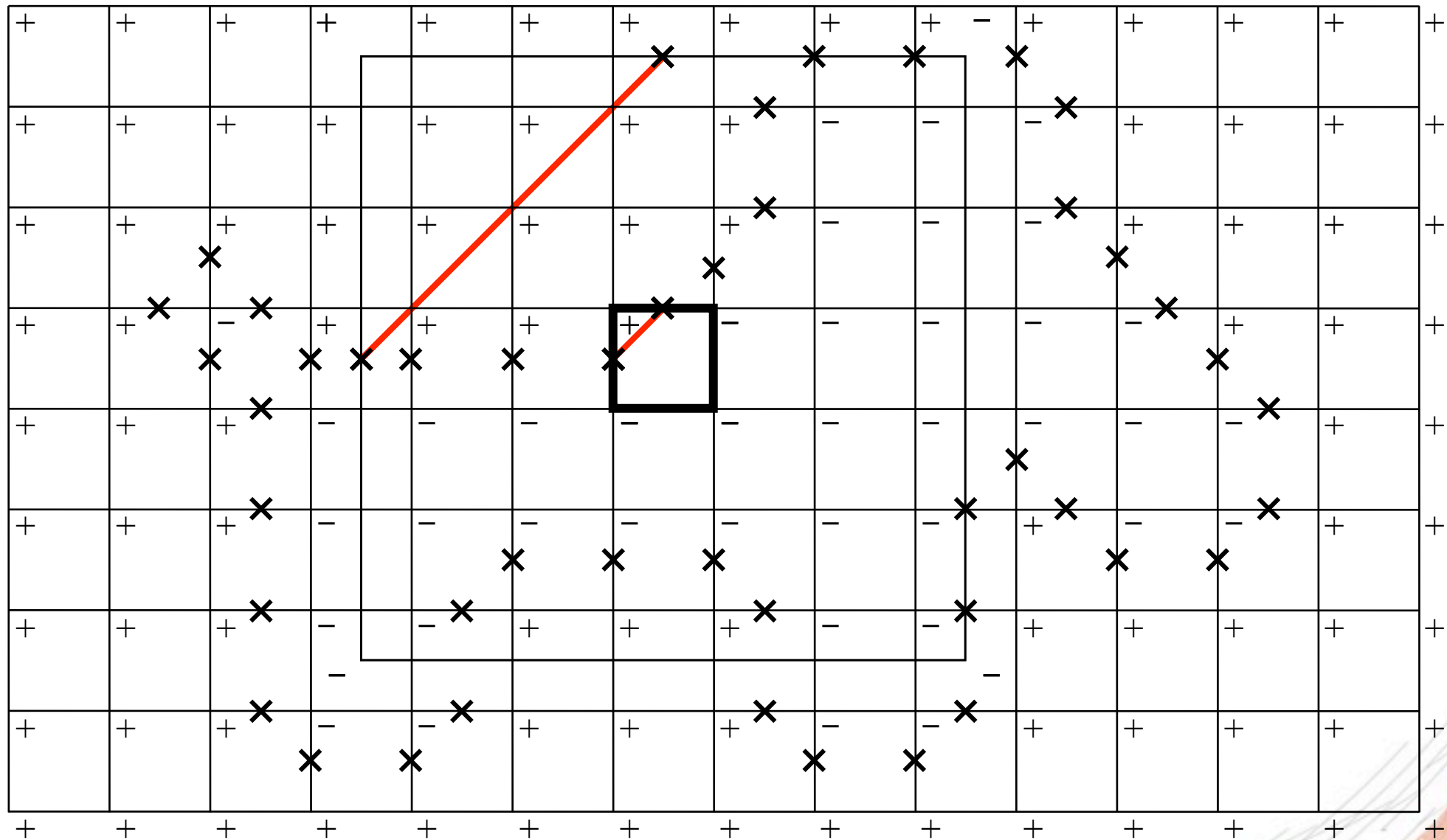




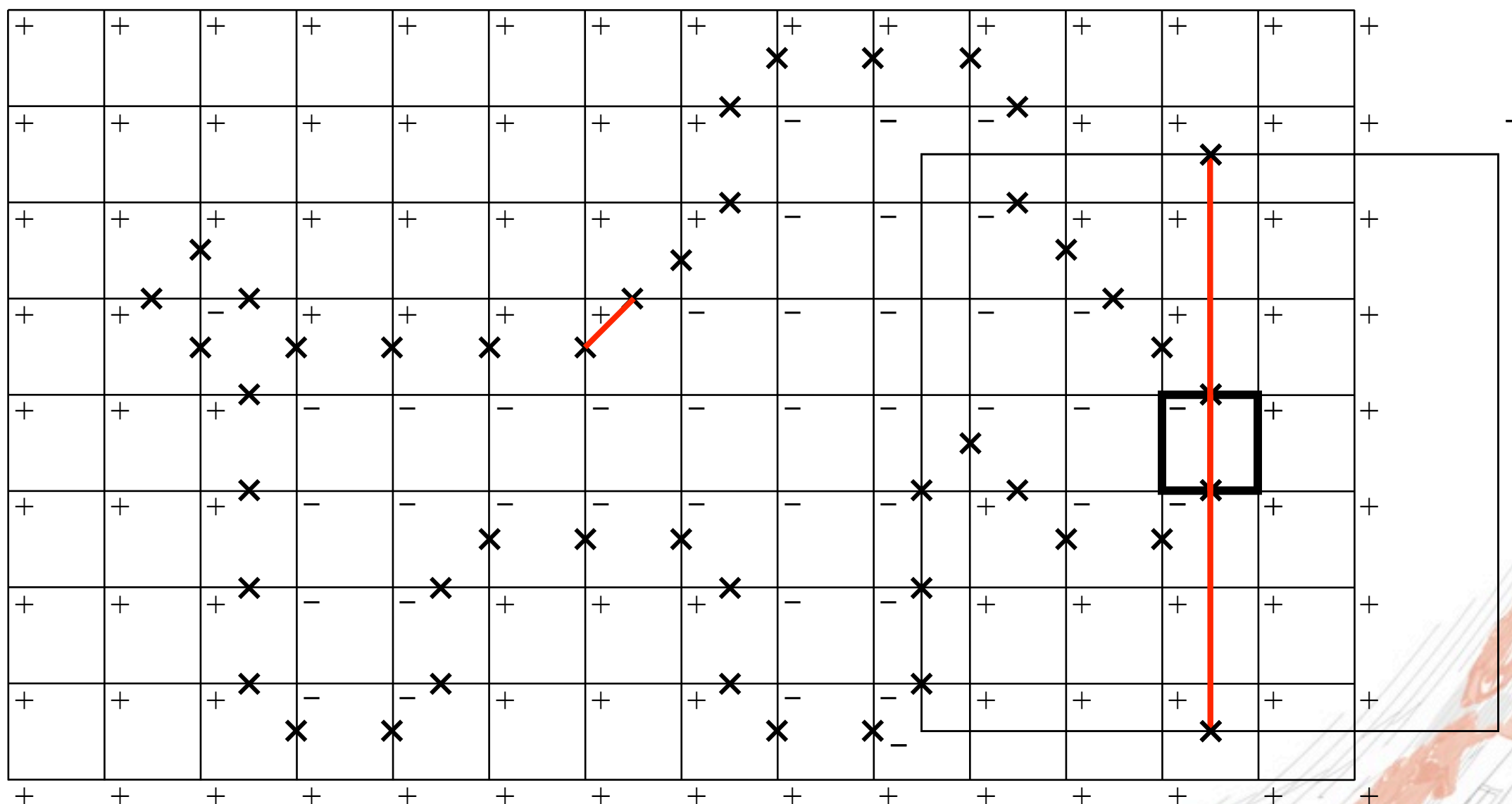
# Calculo dos vértices



# *Marching Squares*

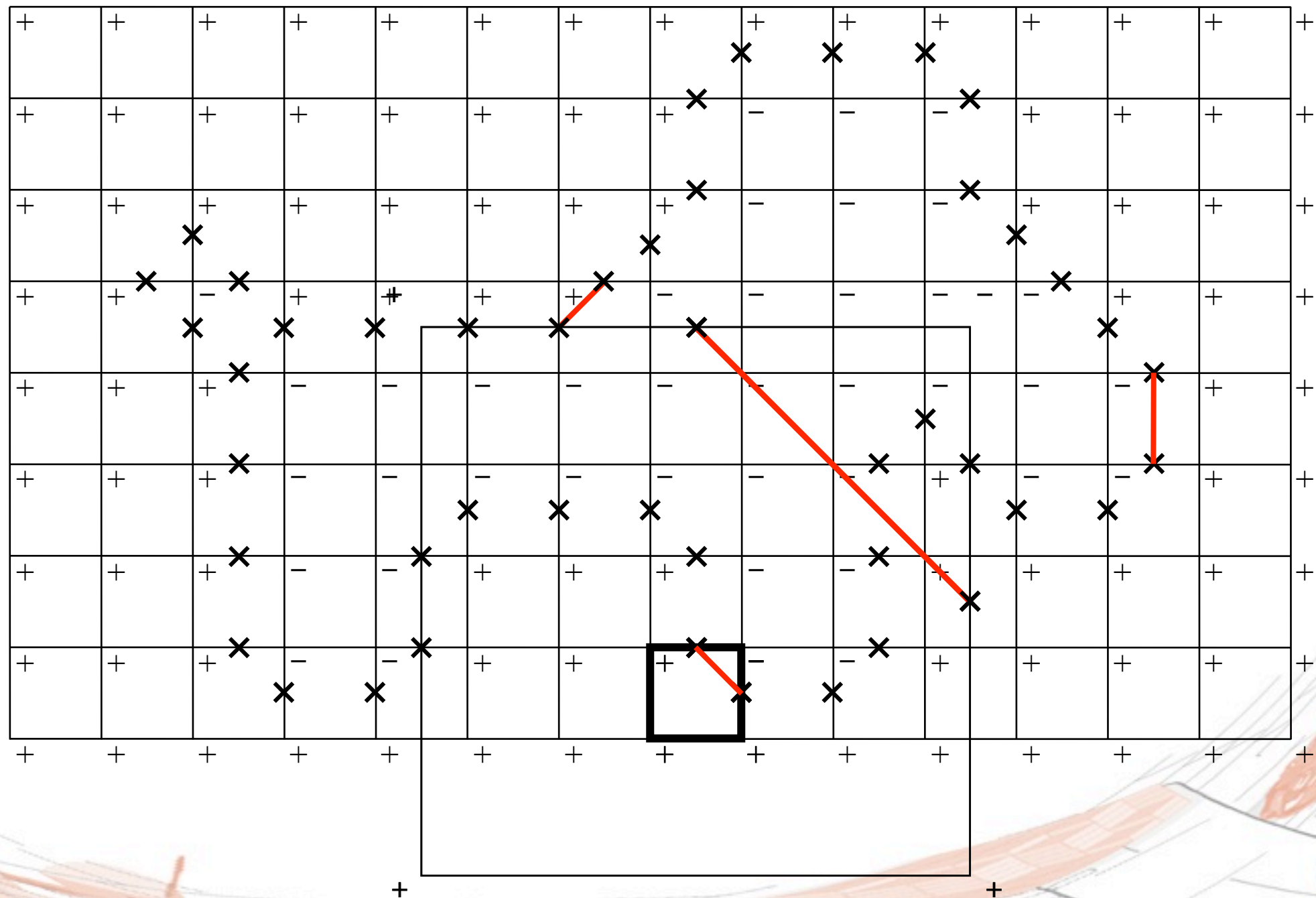


# *Marching Squares*

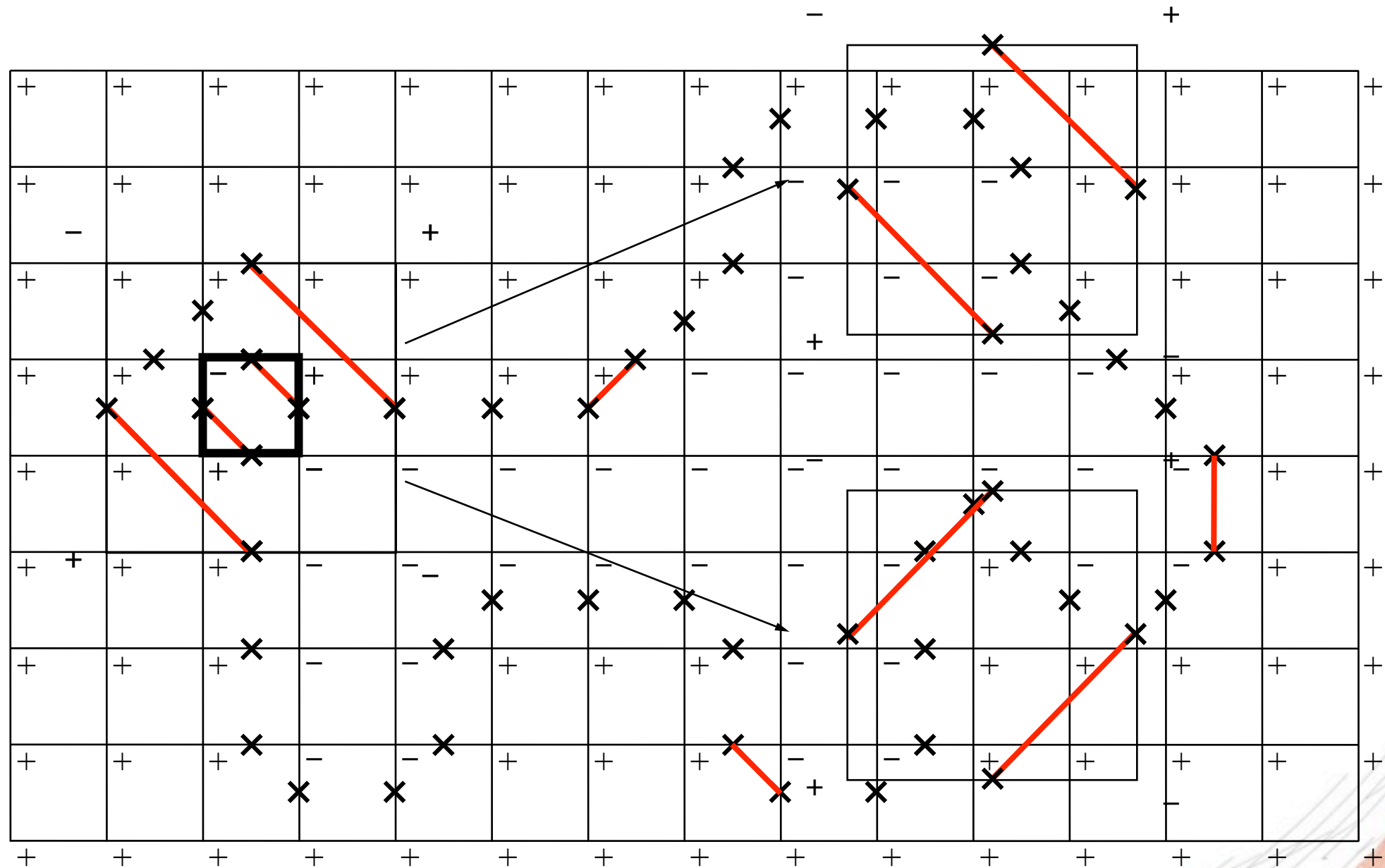




# *Marching Squares*



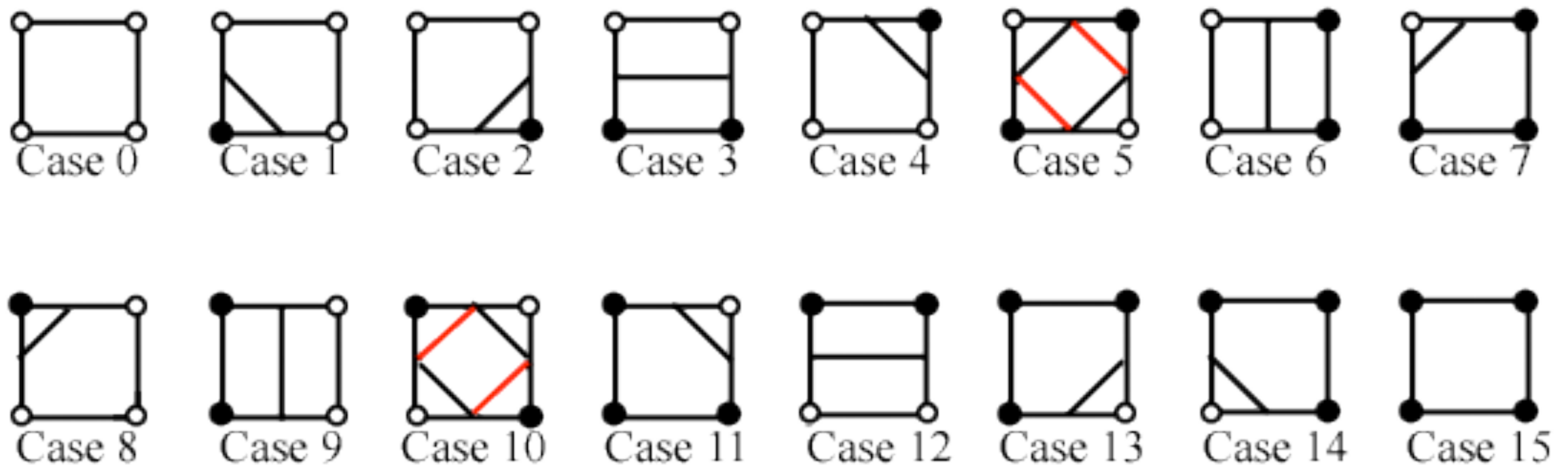
# *Marching Squares*



A 10x10 grid with '+' and '-' signs. A red path of 'x' marks is drawn across the grid. The path starts at (2, 3), goes to (3, 3), (3, 4), (3, 5), (3, 6), (4, 6), (5, 6), (6, 6), (7, 6), (8, 6), (8, 7), (8, 8), (9, 8), (9, 9), (10, 9), (10, 10), (11, 10), (11, 11), (12, 11), (12, 12), (13, 12), (13, 13), (14, 13), (14, 14), (15, 14), (15, 15), (16, 15), (16, 16), (17, 16), (17, 17), (18, 17), (18, 18), (19, 18), (19, 19), (20, 19), (20, 20), (21, 20), (21, 21), (22, 21), (22, 22), (23, 22), (23, 23), (24, 23), (24, 24), (25, 24), (25, 25), (26, 25), (26, 26), (27, 26), (27, 27), (28, 27), (28, 28), (29, 28), (29, 29), (30, 29), (30, 30), (31, 30), (31, 31), (32, 31), (32, 32), (33, 32), (33, 33), (34, 33), (34, 34), (35, 34), (35, 35), (36, 35), (36, 36), (37, 36), (37, 37), (38, 37), (38, 38), (39, 38), (39, 39), (40, 39), (40, 40), (41, 40), (41, 41), (42, 41), (42, 42), (43, 42), (43, 43), (44, 43), (44, 44), (45, 44), (45, 45), (46, 45), (46, 46), (47, 46), (47, 47), (48, 47), (48, 48), (49, 48), (49, 49), (50, 49), (50, 50), (51, 49), (51, 50), (52, 50), (52, 51), (53, 51), (53, 52), (54, 52), (54, 53), (55, 53), (55, 54), (56, 54), (56, 55), (57, 55), (57, 56), (58, 56), (58, 57), (59, 57), (59, 58), (60, 58), (60, 59), (61, 59), (61, 60), (62, 60), (62, 61), (63, 61), (63, 62), (64, 62), (64, 63), (65, 63), (65, 64), (66, 64), (66, 65), (67, 65), (67, 66), (68, 66), (68, 67), (69, 67), (69, 68), (70, 68), (70, 69), (71, 69), (71, 70), (72, 70), (72, 71), (73, 71), (73, 72), (74, 72), (74, 73), (75, 73), (75, 74), (76, 74), (76, 75), (77, 75), (77, 76), (78, 76), (78, 77), (79, 77), (79, 78), (80, 78), (80, 79), (81, 79), (81, 80), (82, 80), (82, 81), (83, 81), (83, 82), (84, 82), (84, 83), (85, 83), (85, 84), (86, 84), (86, 85), (87, 85), (87, 86), (88, 86), (88, 87), (89, 87), (89, 88), (90, 88), (90, 89), (91, 89), (91, 90), (92, 90), (92, 91), (93, 91), (93, 92), (94, 92), (94, 93), (95, 93), (95, 94), (96, 94), (96, 95), (97, 95), (97, 96), (98, 96), (98, 97), (99, 97), (99, 98), (100, 98), (100, 99), (101, 99), (101, 100), (102, 100), (102, 101), (103, 101), (103, 102), (104, 102), (104, 103), (105, 103), (105, 104), (106, 104), (106, 105), (107, 105), (107, 106), (108, 106), (108, 107), (109, 107), (109, 108), (110, 108), (110, 109), (111, 109), (111, 110), (112, 110), (112, 111), (113, 111), (113, 112), (114, 112), (114, 113), (115, 113), (115, 114), (116, 114), (116, 115), (117, 115), (117, 116), (118, 116), (118, 117), (119, 117), (119, 118), (120, 118), (120, 119), (121, 119), (121, 120), (122, 120), (122, 121), (123, 121), (123, 122), (124, 122), (124, 123), (125, 123), (125, 124), (126, 124), (126, 125), (127, 125), (127, 126), (128, 126), (128, 127), (129, 127), (129, 128), (130, 128), (130, 129), (131, 129), (131, 130), (132, 130), (132, 131), (133, 131), (133, 132), (134, 132), (134, 133), (135, 133), (135, 134), (136, 134), (136, 135), (137, 135), (137, 136), (138, 136), (138, 137), (139, 137), (139, 138), (140, 138), (140, 139), (141, 139), (141, 140), (142, 140), (142, 141), (143, 141), (143, 142), (144, 142), (144, 143), (145, 143), (145, 144), (146, 144), (146, 145), (147, 145), (147, 146), (148, 146), (148, 147), (149, 147), (149, 148), (150, 148), (150, 149), (151, 149), (151, 150), (152, 150), (152, 151), (153, 151), (153, 152), (154, 152), (154, 153), (155, 153), (155, 154), (156, 154), (156, 155), (157, 155), (157, 156), (158, 156), (158, 157), (159, 157), (159, 158), (160, 158), (160, 159), (161, 159), (161, 160), (162, 160), (162, 161), (163, 161), (163, 162), (164, 162), (164, 163), (165, 163), (165, 164), (166, 164), (166, 165), (167, 165), (167, 166), (168, 166), (168, 167), (169, 167), (169, 168), (170, 168), (170, 169), (171, 169), (171, 170), (172, 170), (172, 171), (173, 171), (173, 172), (174, 172), (174, 173), (175, 173), (175, 174), (176, 174), (176, 175), (177, 175), (177, 176), (178, 176), (178, 177), (179, 177), (179, 178), (180, 178), (180, 179), (181, 179), (181, 180), (182, 180), (182, 181), (183, 181), (183, 182), (184, 182), (184, 183), (185, 183), (185, 184), (186, 184), (186, 185), (187, 185), (187, 186), (188, 186), (188, 187), (189, 187), (189, 188), (190, 188), (190, 189), (191, 189), (191, 190), (192, 190), (192, 191), (193, 191), (193, 192), (194, 192), (194, 193), (195, 193), (195, 194), (196, 194), (196, 195), (197, 195), (197, 196), (198, 196), (198, 197), (199, 197), (199, 198), (200, 198), (200, 199), (201, 199), (201, 200), (202, 200), (202, 201), (203, 201), (203, 202), (204, 202), (204, 203), (205, 203), (205, 204), (206, 204), (206, 205), (207, 205), (207, 206), (208, 206), (208, 207), (209, 207), (209, 208), (210, 208), (210, 209), (211, 209), (211, 210), (212, 210), (212, 211), (213, 211), (213, 212), (214, 212), (214, 213), (215, 213), (215, 214), (216, 214), (216, 215), (217, 215), (217, 216), (218, 216), (218, 217), (219, 217), (219, 218), (220, 218), (220, 219), (221, 219), (221, 220), (222, 220), (222, 221), (223, 221), (223, 222), (224, 222), (224, 223), (225, 223), (22

