Random walks for vector field denoising

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Abstract. In recent years, several devices allow to directly measure real vector fields, leading to a better understanding of fundamental phenomena such as fluid simulation or brain water movement. This turns vector field visualization and analysis important tools for many applications in engineering and in medicine. However, real data is generally corrupted by noise, puzzling the understanding provided by those tools. Those tools thus need a denoising step as preprocessing, although usual denoising removes discontinuities, which are fundamental for vector field analysis. This paper proposes a novel method for vector field denoising based on random walks which preserve those discontinuities. It works in a meshless setting; it is fast, simple to implement, and shows a better performance than the traditional gaussian denoising technique.
Keywords: ***Discrete Vector Field. Denoising. Random Walk. Markov Chain.***

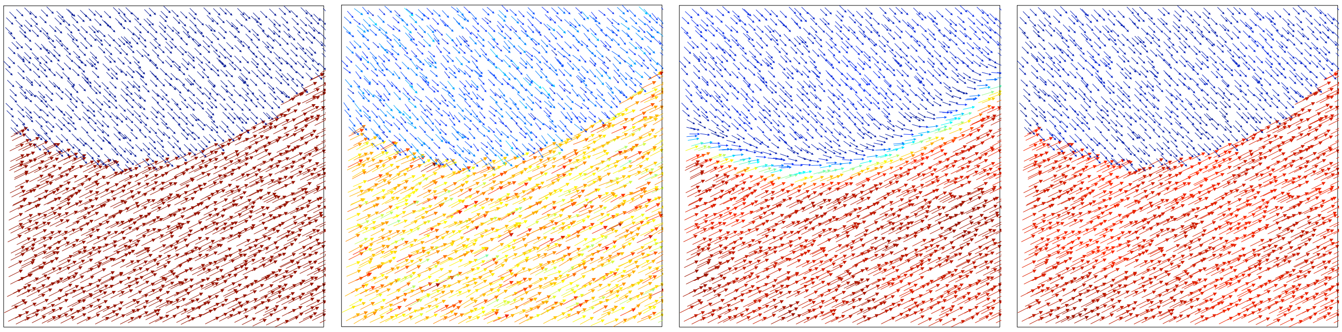

Figure 1: *A simple discontinuous vector field (left) pertubed with a gaussian additive noise (middle left). The gaussian filter (middle right) blurs the interface, while the random walk (right) preserves it.*

1 Introduction

Computer simulations of mechanical phenomena heavily rely on vector field generation, visualization and analysis. In particular, several techniques allow for measuring vector fields, generating fundamental data for understanding a physical behavior. Particle Image Velocimetry (PIV) is concerned with the quantitative investigation of fluids by imaging techniques [11] and has been used on several applications in mechanical engineering, in particular to modern aerodynamics and hydrodynamics research [14]. However, such real data is typically corrupted by noise, which harms further simulation and puzzles the interpretation. The preprocessing of vector fields typically involves a denoising step. Classical denoising approaches rely on local coherence, considering that noise follow one of the vanishing mean model, and thus can be cancelled by averaging a piece of data with its neighbors. This has a smoothing effect which is very well understood for scalar fields such as images.

However, as opposed to scalar physical quantities associated to energies, for which one expects a globally smooth behavior in real experiments, vector fields can present rapidly changing directions. In fact those discontinuities are generally the most interesting part to analyze: they correspond to interfaces in fluid simulation, structural tissues when measuring brain water movement, faults and fractures in geophysical interpretation of soils… On one side, denoising such vector fields by a direct local smoothing would simply remove those discontinuities, and all its valuable information. On the other side, there is still little understanding of the exact noise model induced by the vector field measurement techniques, leaving the vanishing-mean noise as the most reasonable model.

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Contributions

In this work, we propose a vector field denoising technique based on random walks that preserves coherent discontinuities while removing noise under a vanishing-mean *per continuous region* model (Figure 1). Random walk is a stochastic process consisting in taking successive random steps, giving a probability for each direction according to its coherence with the current state. In that sense, it is closely related to Markov chains. This approach leads to a very simple and fast implementation of the denoising, while allowing the handling of unstructured (meshless) data. It shows to have better performance when compared to more pervasive gaussian filtering, while preserving the vector field features, like discontinuities and singularities.