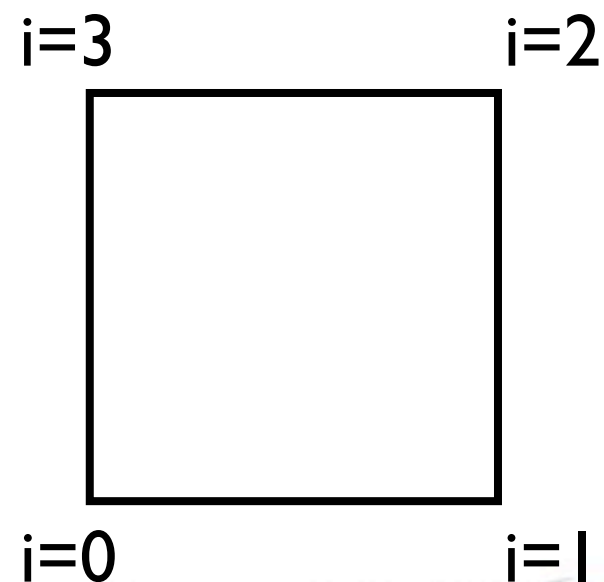


# Tabela de casos



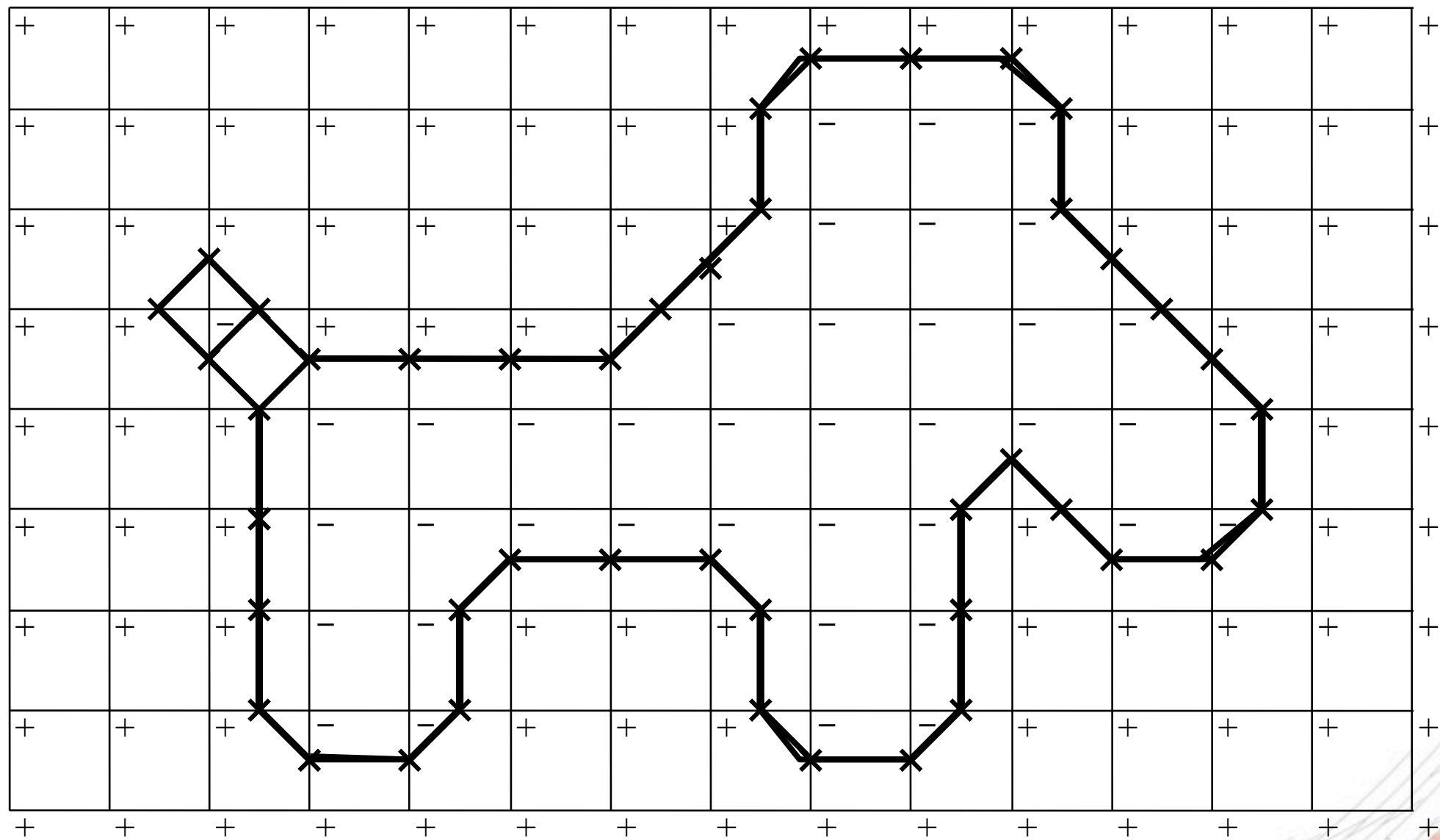
+  $\rightarrow 1 \cdot 2^i$

-  $\rightarrow 0 \cdot 2^i$

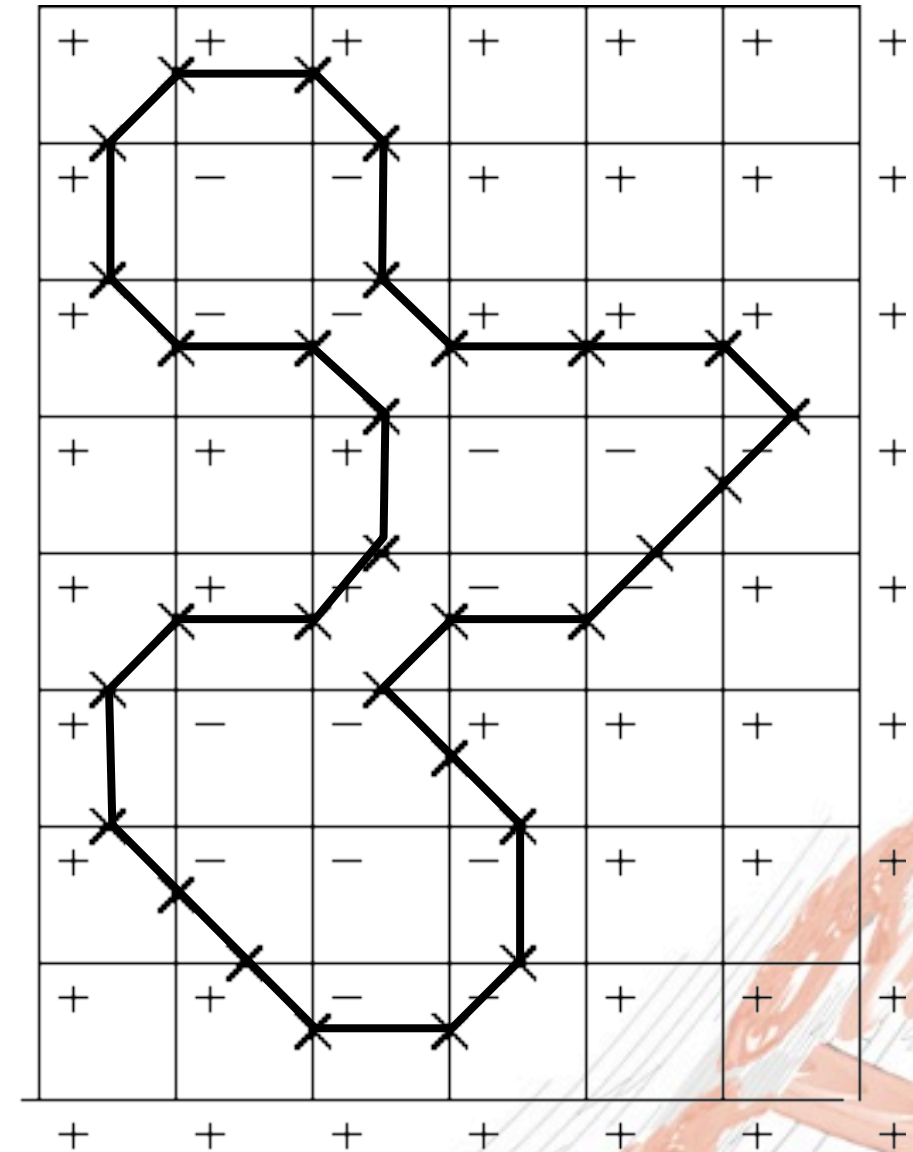
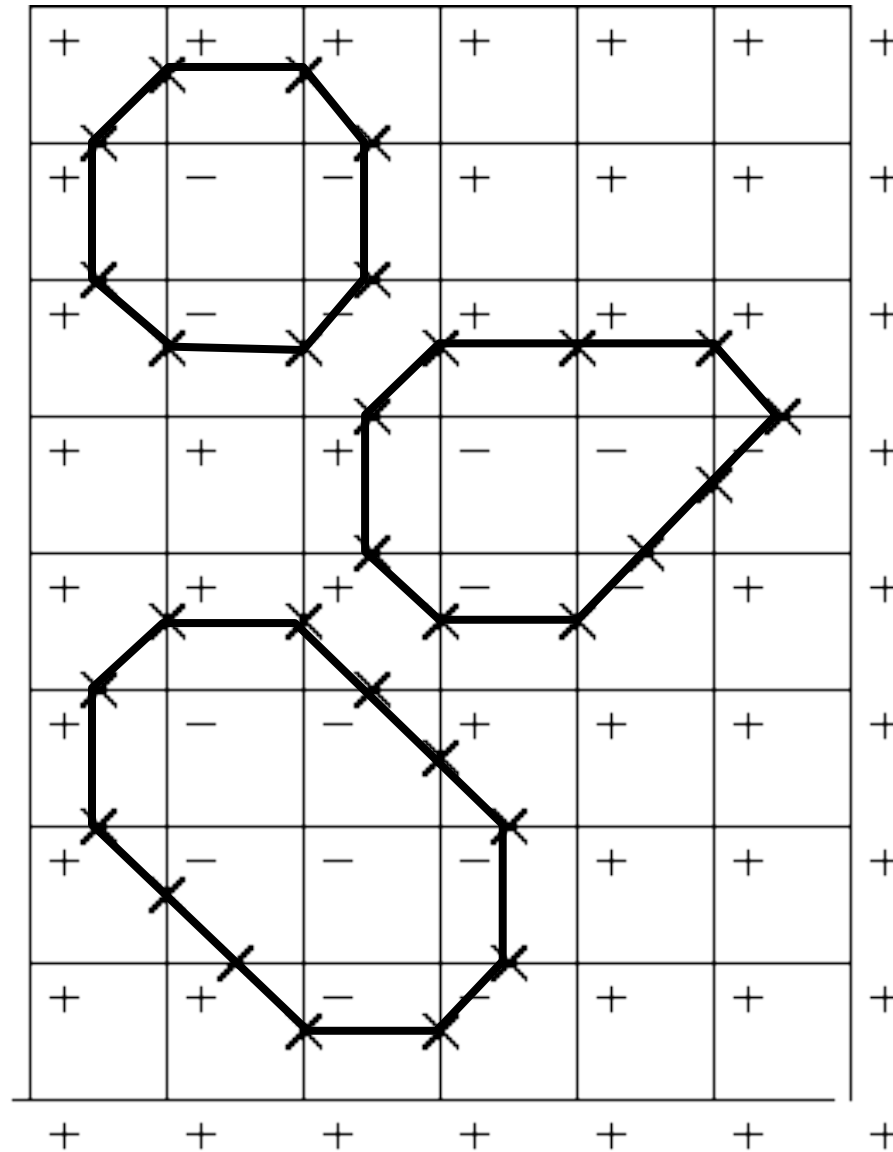
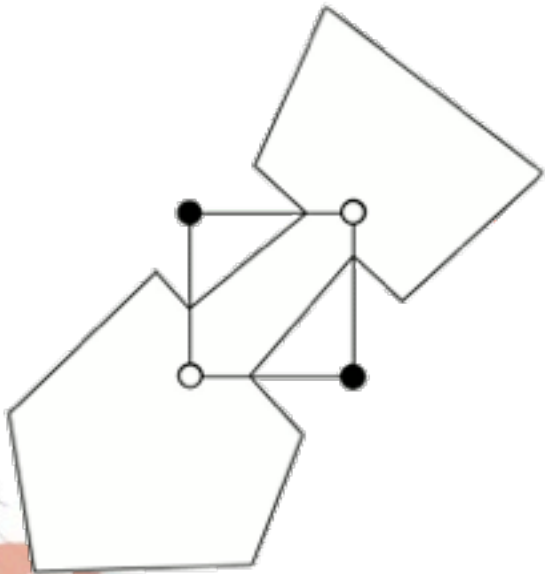
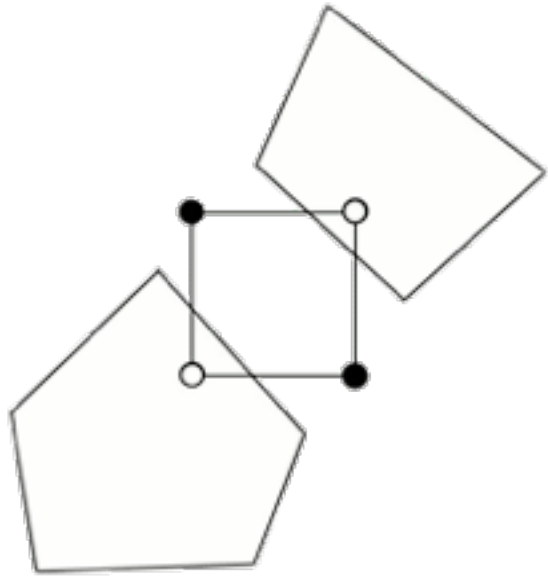


# caso = soma

# Ambigüidades

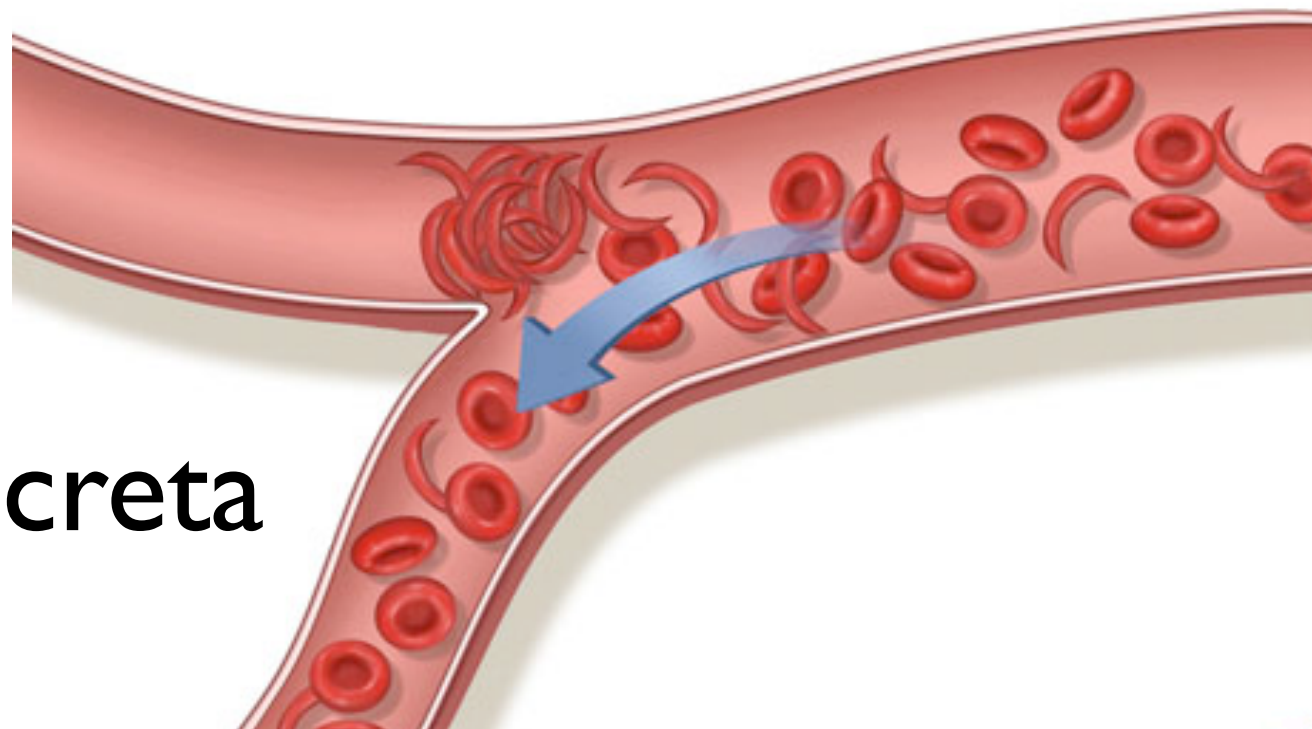


# Ambigüidades de topologia





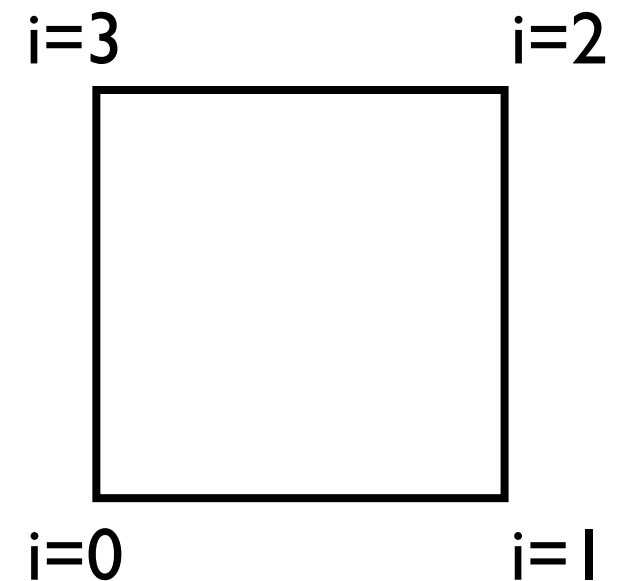
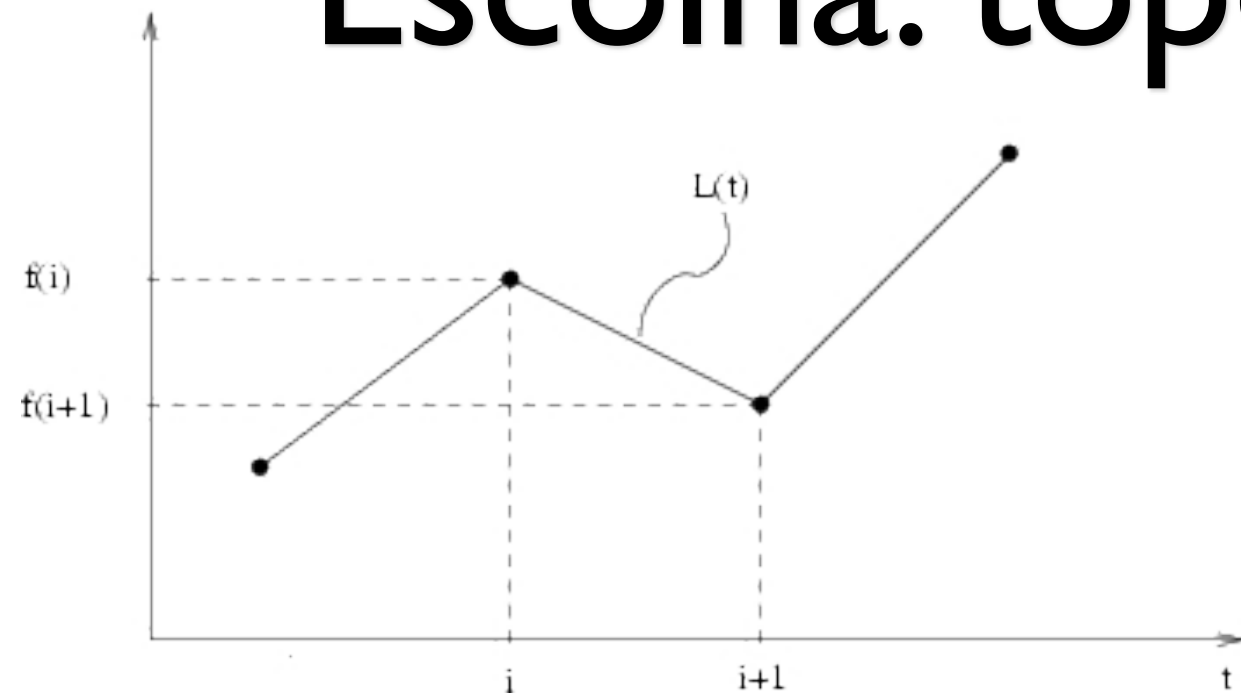
# Catástrofes



Topologia discreta



# Escolha: topologia bilinear



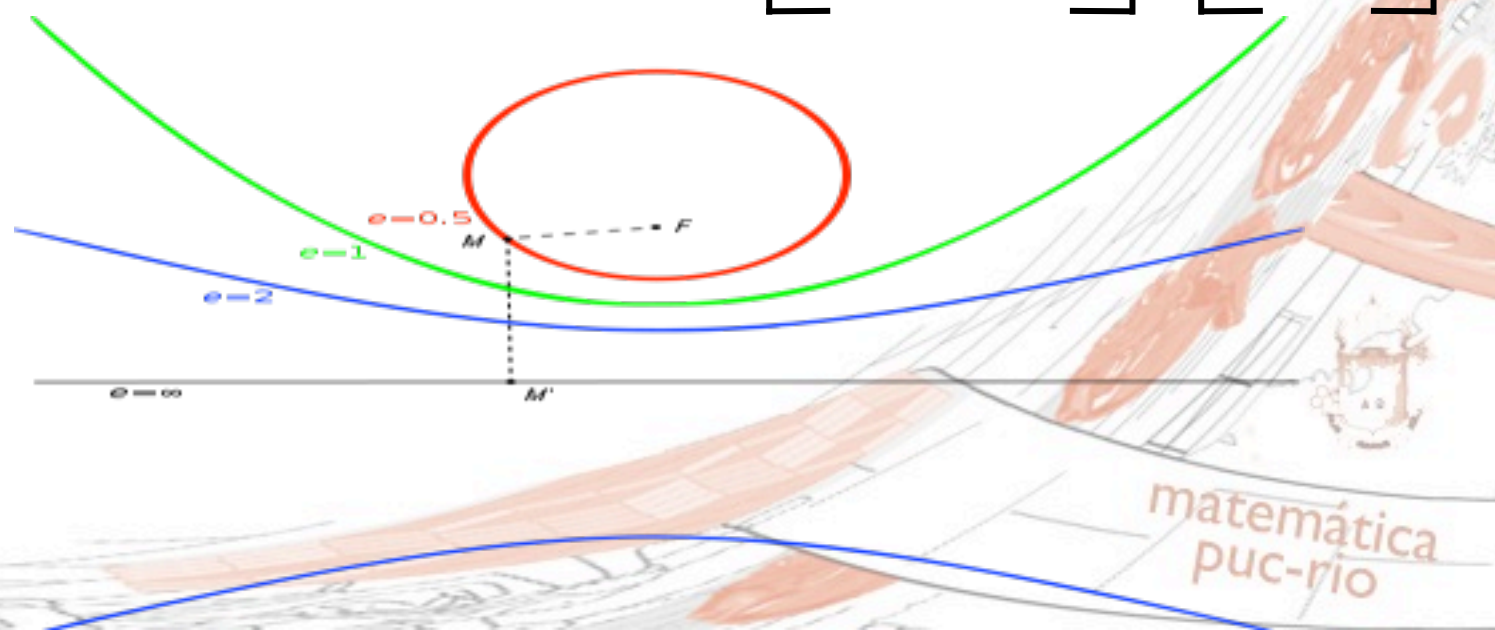
$$f(s,t) = (1-s) \cdot (1-t) \cdot g[0,0] +$$

$$s \cdot (1-t) \cdot g[1,0] +$$

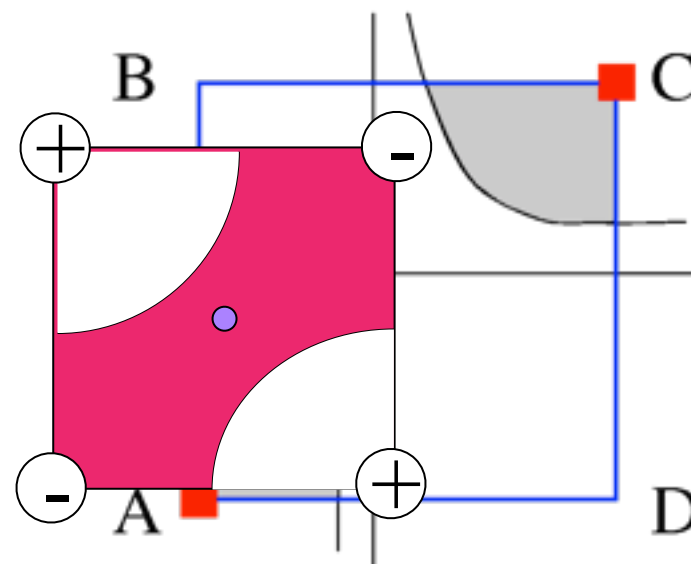
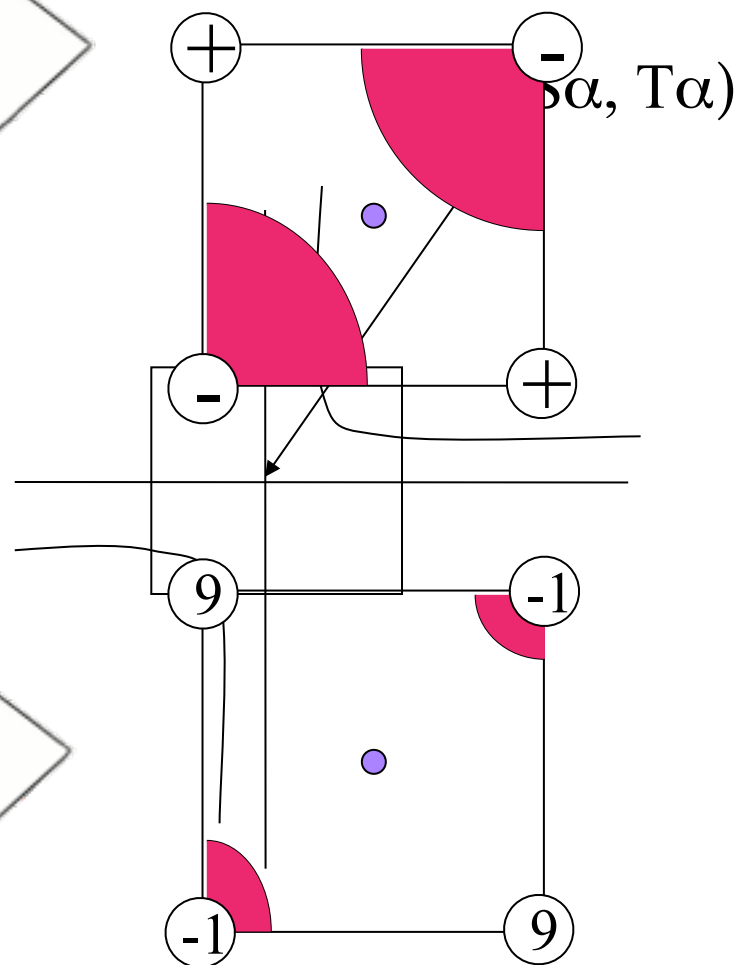
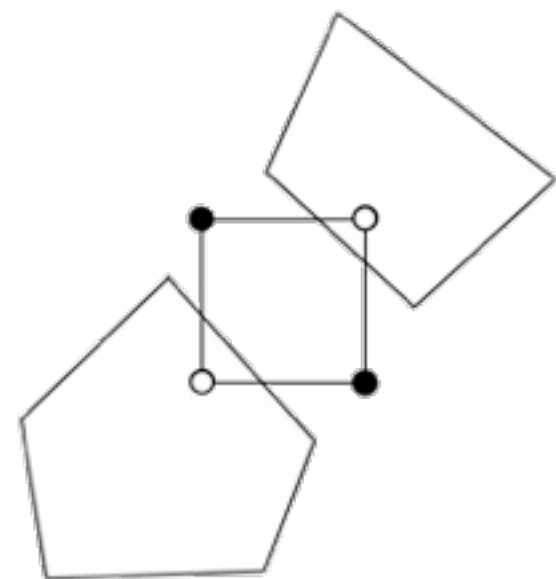
$$(1-s) \cdot t \cdot g[0,1] +$$

$$s \cdot t \cdot g[1,1] .$$

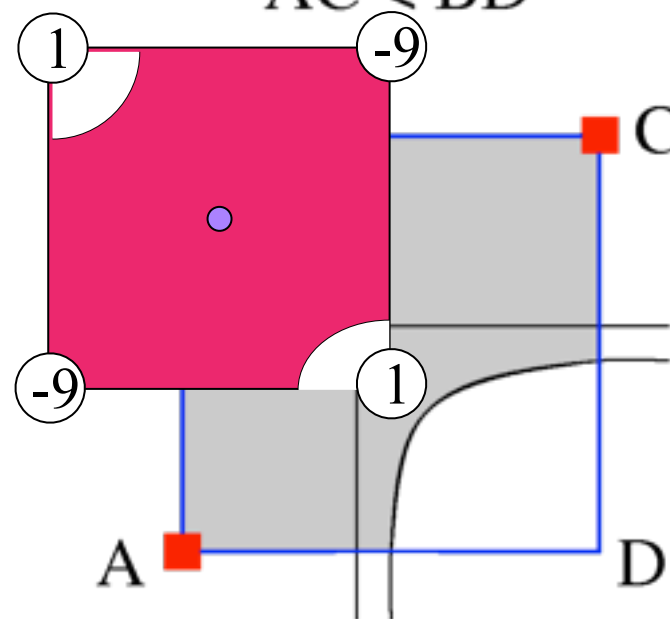
$$f(s,t) = [1-s, s] \begin{bmatrix} v_0 & v_1 \\ v_2 & v_3 \end{bmatrix} \begin{bmatrix} 1-t \\ t \end{bmatrix}$$



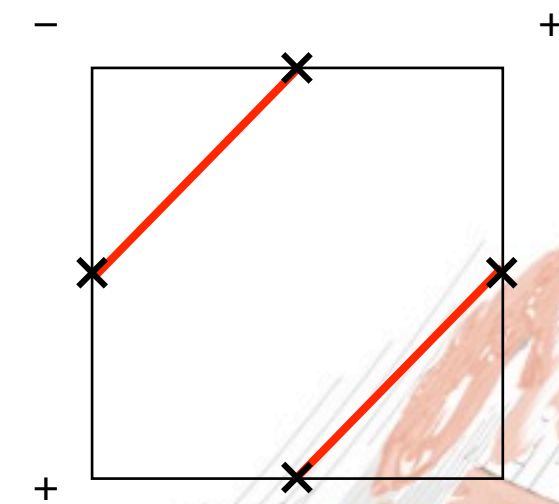
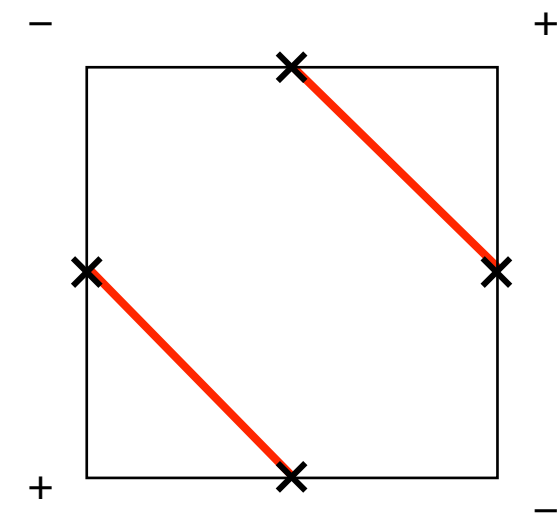
# Resolução de ambigüidades



$AC < BD$

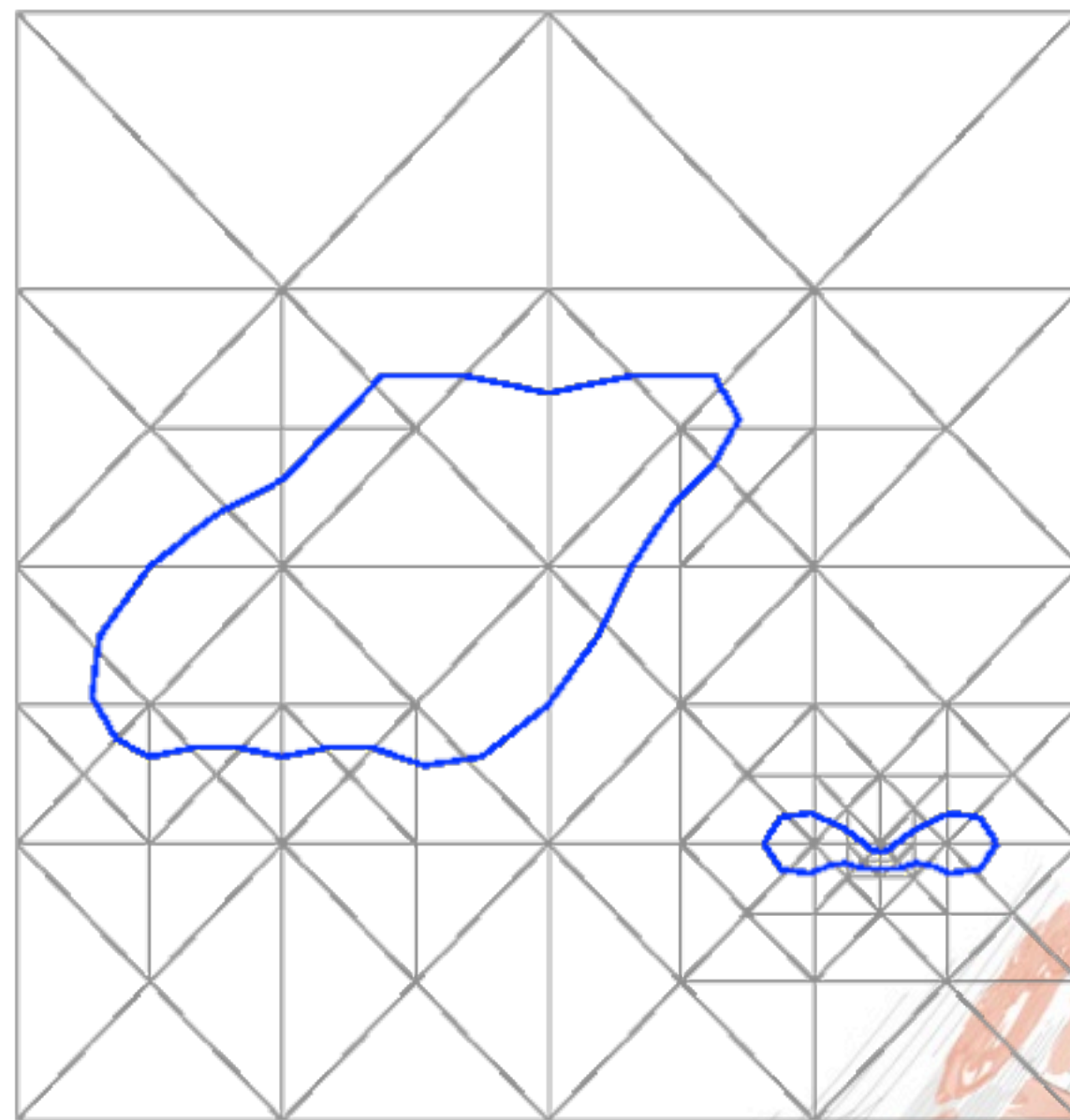
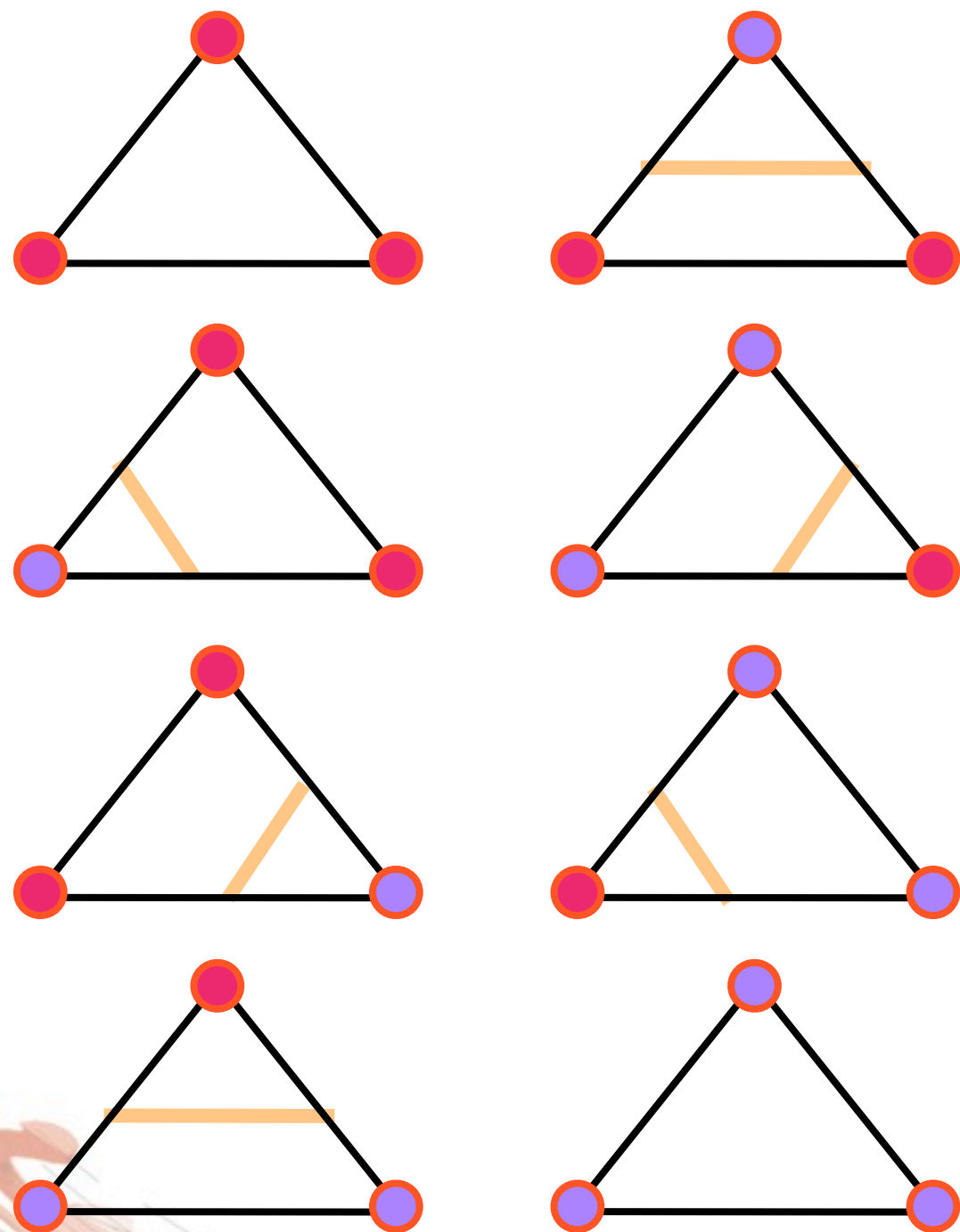