2 Previous and related work

Random walks

Random walk has many applications nowadays not only in visual computing but also in genetics, physics, medicine, chemistry, computer science, just to cite a few. The first work using random walks in computer vision is in the application of texture discrimination [20], and recently has been applied to image segmentation [4]. In the field of image processing, random walk has been used to image enhancement [12] and filtering [17]. The use of random walks in geometry processing was recently proposed by Sun et al in [15, 16] for mesh denoising, and after that it appears an application to mesh segmentation in [5]. This paper is inspired in the work of [12] and [15], we extend their work to deal with meshless data in the plane.

Vector field denoising

In 2005, Westenberg and Erlt in [19] proposed to threshold vector wavelet coefficients to suppress additive noise on a 2D vector field. This work has a disadvantage to work only on a structured grid of points. They compare their work to Gaussian filters. The importance of color image processing is forcing the development of vector filtering techniques on structured grids [10]. Many filters for color images are interested in the reduction of impulsive noise [24, 13, 7]. In geometry processing several works have been proposed to noise reduction on the surface normals [18, 8, 15, 16].

3 Random walks

Random walk (RW), or drunkards walk, was one of the first chance-processes studied in the theory of probability and has gained a lot of attention in several areas in visual computing. The name random walk is used because one may think of it as being a model for an individual walking on a straight line who at each point of time either takes one step to the right with probability p or one step to the left with probability $1 - p$, for example.

Given a graph and a starting node, one selects one of its neighbor at random and moves to this neighbor then selects a neighbor of this node at random and moves to it and so on. This sequence of nodes selected randomly this way is a random walk on the graph. You will see in section 4 that the denoising method to be proposed applies random walks on a graph whose nodes are the input points, and whose links represents the connectivity between them. To do such random walk, a probability has to be assigned to each edge on the graph and this represents the chance to move from a vertex to its adjacent neighbor through an edge.

Actually random walk on graph is a very special case of a Markov process [6]. This work follows the notation presented in [15].

A *Markov process* is a sequence of possibly dependent random variables $(X_1, X_2, X_3, \ldots)$ identified by increasing values of their index, commonly time. Its main property is that any prediction of the next value of the sequence (X_n), knowing the preceding states $(X_1, X_2, X_3, \ldots, X_{n-1})$, may be based only on the last state X_{n-1}. That is, the future value of such a variable is independent of its past history: $P(X_{n+1} = x_{n+1} | X_n = x_n, X_{n-1} = x_{n-1}, \ldots, X_1 = x_1) = P(X_{n+1} = x_{n+1} | X_n = x_n)$.

When a Markov process is a sequence of discrete-valued variables it is called a *Markov Chain* [9]. The possible values of X_n are called the *state space*, which is a countable set and can be either finite or infinite. In this paper, the state space $\mathcal{I}$ is finite and has L possible values. In the denoising method proposed here, L will represent the number of points in the input set.

A *transition probability* from state i to state j at the step n, where $i, j \in \mathcal{I}$, is equal to $P(X_{n+1} = j | X_n = i)$ and is denoted by $p_{i,j}(n)$. A Markov chain is *stationary* when the transition probability does not depend on n, that means: $P(X_{n+1} = j | X_n = i) = P(X_n = j | X_{n-1} = i)$.

The *transition probability matrix* $\Pi(n) \in \mathbb{R}^{L \times L}$ is the matrix whose the entry at the i^{th} row and j^{th} column is $p_{i,j}(n)$. Observe that each of its rows sums one. The probability that the Markov chain reaches the state i at the n^{th} time step is equal to $P(X_n = i)$ and is denoted by $p_i(n)$. The *probability distribution* of the Markov chain over all states at time n is represented by the vector $P(n) = [p_1(n), \ldots, p_L(n)]$. Note that $\sum_{i=1}^{L} p_i(n) = 1$.

Given an initial probability distribution, denoted by $P(0)$, the distribution of the Markov chain in the first step is $P(1) = P(0)\Pi(1)$, and in the second step is $P(2) = P(1)\Pi(2) = P(0)\Pi(1)\Pi(2)$. So, after n steps, the distribution of the Markov chain is $P(n) = P(0)\Pi^n$ where $\Pi^n = \Pi(1) \cdots \Pi(n)$ is the *n-step transition probability matrix*. The entry at the i^{th} row and j^{th} column of Π^n is the probability of moving from state i to the state j after n steps, and is denoted by $p^n_{i,j}$. Observe that if the MC is stationary, $\Pi(1) = \Pi(2) = \ldots = \Pi(n)$, so $\Pi^n = (\Pi(1))^n$.