$AC > BD$



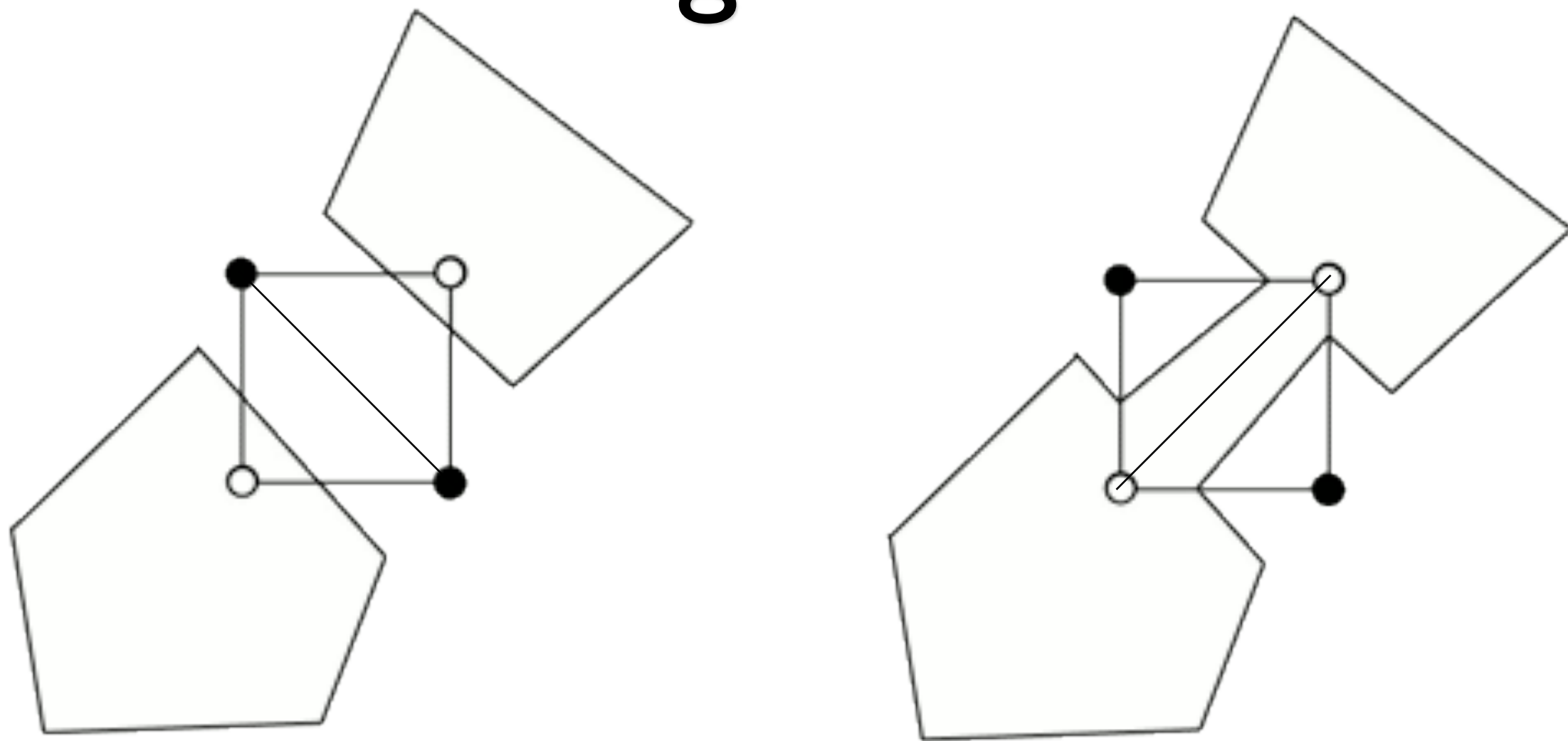


# *Marching Triangles*



2 casos distintos

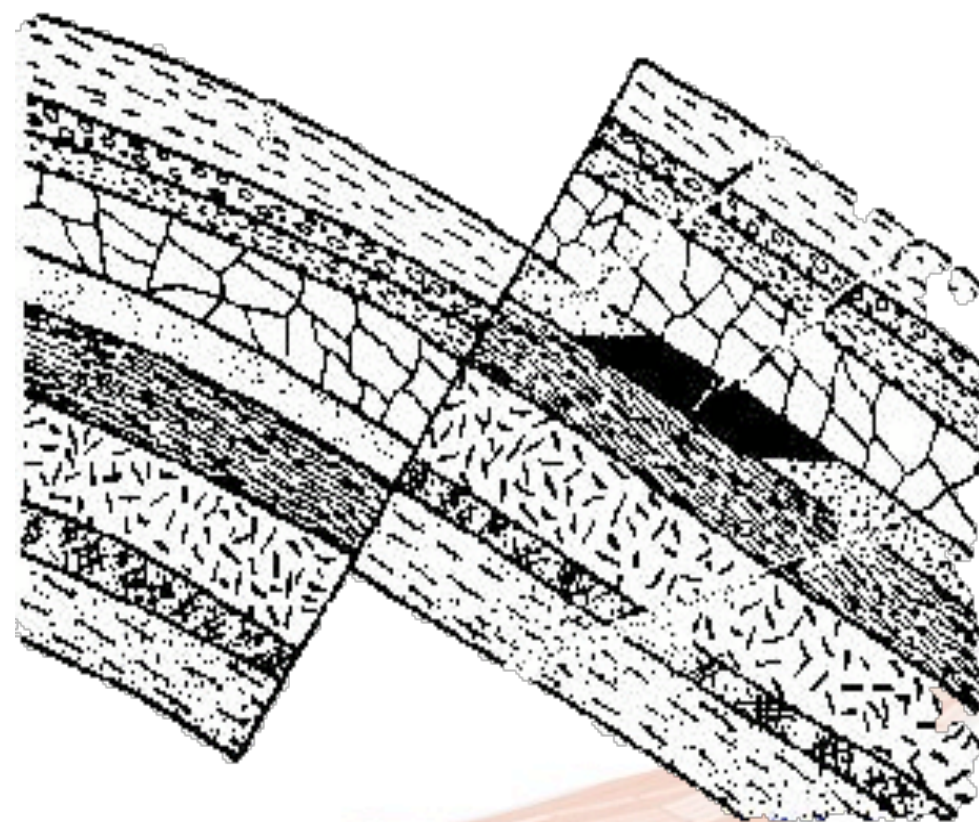
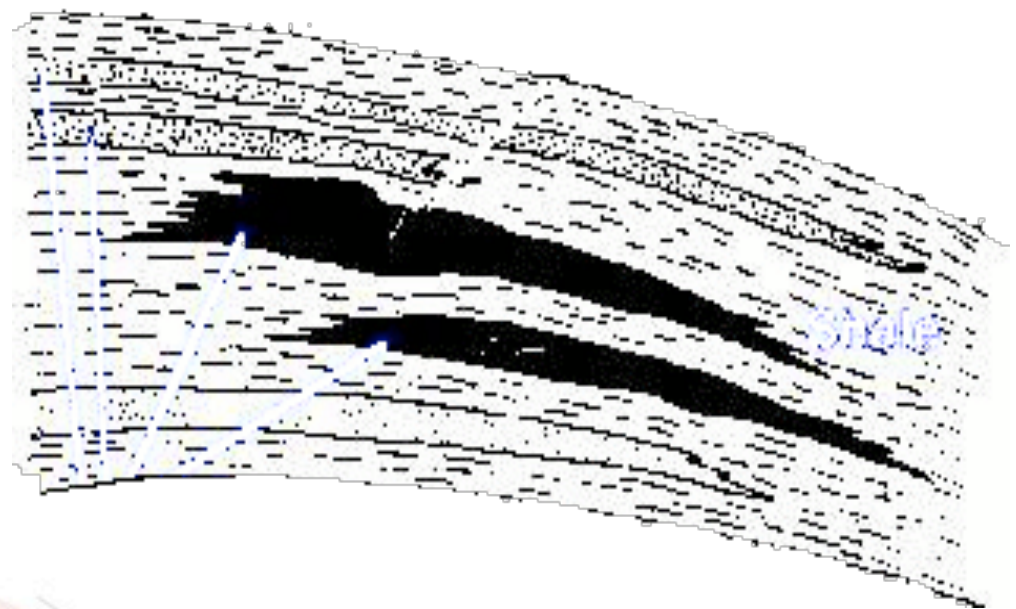
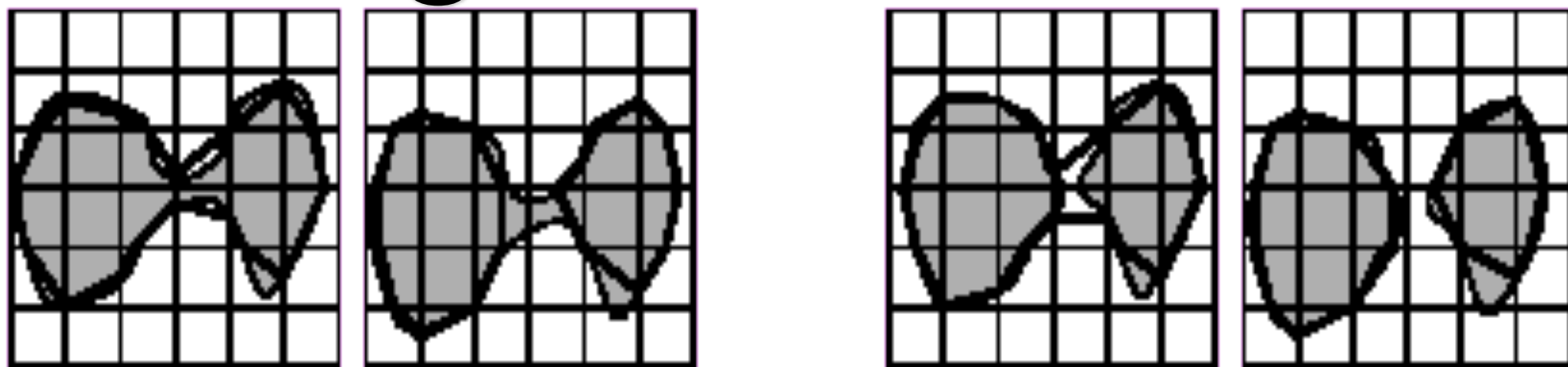
# Ambigüidades?



depende da triangulação,  
não do dado!

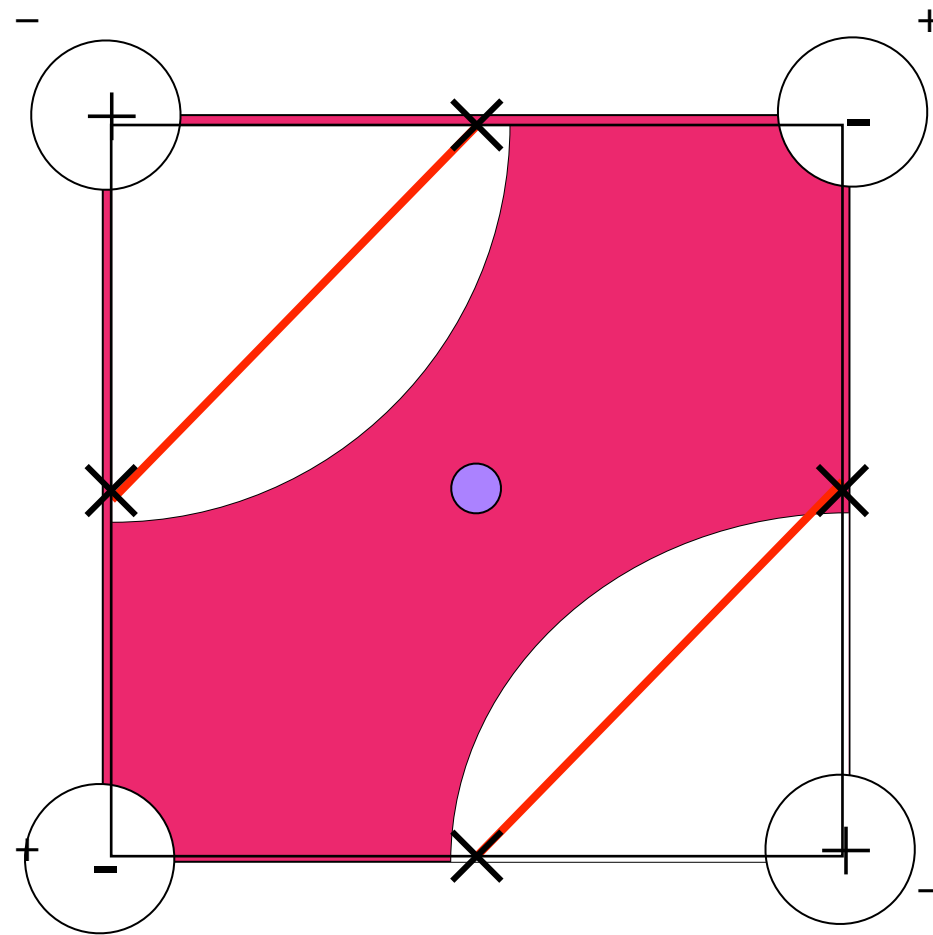


# Ambigüidades de resolução



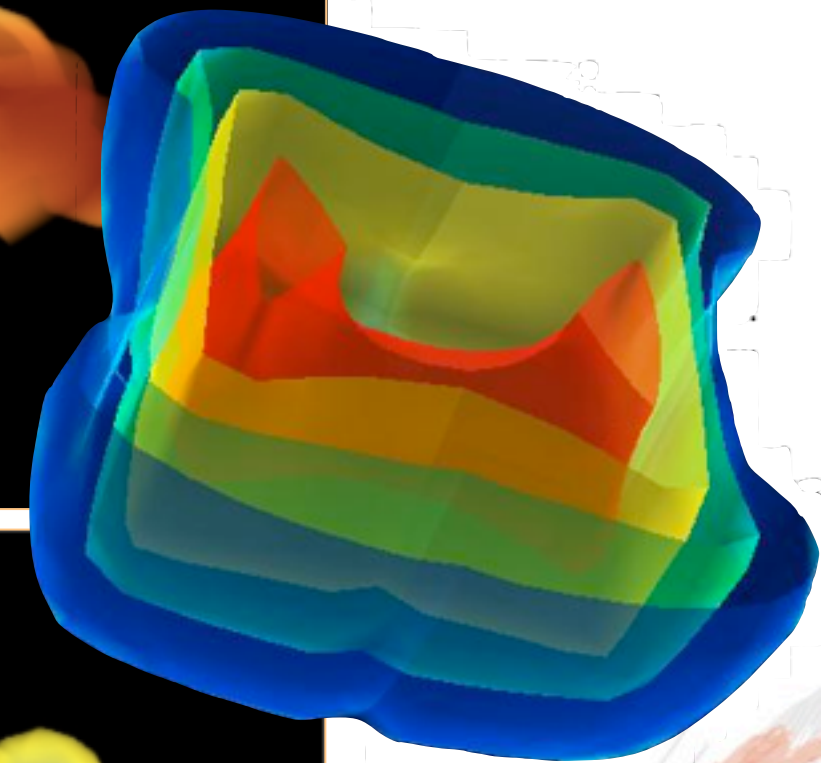
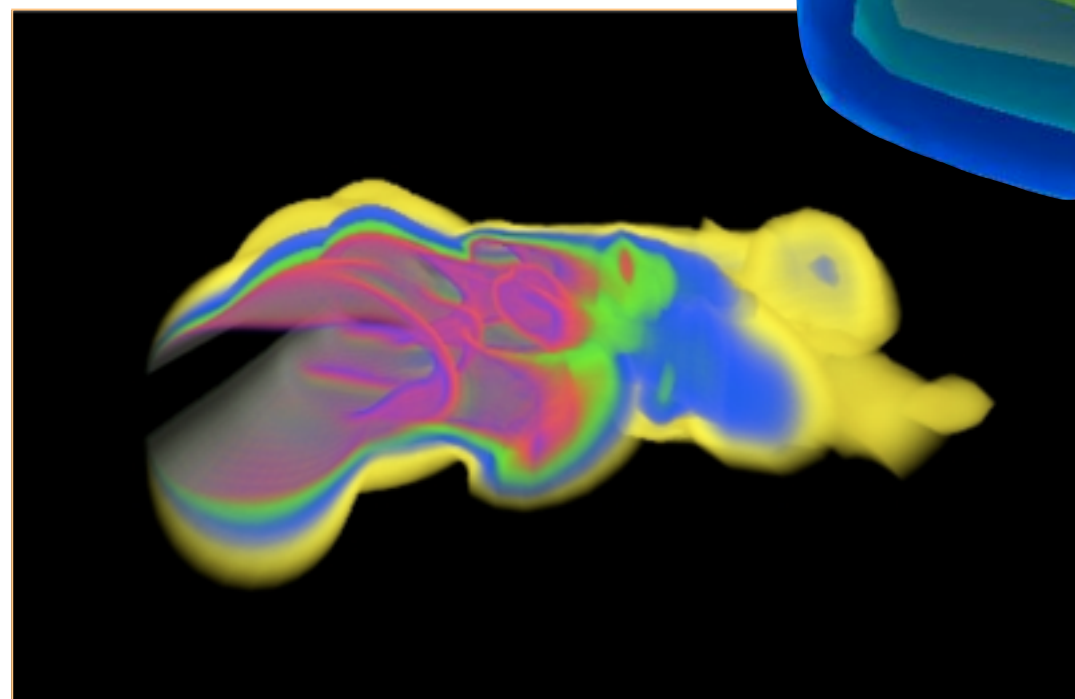
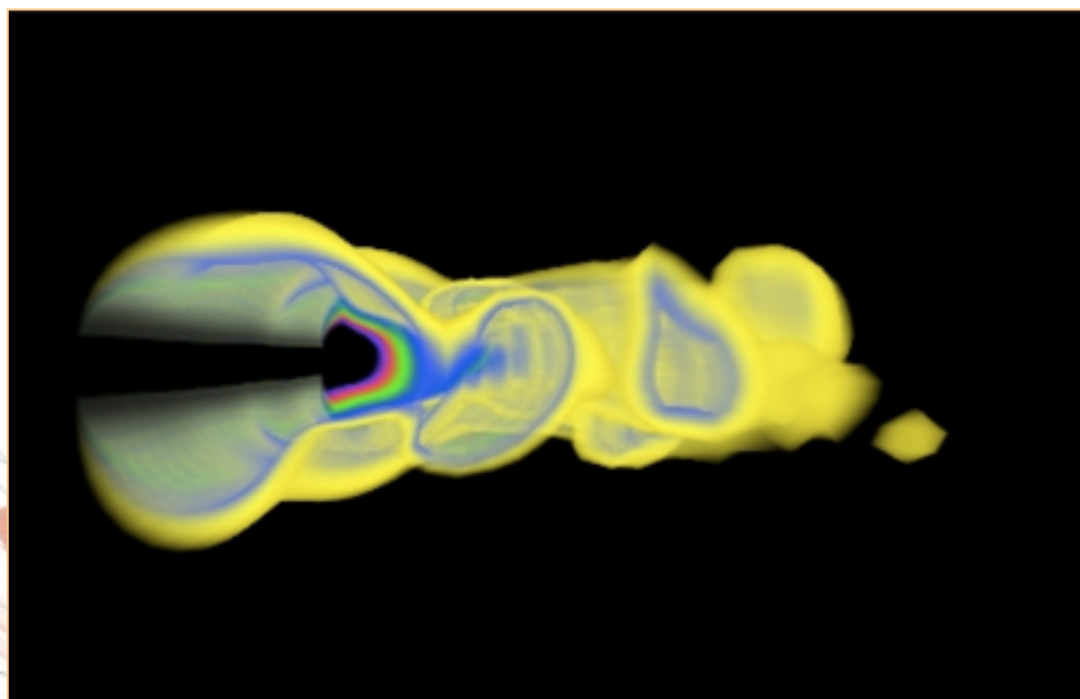
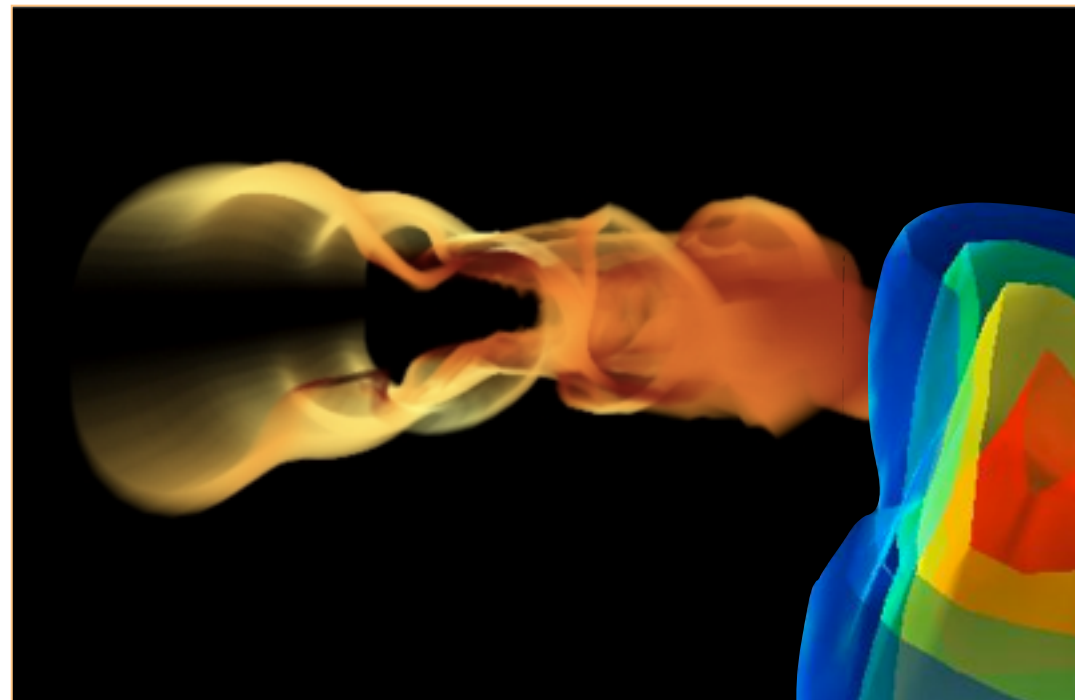
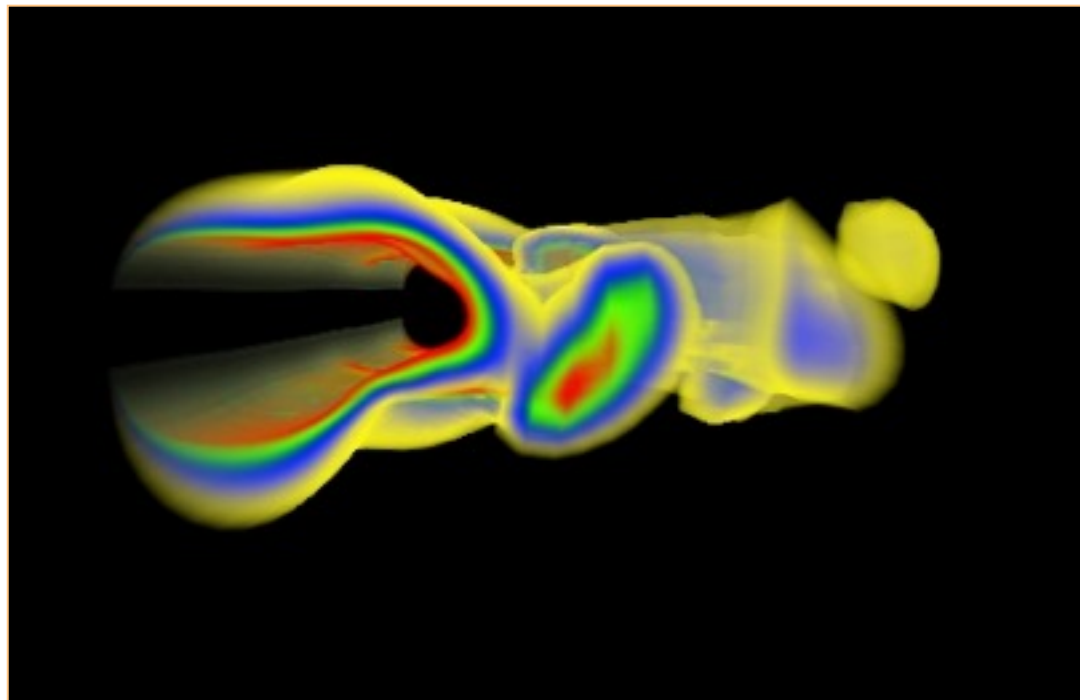


# Observação

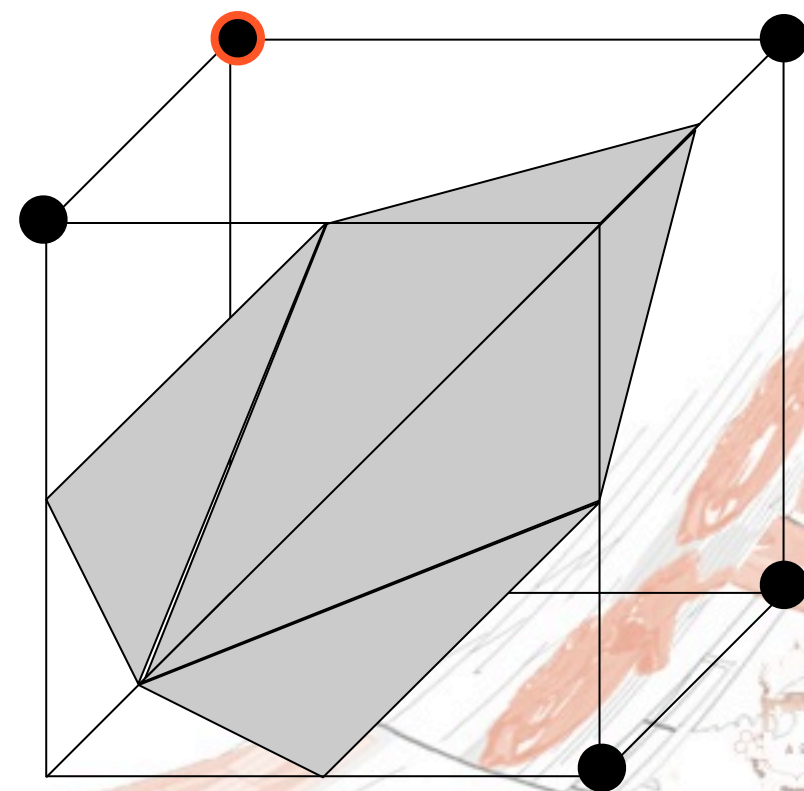
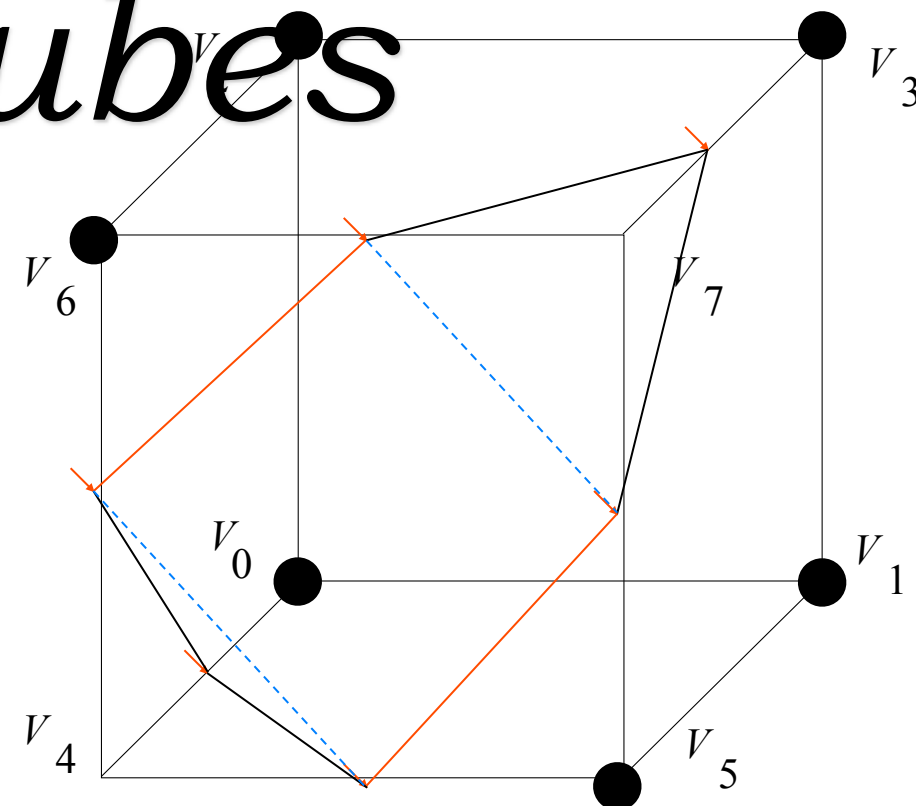
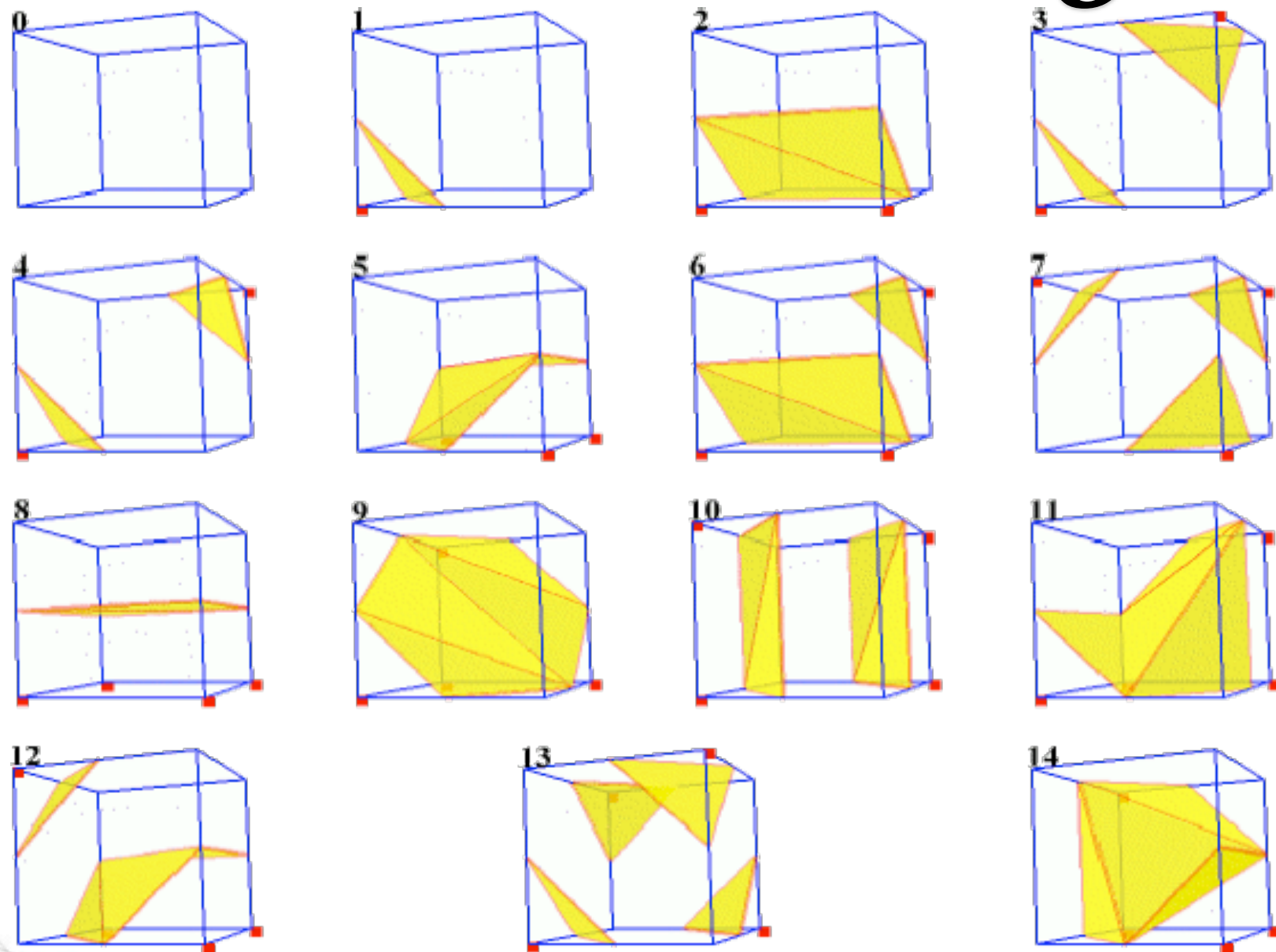


Interpolação bi-linear mas  
aproximação linear por segmentos

# Caso 3D



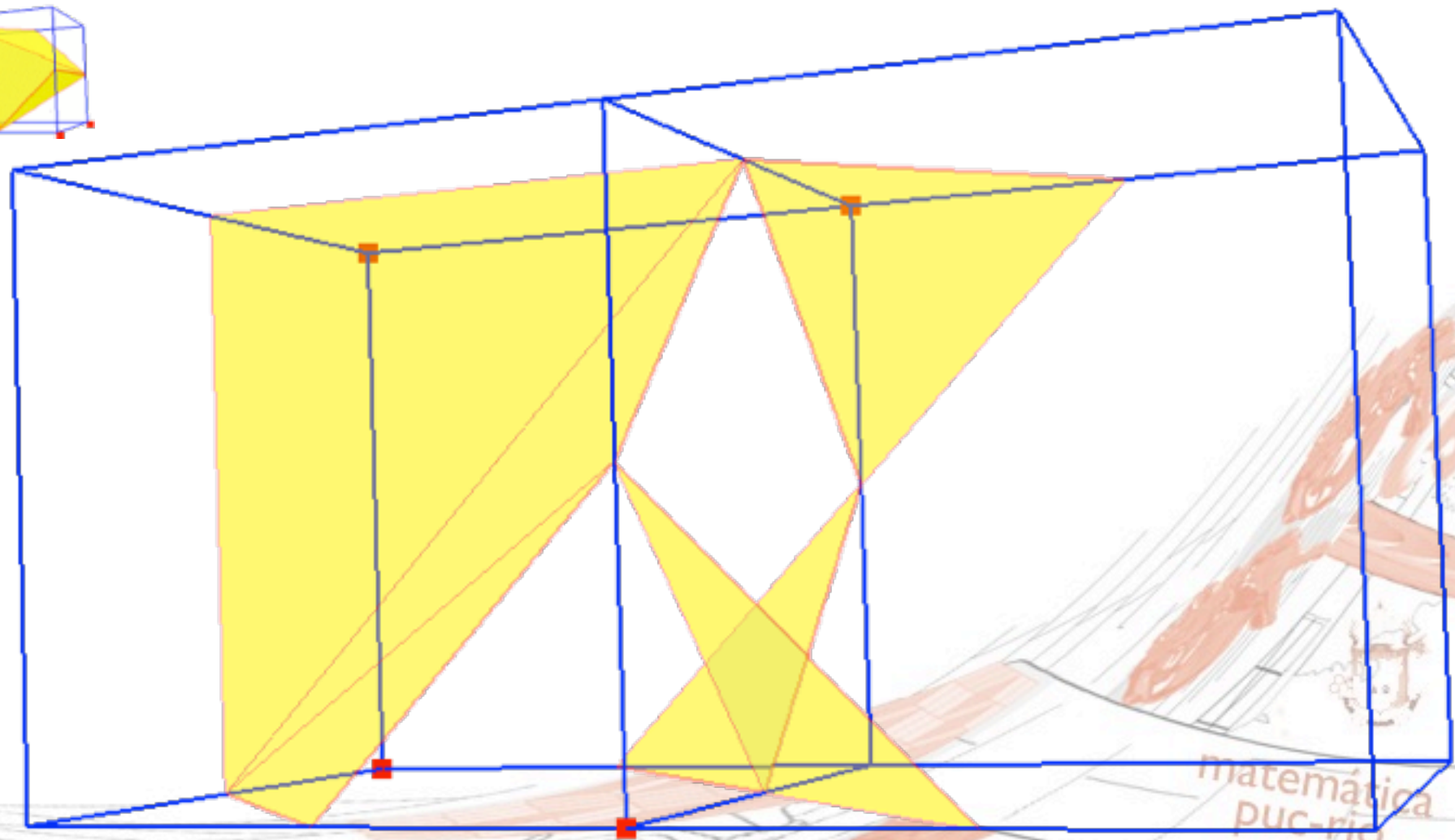
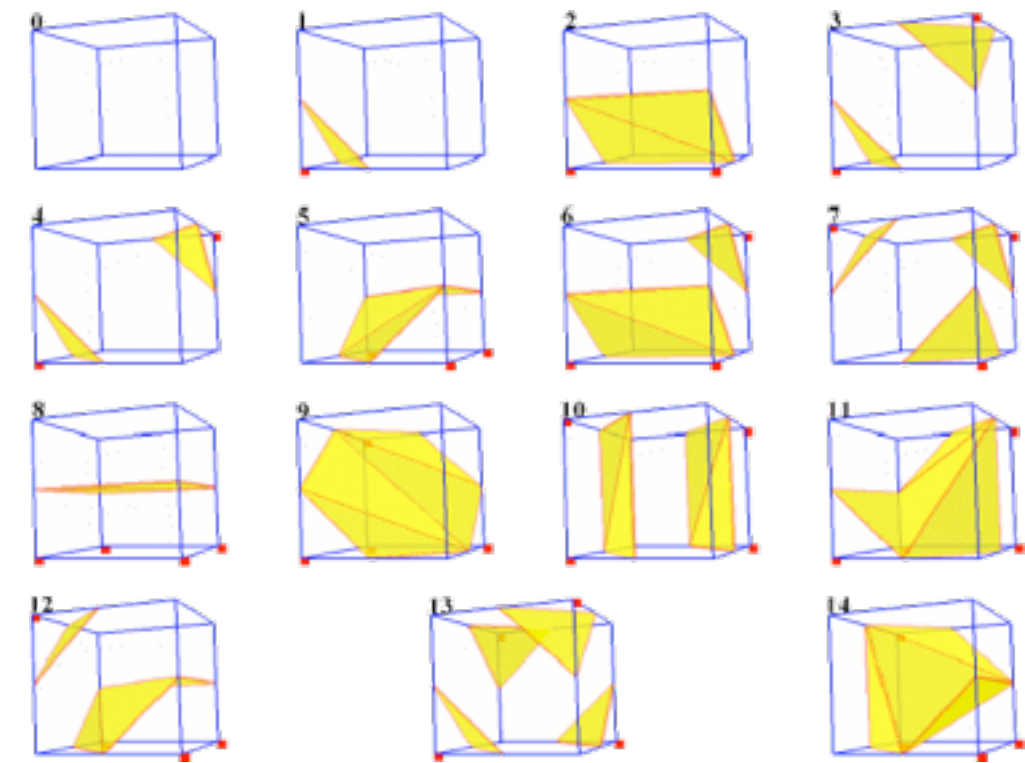
# Marching Cubes



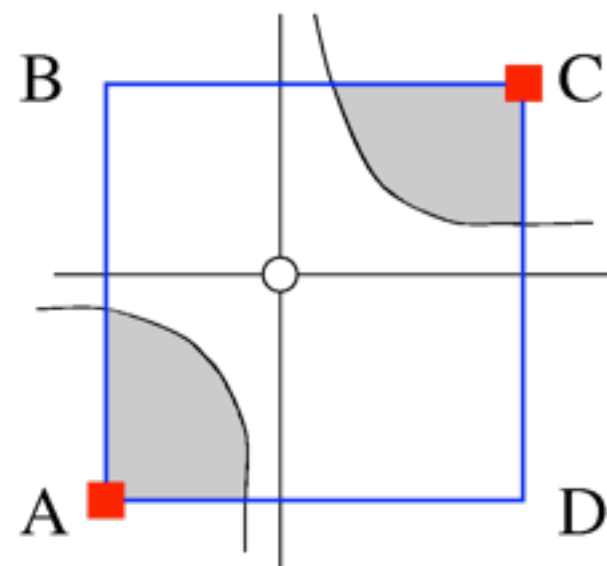
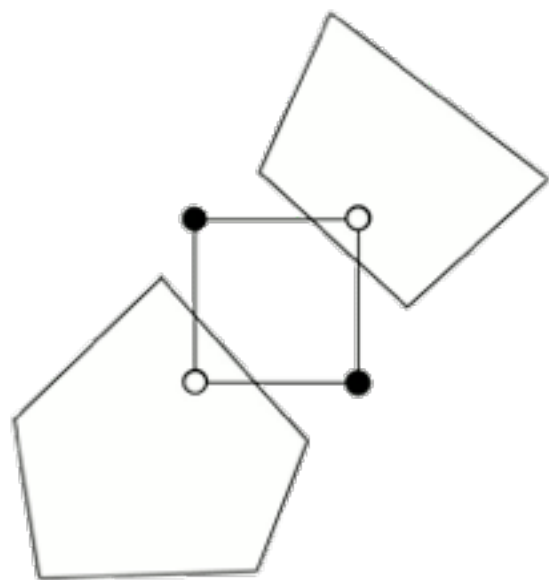
15 casos originais



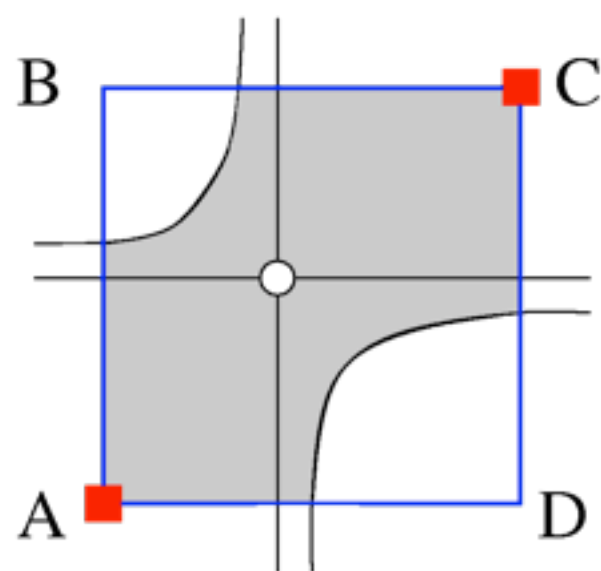
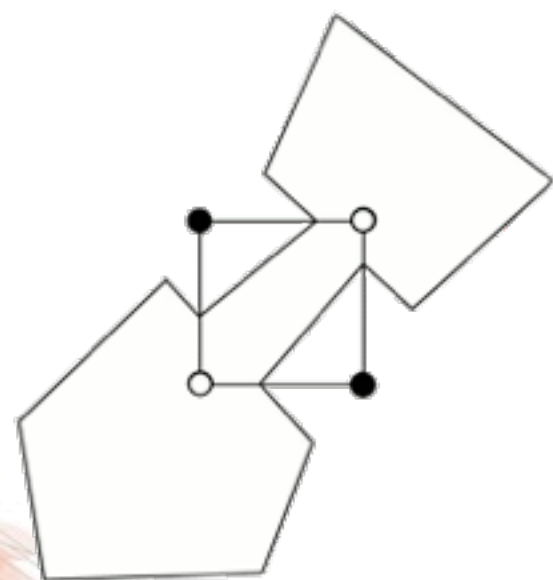
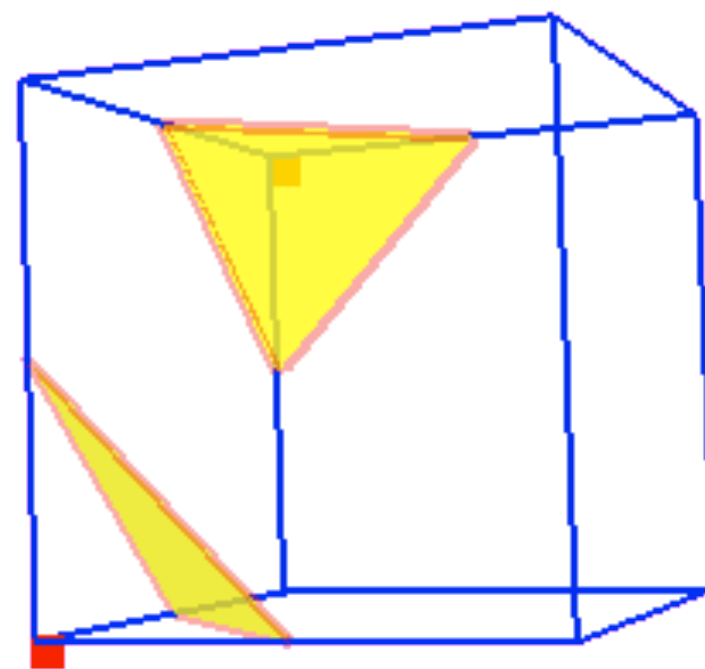
# Descontinuidades



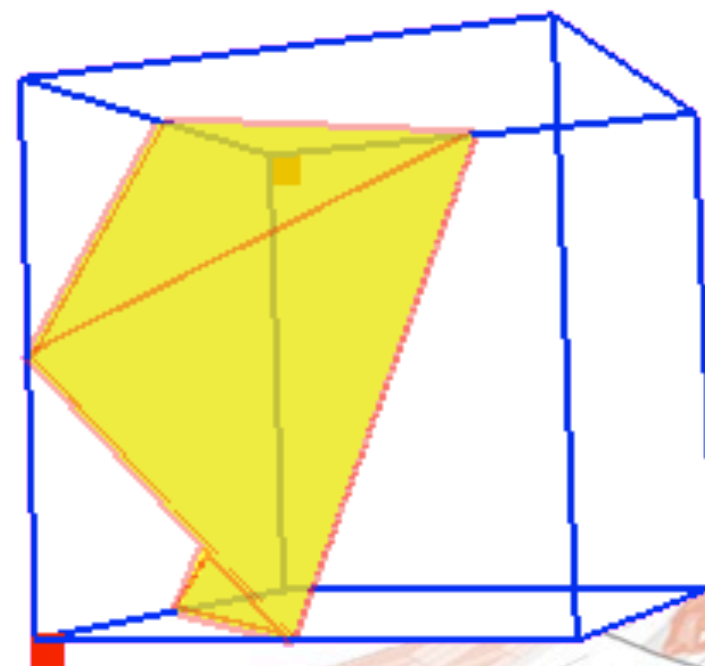
# Ambigüidades



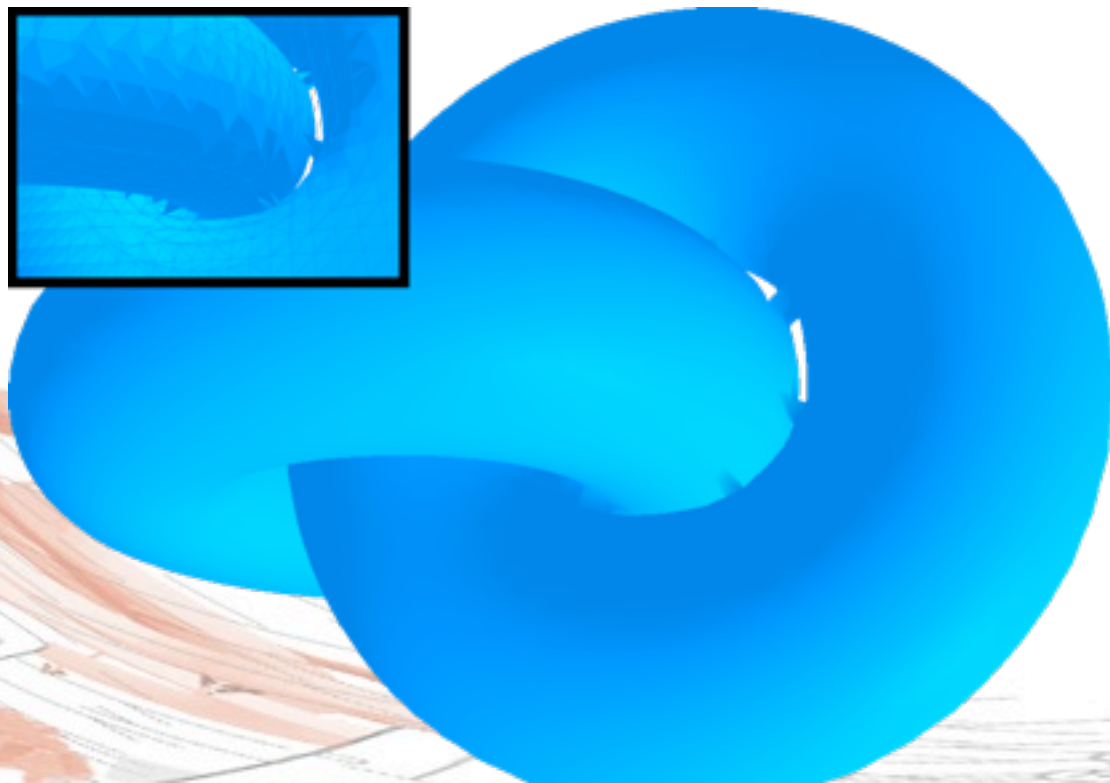
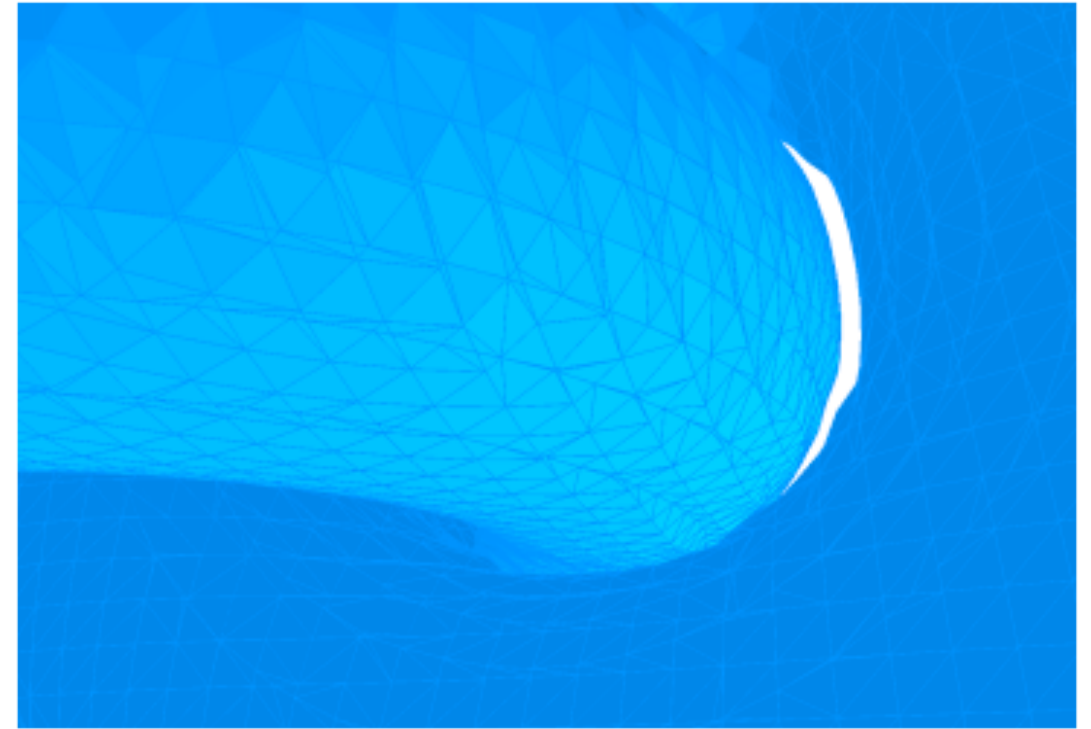
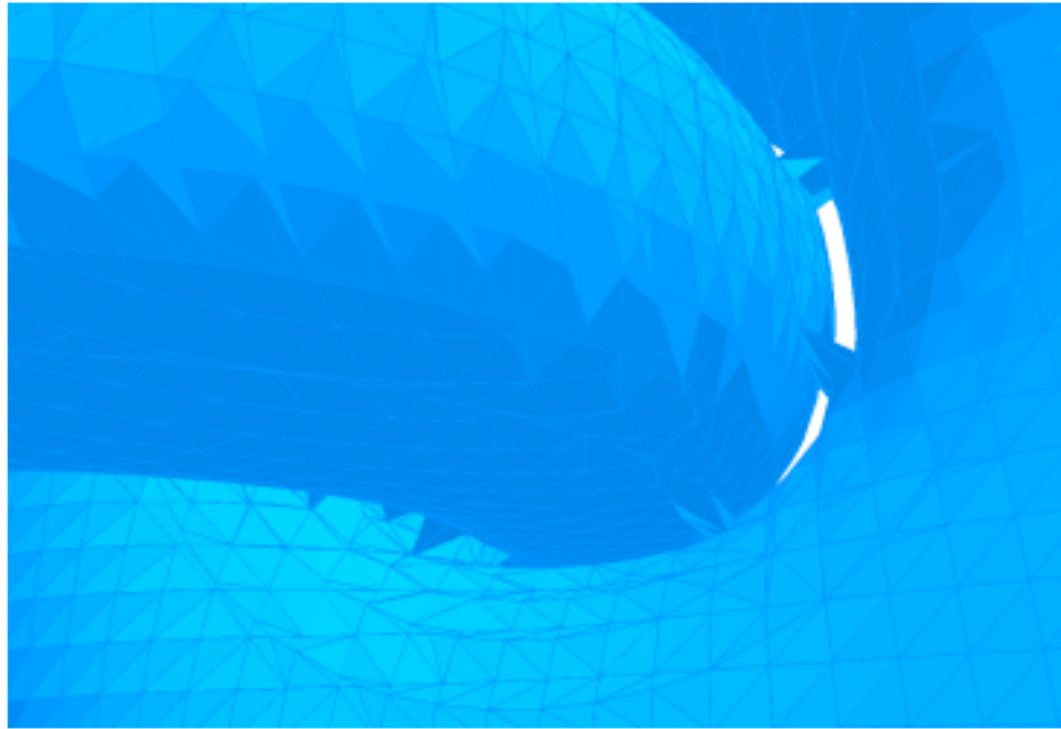
$$AC < BD$$