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4 Denoising vector fields

(a) Problem description

Given a set of L unstructured points $\mathcal{P} = \{\mathbf{p}_1, \mathbf{p}_2, \ldots, \mathbf{p}_L\}$, where each point $\mathbf{p}_i \in \Omega \subset \mathbb{R}^2$ base a vector $\mathbf{v}_i \in \mathbb{R}^2$, and denote the set of vectors $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_L\}$ by $\mathcal{V}$ (see Figure 2). A *vector field* is a map $\mathcal{F} : \Omega \subset \mathbb{R}^2 \to \mathbb{R}^2$ that associates to each point $\mathbf{p} \in \Omega$ a vector $\mathcal{F}(\mathbf{p})$. It is supposed that the vectors $\mathbf{v}_i \in \mathcal{V}$ are sampled from an unknown map $\mathcal{F}$ and corrupted by an additive random noise. The problem is to develop a method that suppress the noise from the samples that mantains the relevant features of the vector field.

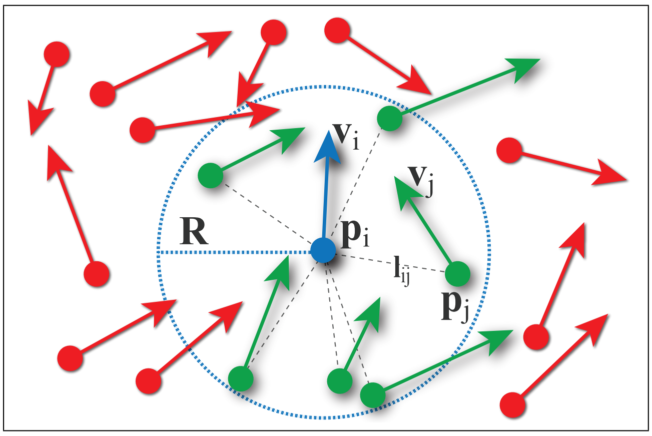

Figure 2: *The support region and the induced graph for the random walk.*

(b) Graph definition

Since the points on $\mathcal{P}$ are unstructured, the first step of the random walk based method is to define a graph G on which a random walk will be processed.

The nodes of G

There is a bijective mapping from the nodes of graph G to the set of input points $\mathcal{P}$, which associates each node n_i on G the corresponding point $\mathbf{p}_i$ on $\mathcal{P}$.

The links of G

The connectivity between the nodes of G depends on a model parameter R, which represents the radii of balls centered at each point of $\mathcal{P}$, the nodes that correspond to points that are inside the ball centered at $\mathbf{p}_i$ are adjacent[1] to n_i. From now on, the graph defined for a given radius R will be denoted by G_R. More precisely, to define the links of G_R, the following rule is adopted:

"Node n_i is adjacent to node n_j through a link l_{ij} if and only if $||\mathbf{p}_i - \mathbf{p}_j|| < R$."

The *first neighborhood* of a node n_i, denoted by $\mathrm{N}(n_i)$, is the set of all adjacent nodes of n_i on G_R.

[1] On a graph G, two of its nodes n_i and n_j are adjacent if there is a link joining them.

(c) Random walks for vector fields denoising

The basic principle used in [12, 15, 16], translated to a graph setting, is that the probability for moving from one node to its neighbor on the graph depends on how similar they are. Suppose that a single virtual particle i is located at every node $n_i \in G_R$, and that each particle i knows not only the position $\mathbf{p}_i$ but also the vector $\mathbf{v}_i$. At each step of the random walk the particle moves from n_j to one of its neighbors or stay at its current position. After the application of n steps of these random walks, the L particles are redistributed on the graph according to the transition matrix Π^n. Such matrix induces a weighted average filter to be applied to each vector $\mathbf{v}_i \in \mathcal{V}$. The *random walk filter* computes, for each node n_i of the graph G_R, a new vector $\mathbf{v}_i$, denoted by $\mathbf{v}'_i$, and is computed according to:

$$\mathbf{v}'_i = \sum_{j \in \mathcal{I}} p^n_{i,j} \mathbf{v}_j, \tag{1}$$

where $\mathcal{I} = \{1, 2, \ldots, L\}$ and $p^n_{i,j}$ is the probability of moving from state i to the state j after n steps, which is the entry at the i^{th} row and j^{th} column of Π^n. The main question now is how to define the transition matrices.

(d) Similarity functions for vector fields

The idea to define the transition matrix $\Pi(n)$ is based on the fact that larger is the "difference" between two vectors, less similar they are. In [15], the authors suggest a set of similarity functions whose independent variable is the norm of the difference d between the normals of adjacent faces, like for example $s(d) = \frac{1}{C} e^{-\alpha d^2}$, where $\alpha \in (0, \infty)$ is a scale parameter and C is a normalization constant. When α is small, only faces with very close normals are considered similar. Thus, using such kind of similarity function, one has the property that larger is the difference between the normal vectors, smaller is the probability one should use to move a particle between the nodes. This function s are adopted in all examples of this paper.

Specifically to the application of this paper which is to suppress the noise from a vector field in an unstructured set of points., a more specific measure of similarity is suggested. Eibl and Brundle in [3] propose three different measures for two given pairs of point/vector $\mathbf{f}_i = (\mathbf{p}_i, \mathbf{v}_i)$, $\mathbf{f}_j = (\mathbf{p}_j, \mathbf{v}_j)$:

- *Squared Euclidian distance* $\rightarrow d_1^2 = ||\mathbf{p}_i - \mathbf{p}_j||^2 + ||\mathbf{v}_i - \mathbf{v}_j||^2$
- *Mahanalobis distance* $\rightarrow d_2^2 = (\mathbf{f}_i - \mathbf{f}_j)\Sigma^{-1}(\mathbf{f}_i - \mathbf{f}_j)^T$, where Σ is the covariance matrix of the coordinates of $\mathbf{f}_k$'s.
- *Weighted additive distances* $\rightarrow$ Given weights w_p, w_θ, w_r and w_β, $d_3^2 = w_p\|\mathbf{p}_i - \mathbf{p}_j\|^2 + w_\theta(\angle(\mathbf{v}_i, \mathbf{v}_j))^2 + w_r(\|\mathbf{v}_i\| - \|\mathbf{v}_j\|)^2 + w_\beta(\angle(\mathbf{p}_j - \mathbf{p}_i), \frac{1}{2}(\mathbf{v}_j + \mathbf{v}_i))^2$, where $\angle(\cdot, \cdot)$ is the angle between two vectors. Those weights balance the effects of each distances: the Euclidean distance from the base points, the vectors angle and norm difference and the difference of the points segment with the vector average direction.