$$AC > BD$$

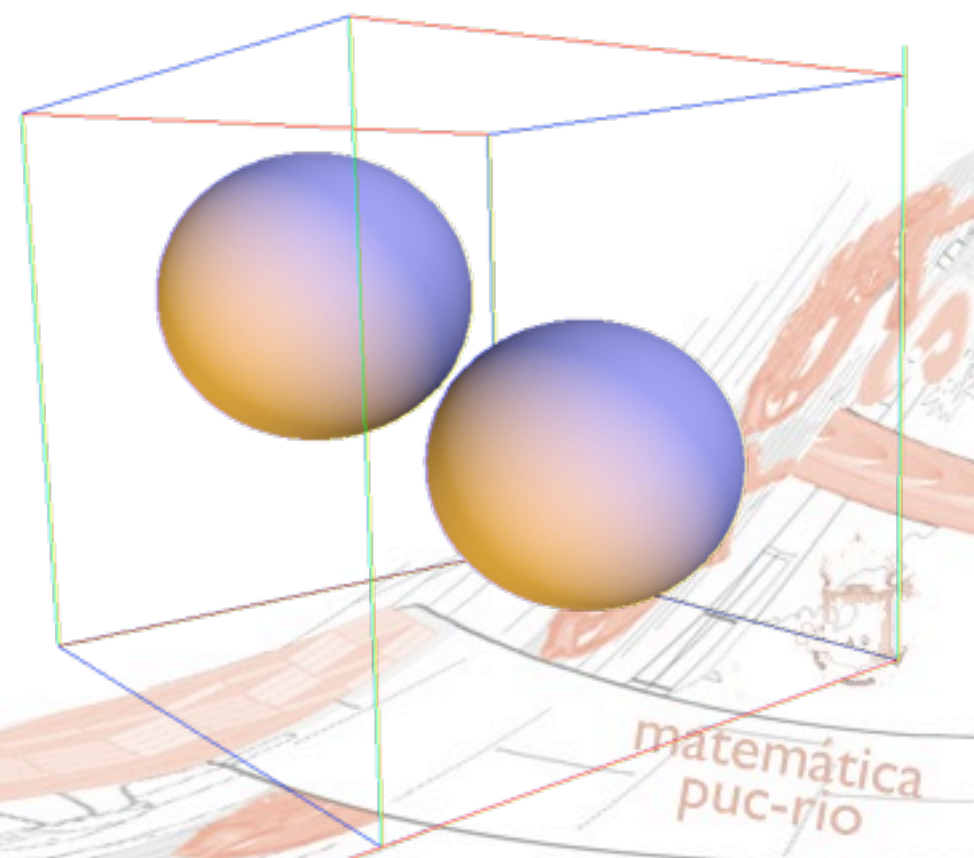
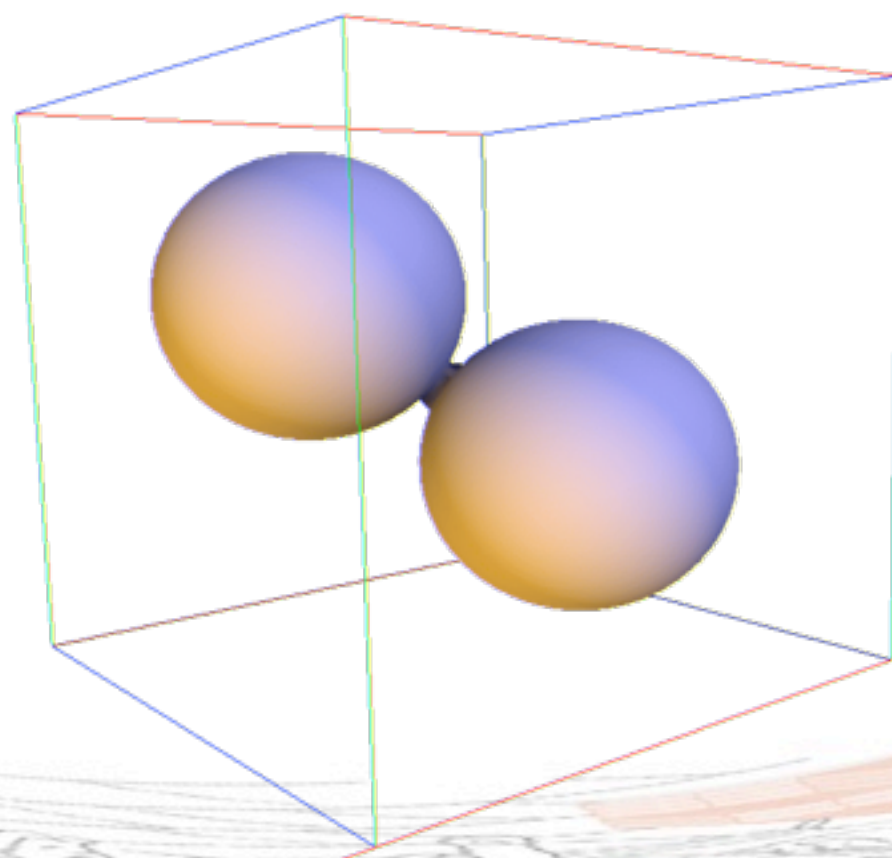
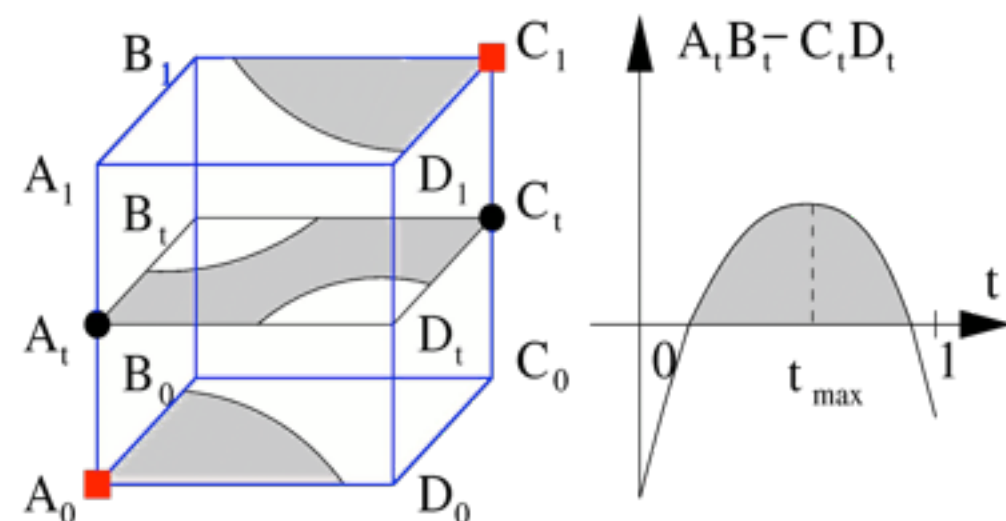
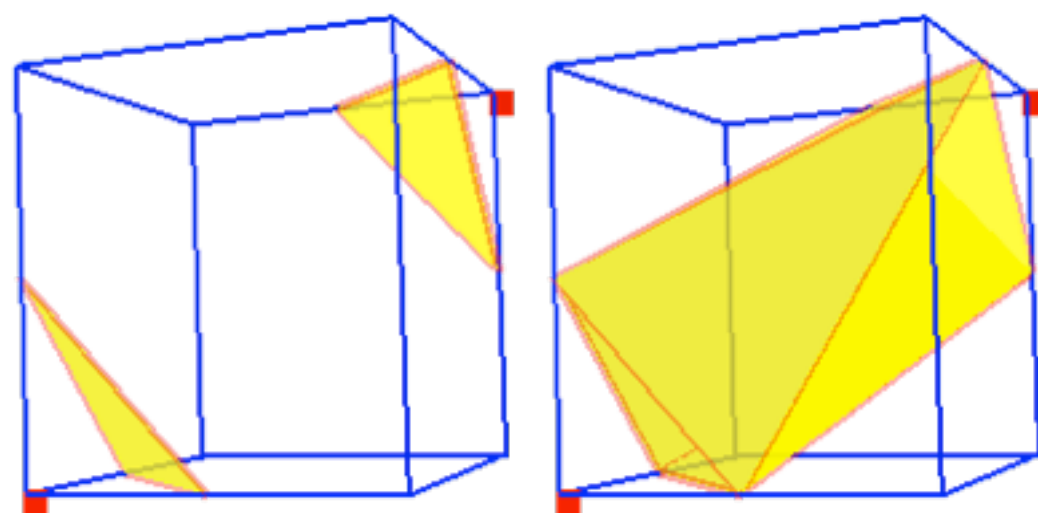


# Topologia global

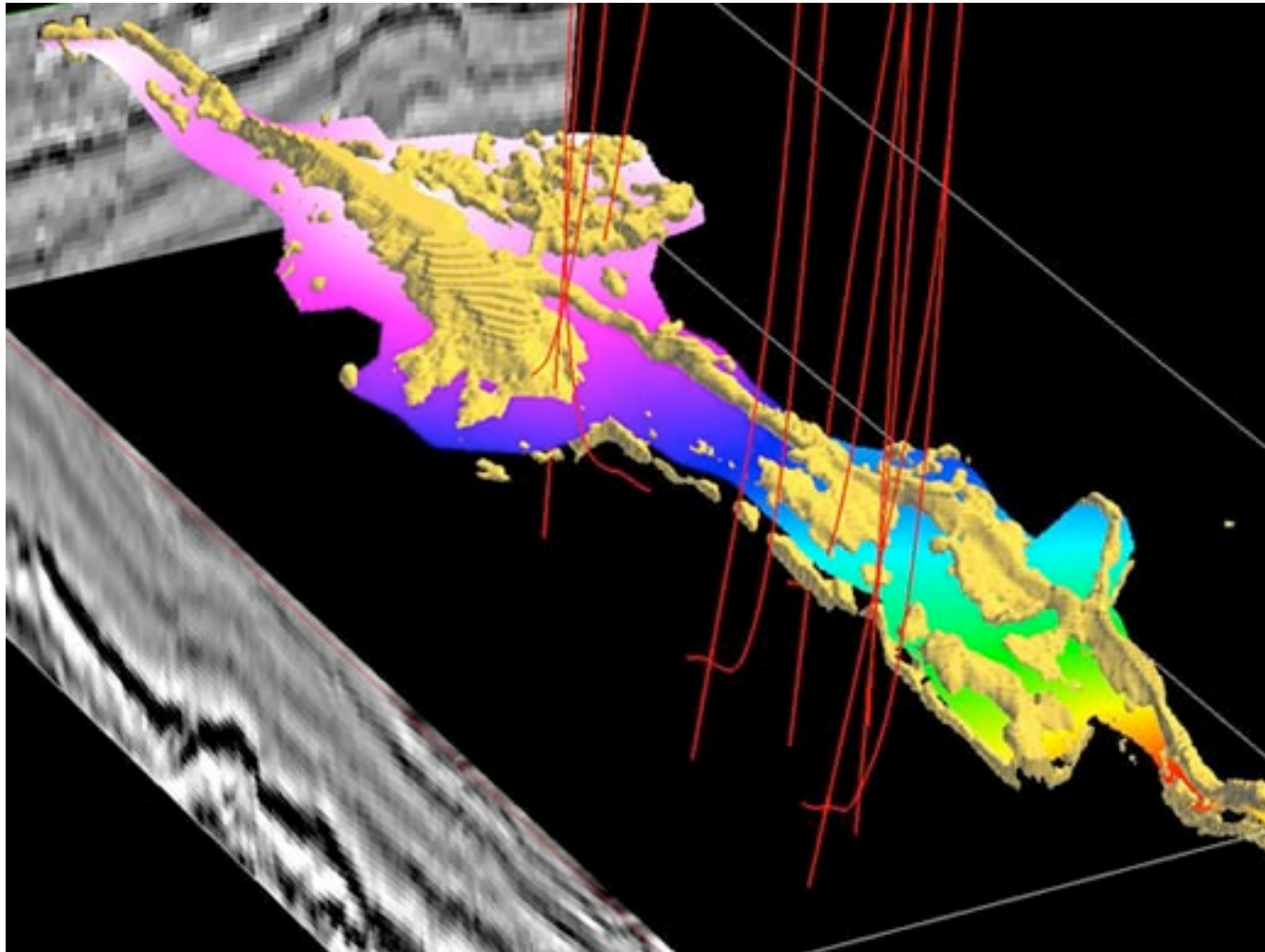




# Outras ambigüidades 3D

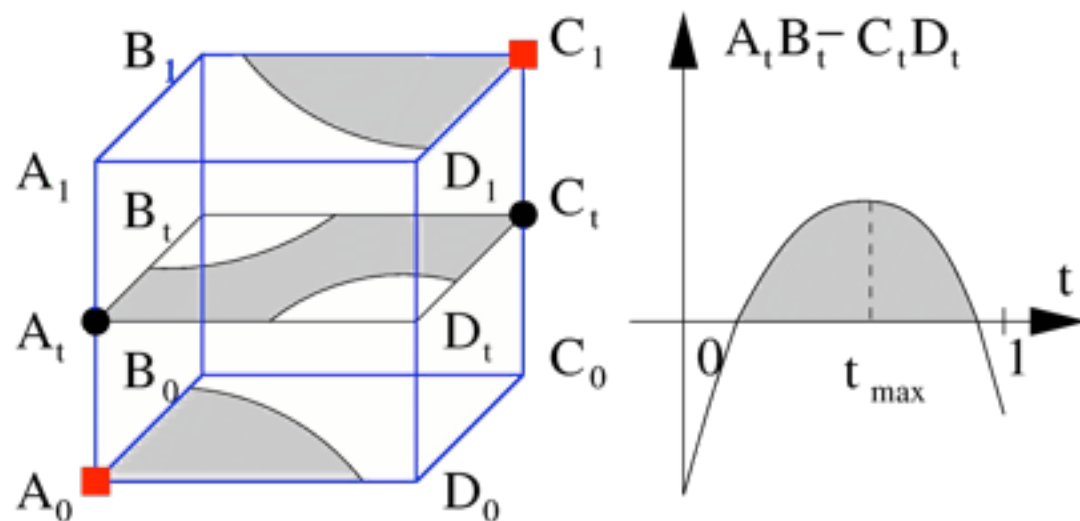
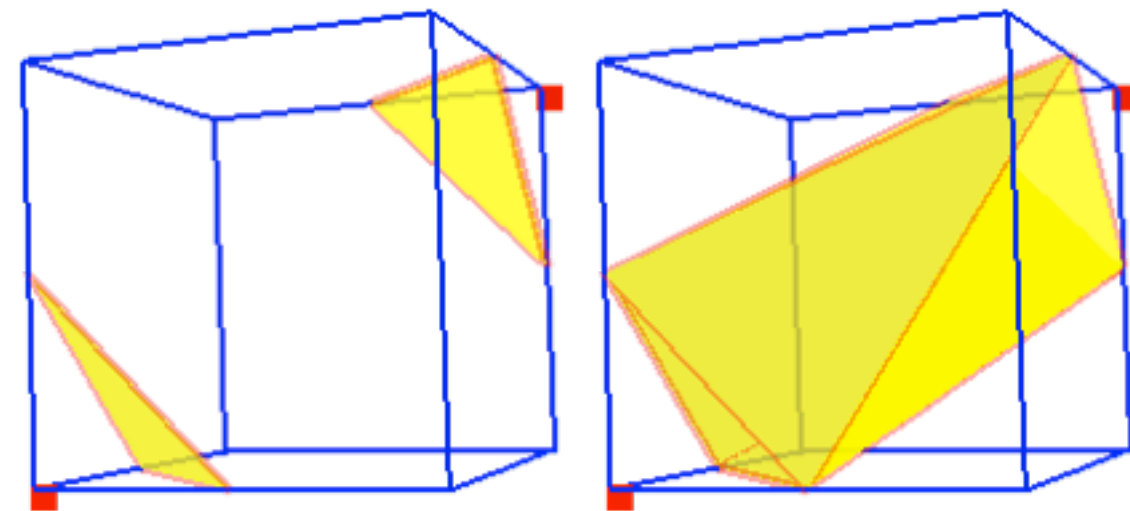