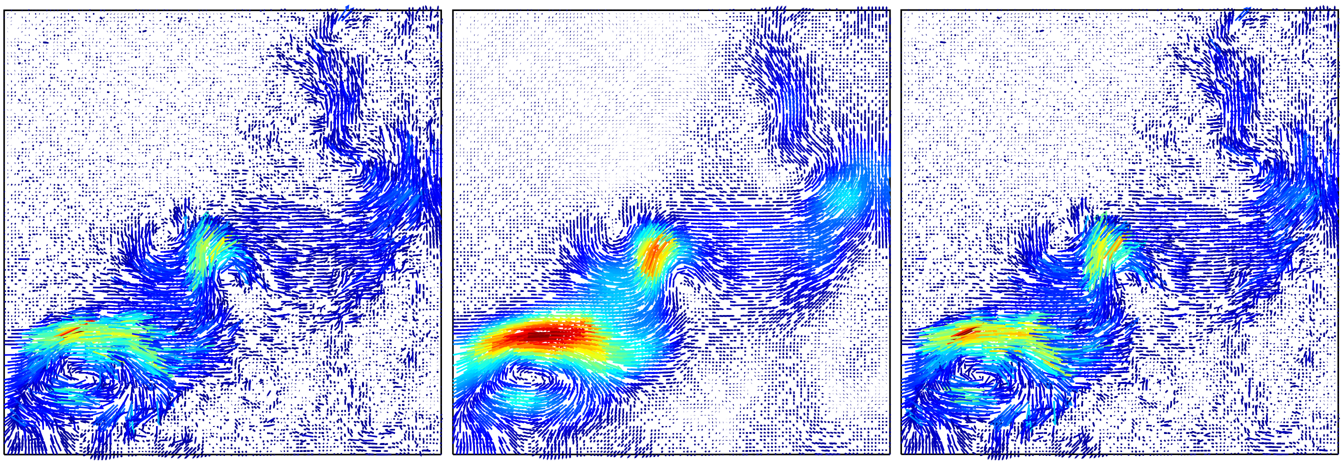

Figure 3: *A Particle Image Velocimetry (PIV) example of a fluid flow (left), filtered by a gaussian filter (middle) and by our random walk (right): the random walk better preserves the magnitude of the field.*

They apply such measures to vector field segmentation.

After several experiments, the authors decided to adopt the *weighted additive distances*. As a conclusion, the transition probability to move the particle from the node n_i to the node n_j at the n^{th} step is given by:

$$p_{i,j}(n) = \begin{cases} \frac{1}{C} e^{-\alpha d_{i,j}^2} & \text{if } n_j \in N(n_i), \\ 0 & otherwise, \end{cases} \tag{2}$$

where $d_{i,j}^2$ is the weighted additive distance between $(\mathbf{p}_i, \mathbf{v}_i)$ and $(\mathbf{p}_j, \mathbf{v}_j)$ and the value of the normalization constant is

$$C = \sum_{n_j \in \mathrm{N}(n_i)} e^{-\alpha d^2}.$$

(e) Filtering

There are two ways to implement Equation (1). One is what Sun et al. [15] called the *batch* scheme, and the alternative one is what they called the *progressive* scheme. In the batch scheme the entries $p_{i,j}^n$ are computed by growing the neighborhood of the nodes, and computing for each step all transition probabilities, and use them at the end to compute the weighted average. In the progressive scheme, the algorithm runs step by step. It traverses only the first neighbors of the spot vertex and computes the probabilities for its neighbors. Here the authors suggest using progressive scheme, since it shows to be faster than the batch one in the greater majority of the experiments: actually the denoising requires only a few iterations.

(f) Parameters of the method

Besides the radius R used to construct the connections between the nodes of the graph, and the number n of steps for the random walk, there are more four parameters, the ones for the weighted additive distances: w_p, w_θ, w_r and w_β.

A suggestion for the weight w_p is $1/(2R^2)$, in order to give more weights to the points that are more close to each other in the ball of radius R. Notice that the term $w_p||\mathbf{p}_i - \mathbf{p}_j||_2^2$ naturally incorporates the distance between the base points, which is a nice advantage when the set of input points are unstructured. To fix parameter w_θ independently of the experiment, we optimize it for an average configuration: when the angles $(\angle(\mathbf{v}_i, \mathbf{v}_j))$ are uniformly distributed in the interval $[0, \pi]$. Then one can set as default w_θ to be the variance of this distribution, i.e. $w_\theta = \pi/12$. Finally, if σ^2 is the variance of the lengths of the vector, then a suggestion for the value of w_r is $100/(2\sigma^2)$, since in this case it is considering the Gaussian distribution with variance equal to the total variance over ten. Since the application is on denoising, as default the weight w_β is set to zero, because it usually destroys the interface of discontinuity of the vector field if it exists.