# Catástrofe

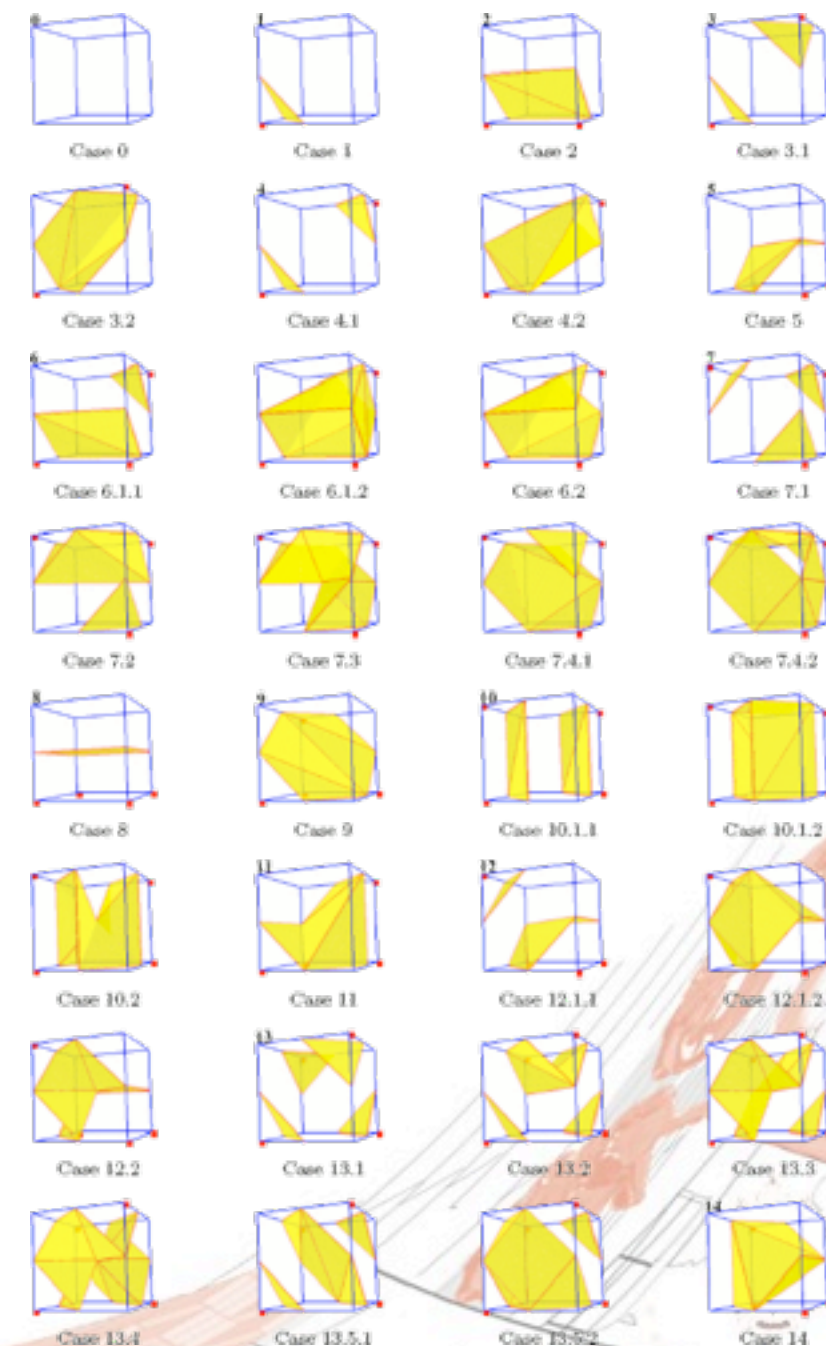




# Topologia trilinear



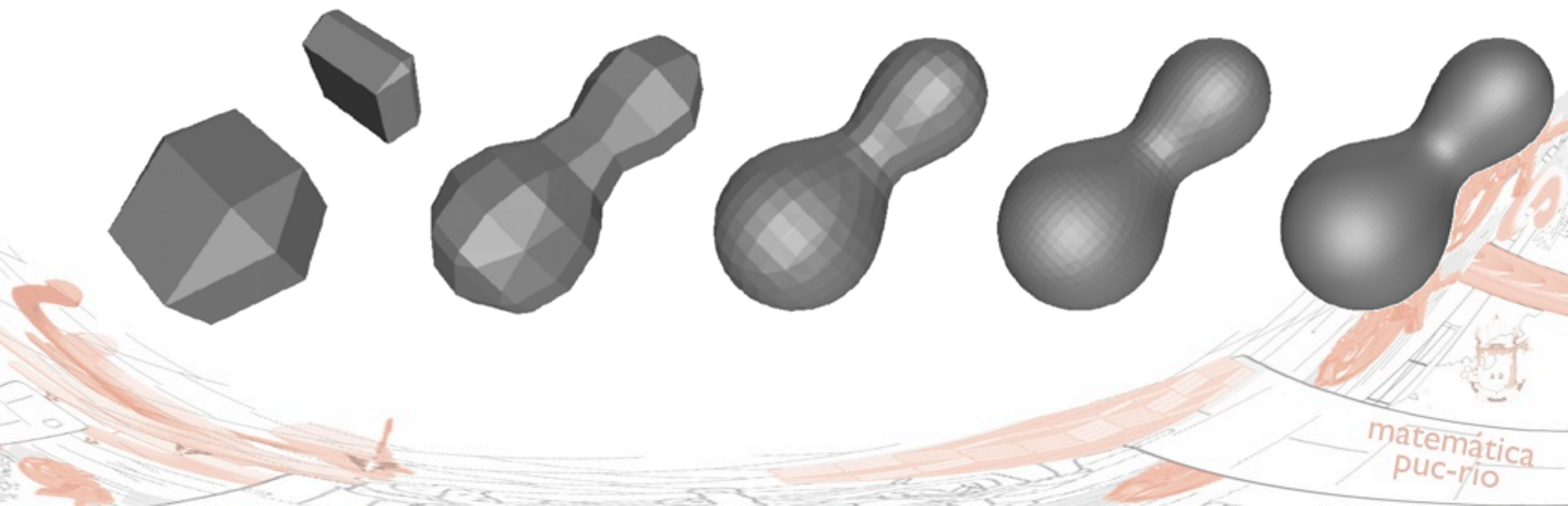
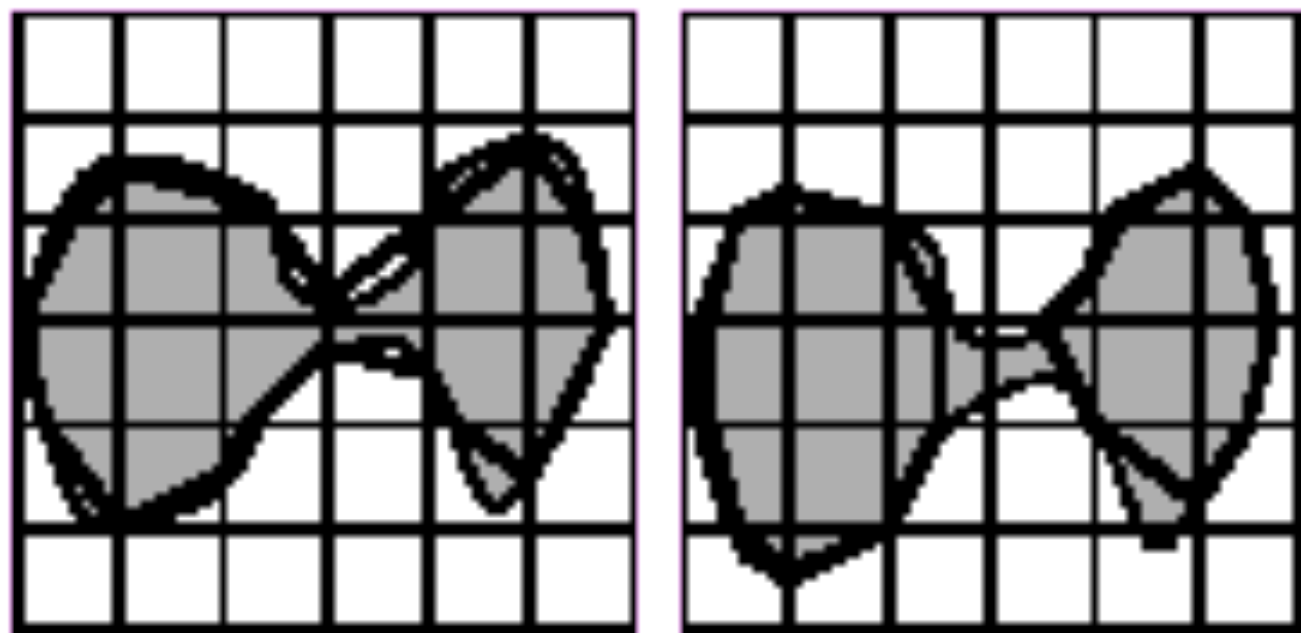
$$\rho(u, v, w) = \begin{matrix} (1-u) & (1-v) & (1-w) & \rho_{000} & + \\ (1-u) & (1-v) & w & \rho_{001} & + \\ (1-u) & v & (1-w) & \rho_{010} & + \\ (1-u) & v & w & \rho_{011} & + \\ u & (1-v) & (1-w) & \rho_{100} & + \\ u & (1-v) & w & \rho_{101} & + \\ u & v & (1-w) & \rho_{110} & + \\ u & v & w & \rho_{111} & \end{matrix}$$



33 casos

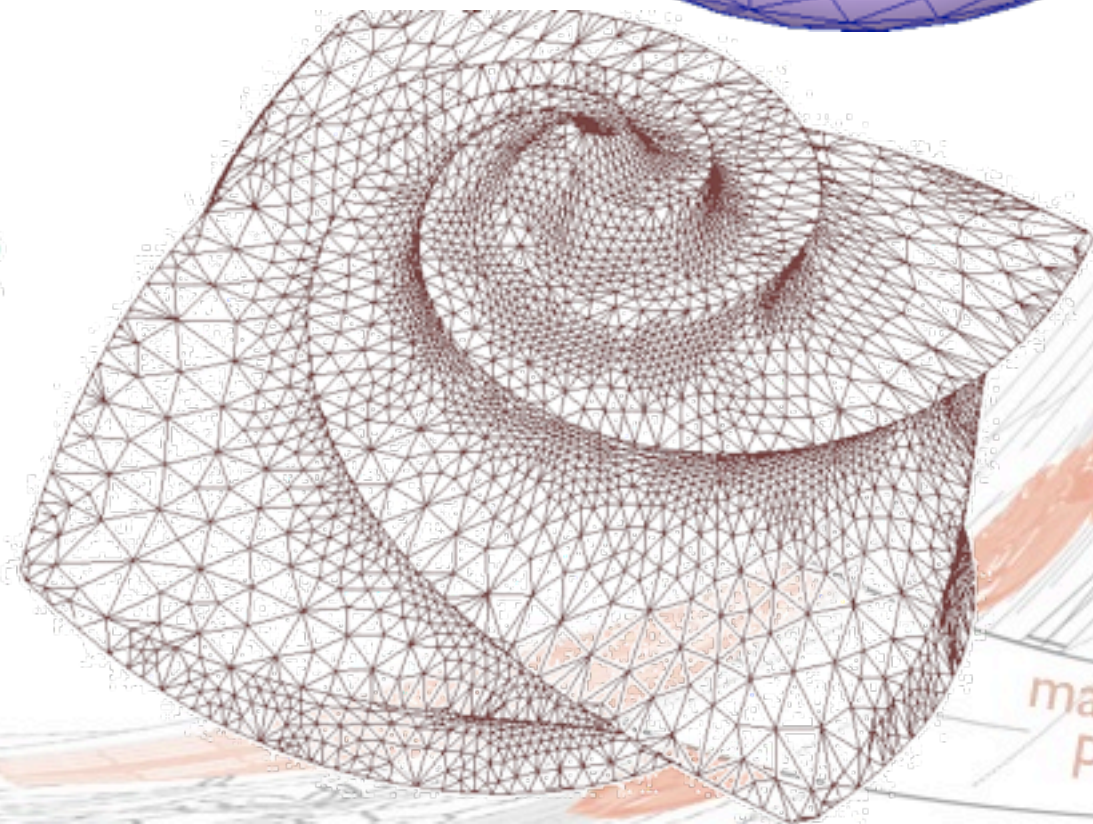
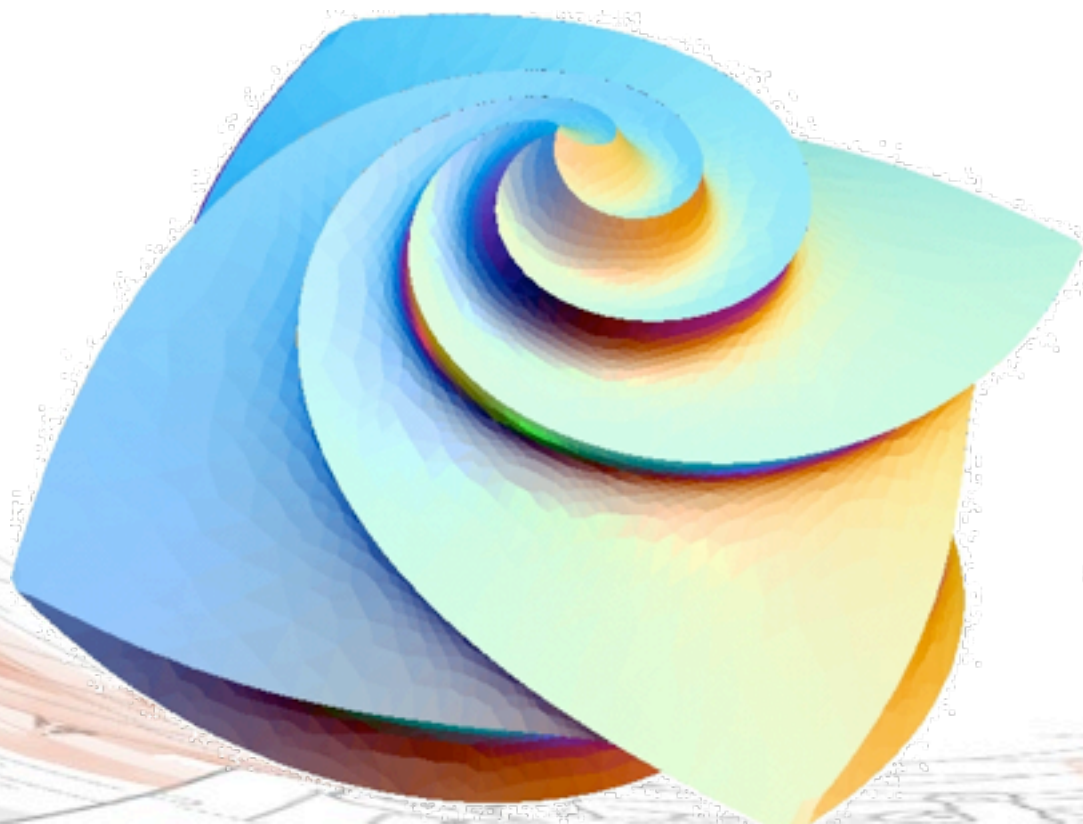
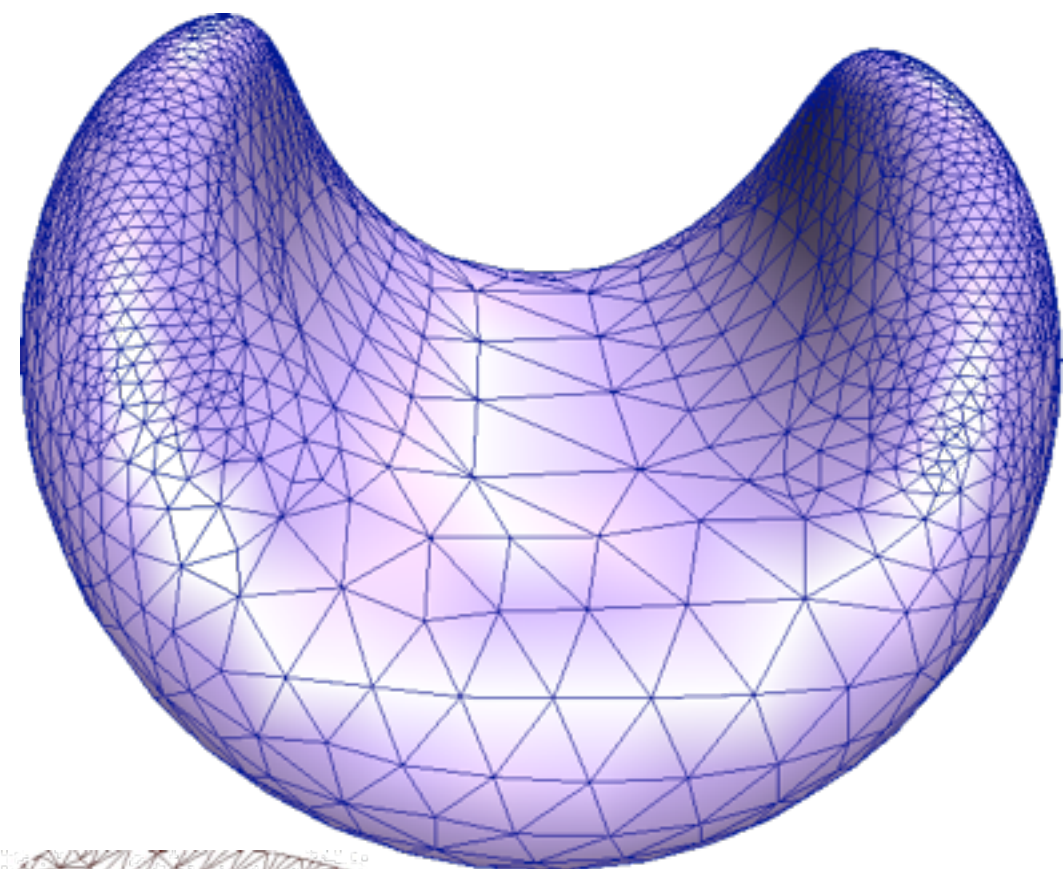
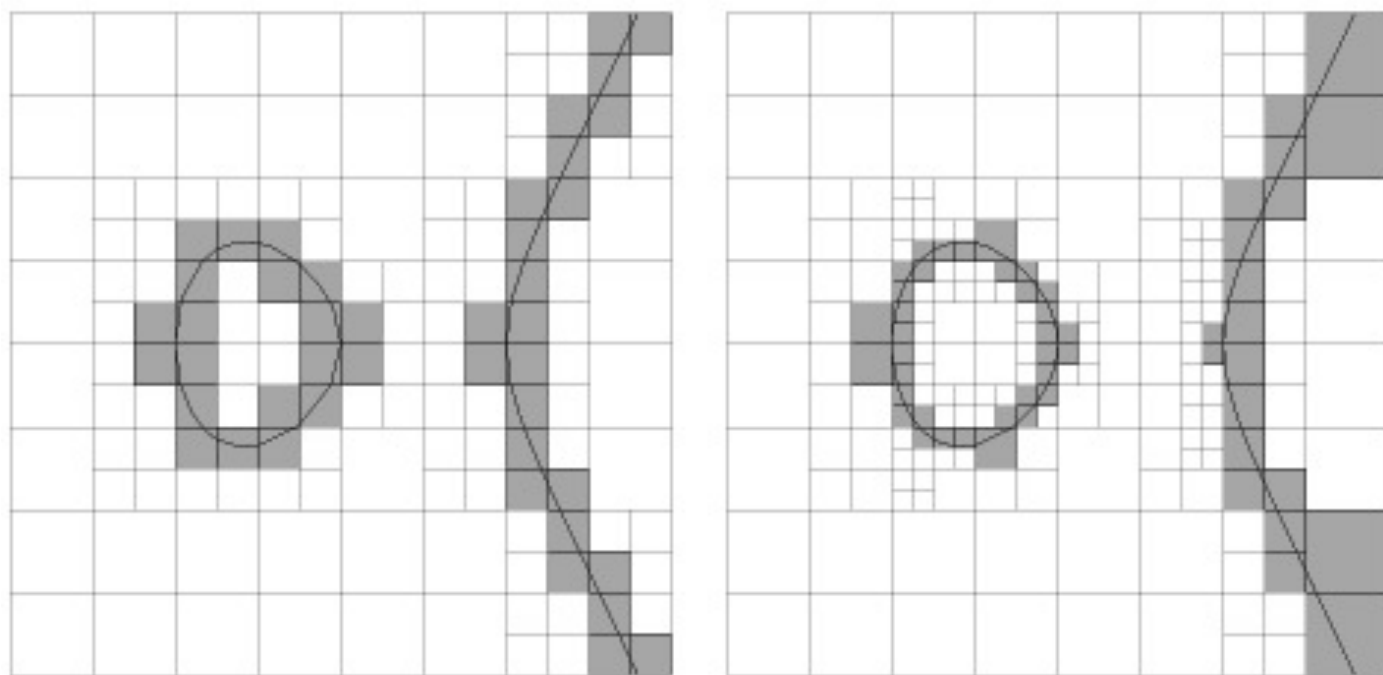


# Ambigüidades de resolução

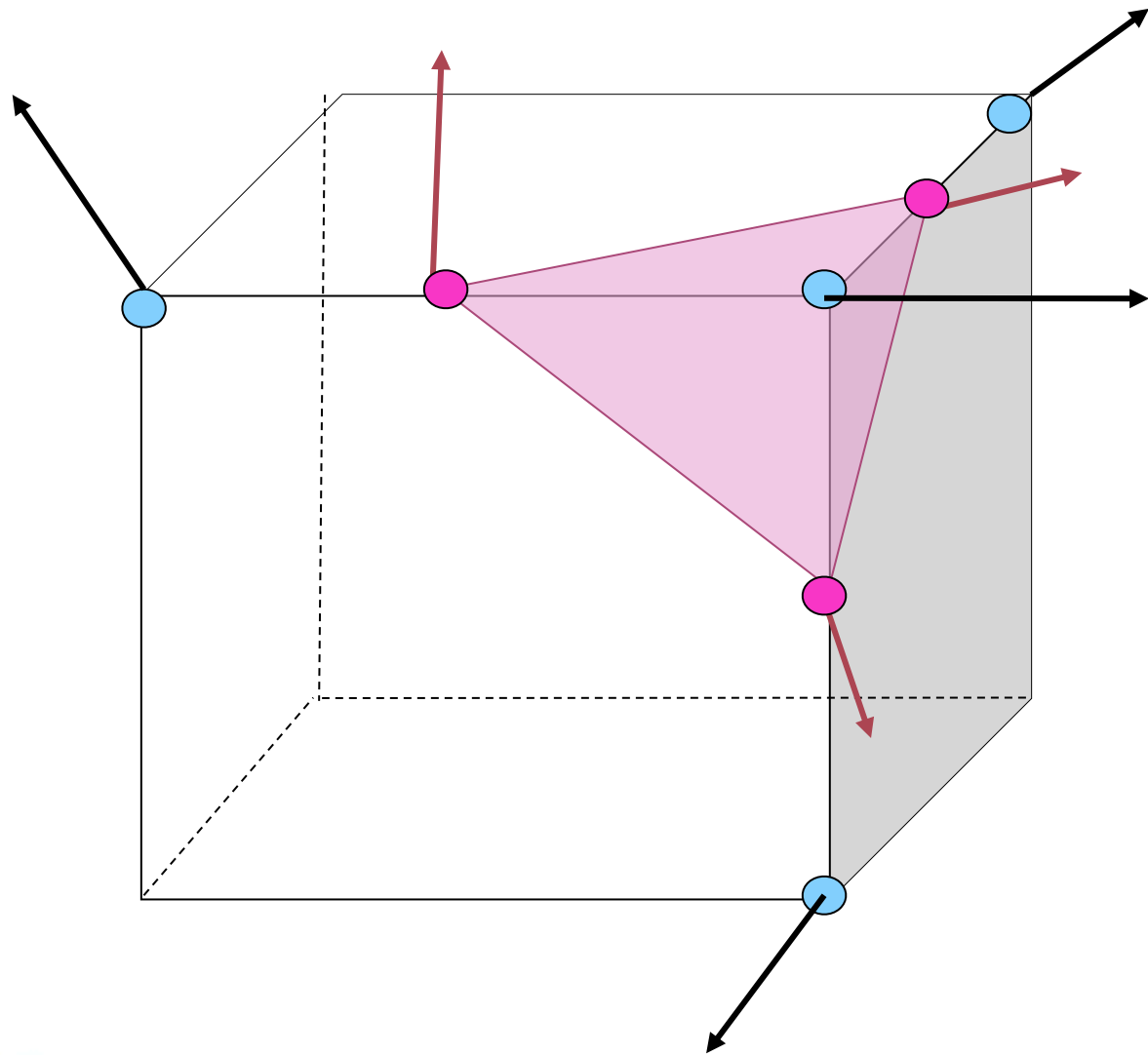




# É a geometria local?



# Cálculo da normal



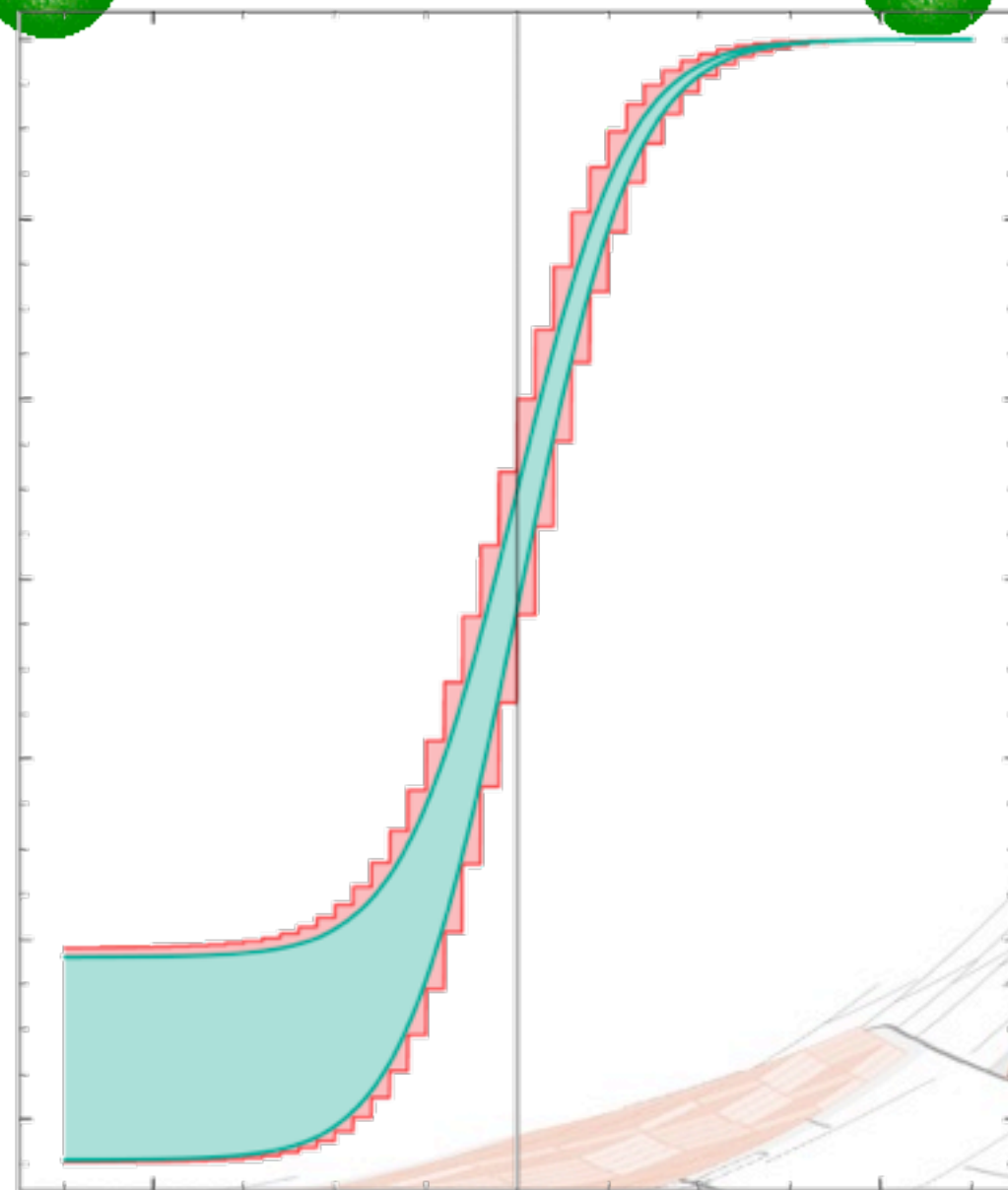
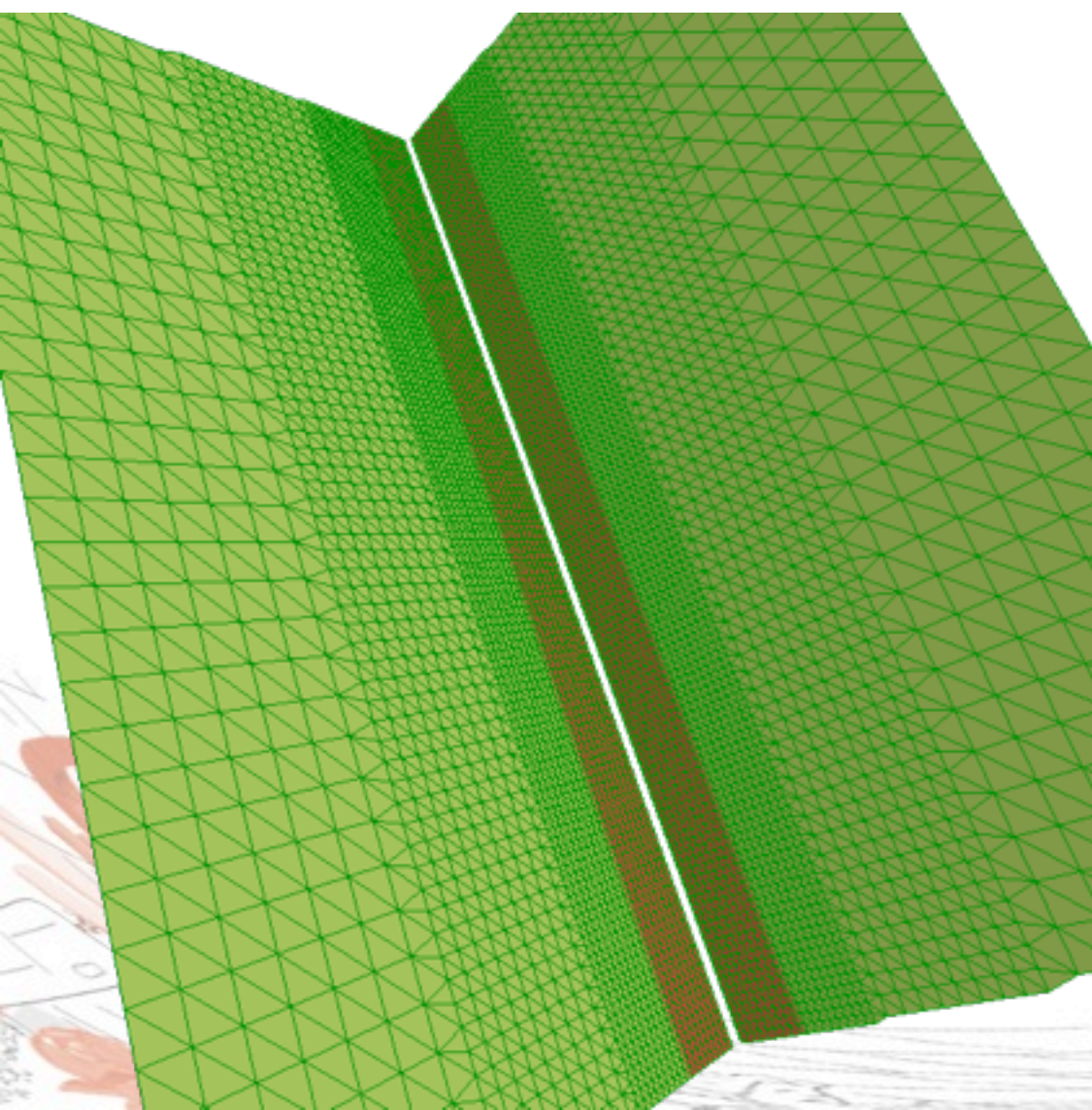
$$\frac{\partial f_{ijk}}{\partial x} \approx \frac{f_{i+1,j,k} - f_{i-1,j,k}}{\Delta x}$$

$$\frac{\partial f_{ijk}}{\partial y} \approx \frac{f_{i,j+1,k} - f_{i,j-1,k}}{\Delta y}$$

$$\frac{\partial f_{ijk}}{\partial z} \approx \frac{f_{i,j,k+1} - f_{i,j,k-1}}{\Delta z}$$



# Adaptação topológica



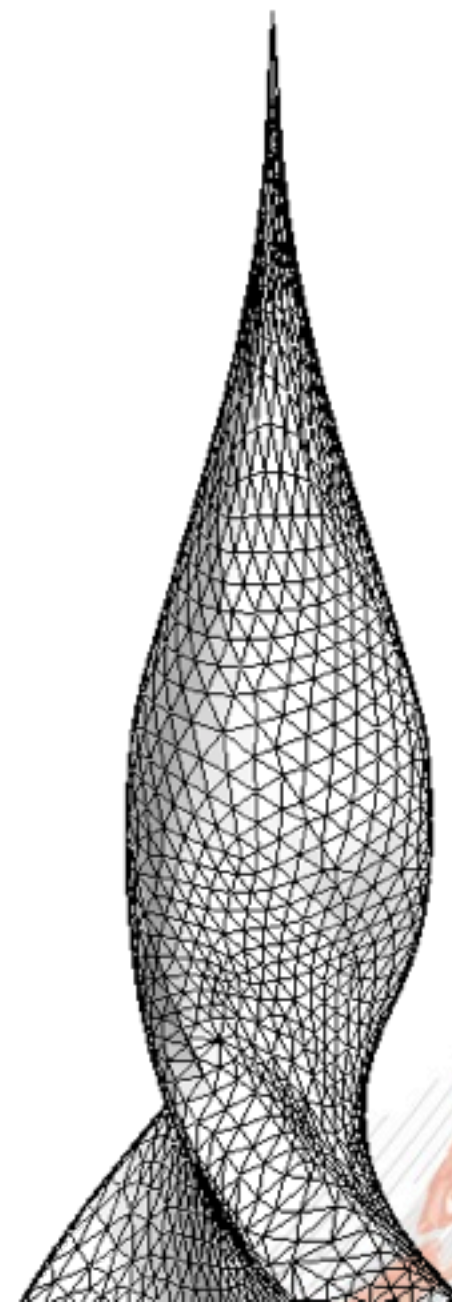
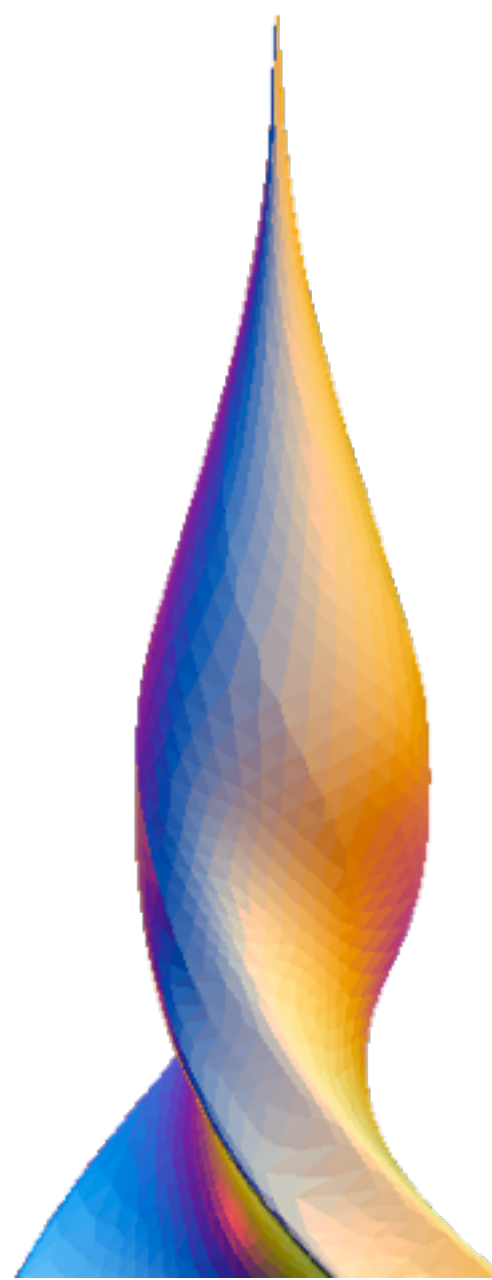
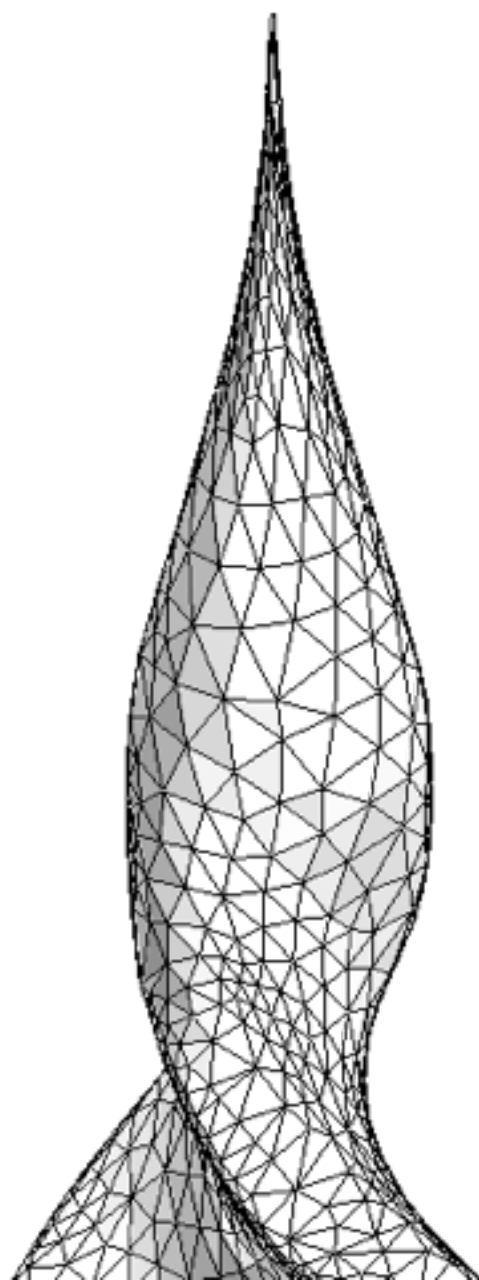
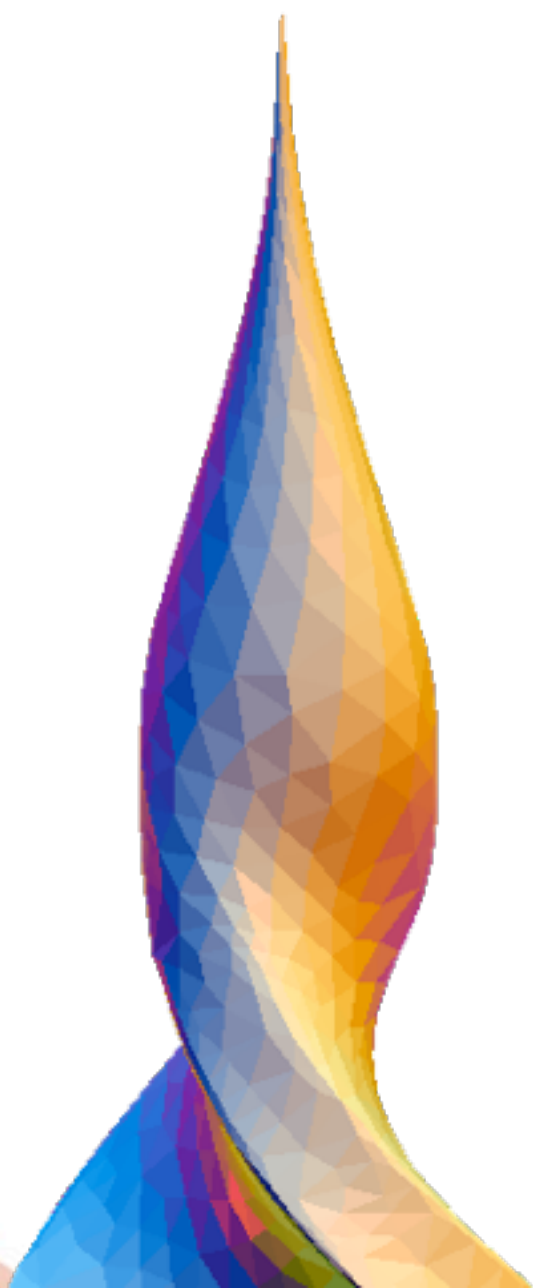


# Isosuperfícies em quadrees



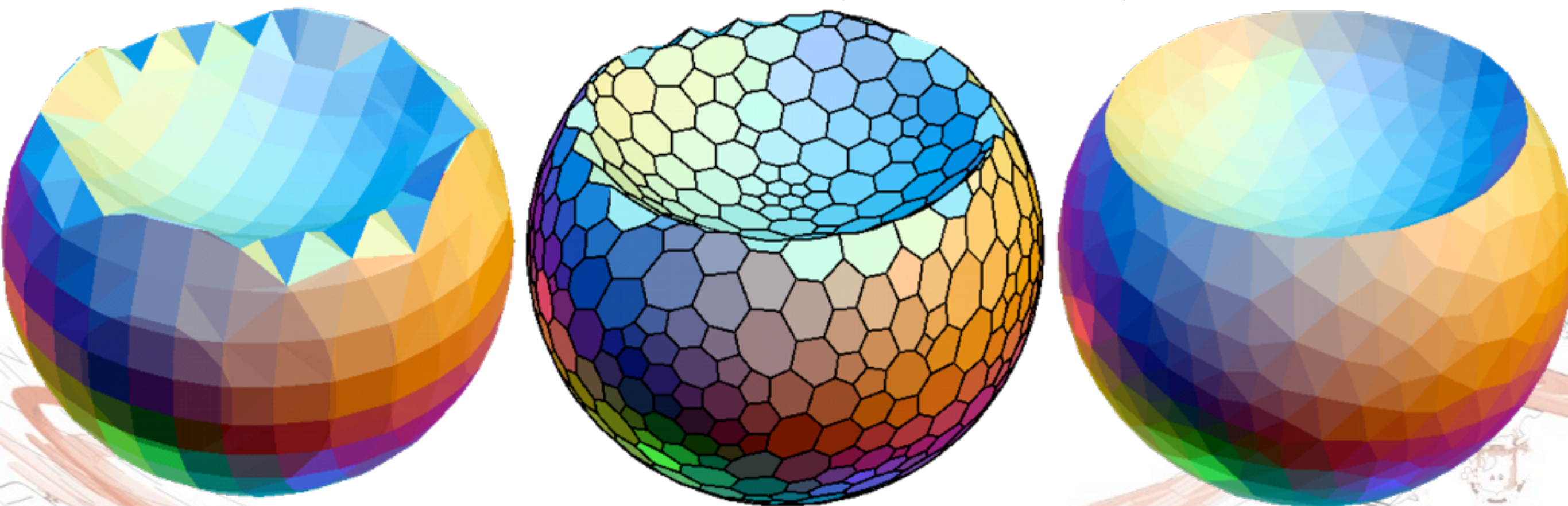
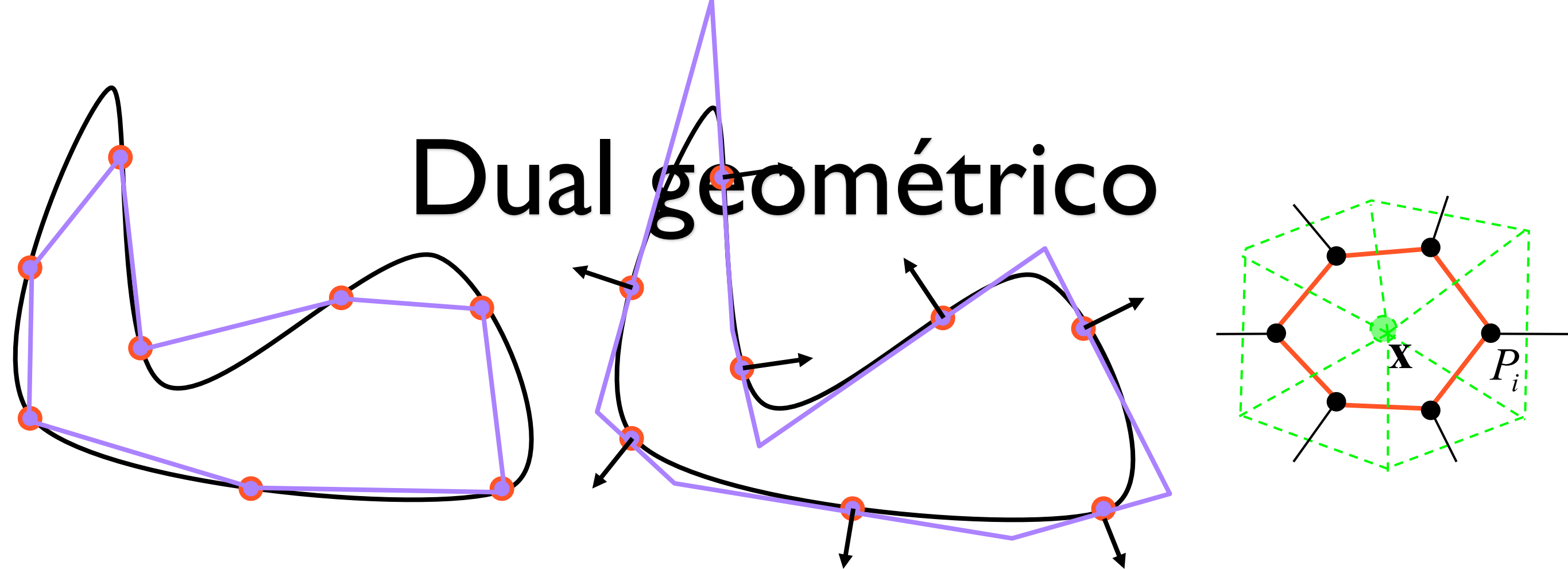


# Arestas vivas





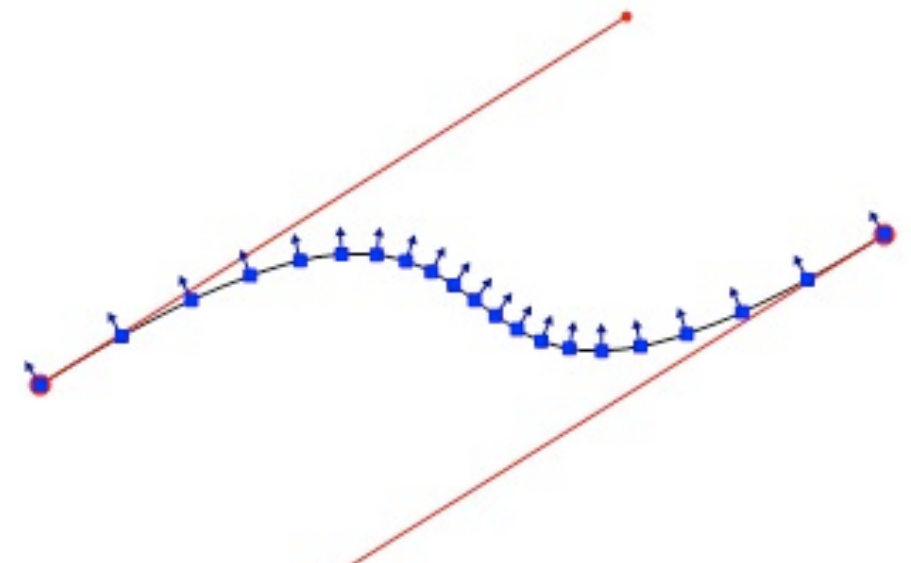
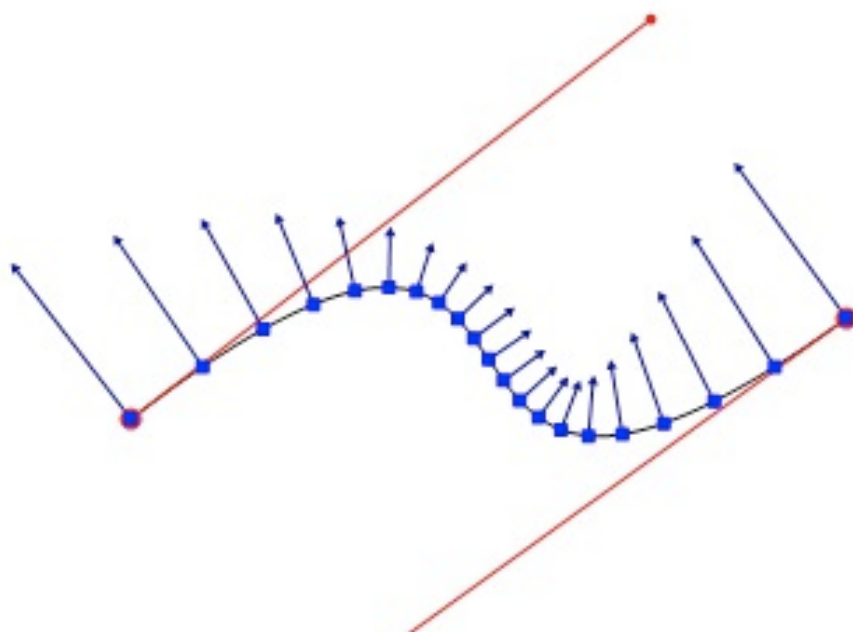
# Dual geométrico



# Invariância?

transformações geométricas

Amostragem no grid







# Conclusão

Mistura de conceitos:

- geometria descritiva (realidade)
- geometria diferencial (interpretação)
- geometria discreta (aproximação?)
- métodos numéricos (técnica)

Métodos versáteis, em teoria...





# SIBGRAPI 2011

XXIV Conference on Graphics, Patterns and Images

**Maceió – Alagoas – Brazil**

**August 28<sup>th</sup> to 31<sup>st</sup>, 2011**

**Tutorials**

**Technical Papers**

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**Undergraduate Work**

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UFAL

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Sociedade Brasileira  
de Computação

**Cooperation**



Eurographics

# SIBGRAPI 2011

XXIV Conference on Graphics, Patterns and Images

The XXIV Conference on Graphics, Patterns and Images, Sibgrapi 2011, will be held in the beautiful Maceió, Alagoas, Brazil, between August 28<sup>th</sup> and 31<sup>st</sup>. Sibgrapi 2011 is being organized by the Institute of Mathematics of the Universidade Federal de Alagoas (UFAL).

SIBGRAPI started in 1988 and is the most important Brazilian Meeting in Computer Graphics, Image Processing and Computer Vision. It is annually promoted by the Sociedade Brasileira de Computação (SBC). The Sibgrapi proceedings are available on-line at IEEE Xplore since 1997 and have also been published by IEEE Computer Society Press. Since 2009, Sibgrapi further cooperates with Eurographics Association.

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# Obrigado pela atenção

Curvas e Superfícies Implícitas:  
Noções de Geometria Diferencial e Discreta

errata e mais referências: <http://thomas.lewiner.org/>