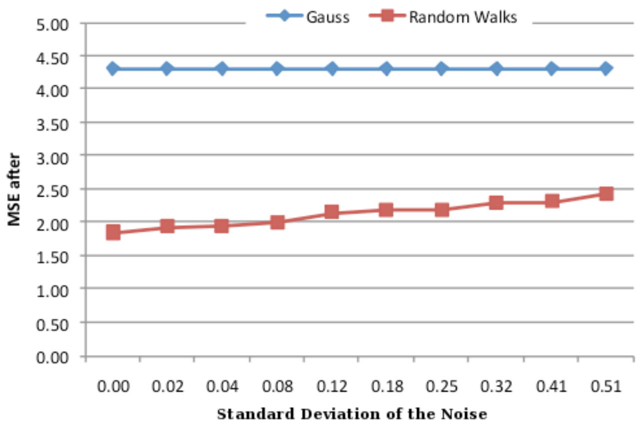

Figure 4: *Performance on the PIV model of Figure 3.*

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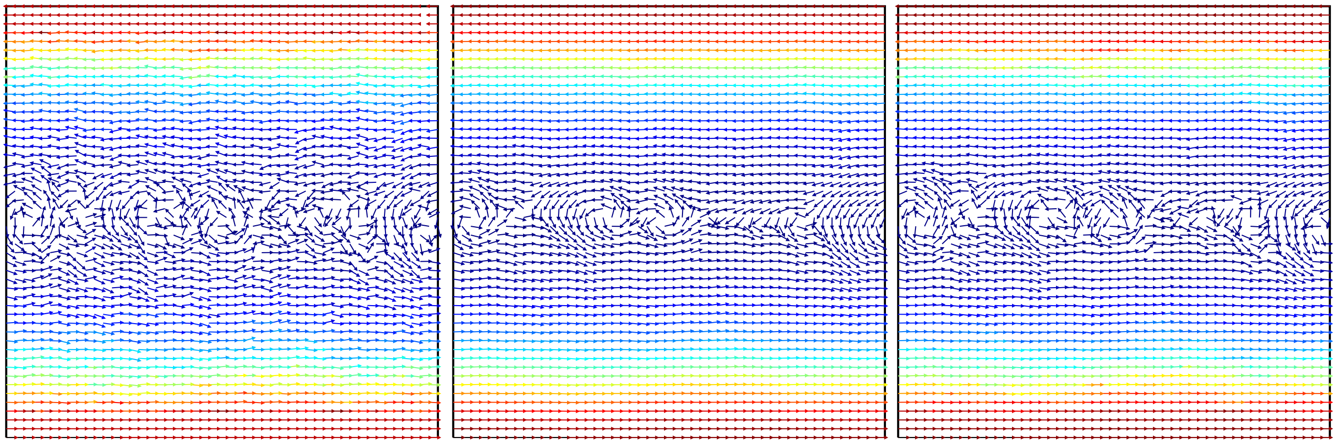

Figure 5: *A shear band simulation of a granular flow (left): the gaussian filter (middle) removes the shear band, while the random walk (right) stresses it.*

5 Results

We tested our de-noising method on three kind of models: a simple noisy discontinuity test, where we expect the random walk to outperform the gaussian filter, measured vector field of physical systems and simulation models. For all examples on this section, we choose the parameters according to the suggestions presented in the previous section. We compare our method to a Gaussian filter, which corresponds to a particular case by setting $w_p = (2R^2)^{-1}$, and $w_\theta = w_r = w_\beta = 0$. For all examples, we set $n = 2$ for both filtering methods.

This simple discontinuity test is constructed using a synthetic vector field:

$$\mathcal{F}(x,y) = \begin{cases} (2,\ 1) & \text{if } (10y < (x+1)^2), \\ (1,-1) & \text{otherwise.} \end{cases}$$

The samples are created by evaluating this map on 900 base points that are randomly generated using a stratified distribution in the grid $[-3,3] \times [-3,3]$ [2]. To each component of the sampled vectors we add an independent and identically distributed random gaussian noise with mean 0 and standard deviation equals to 0.05. Figure 1 shows that the gaussian filter blurs the interface, while the random walk nicely preserves it.

We perform a second test of our approach on a real data acquired from a PIV device. The left picture of Figure 3 shows the original data on $\Omega = [-1,1] \times [-1,1]$ with 15607 points. This sampled velocity field corresponds to a flow of a gas that is continuously injected horizontally on the bottom left corner. This gas flows on the domain from left to right until it meets an wall, represented on the image by its right edge. To this data we again add to each component of the sampled vectors, an independent and identically distributed random gaussian noise with mean 0 and standard deviation equals to 0.1. The resulted vector fields after applying a gaussian and a random walk filter are illustrated, respectively, by the middle and the right pictures on Figure 3. For the same data, we performed another tests. We vary the standard deviation of the additive gaussian noise from 0.0 to 0.51. Figure 4 shows a graph that represents the vector magnitude Mean Squared Error (MSE) measured on the noised image as a function of the standard deviation of the additive noise. As one can see, the proposed filter conserves better the norm of the vector field.

We also checked our method on a simulation of shear bands in granular flows [1]. The vectors on this example are placed on a 50×50 grid. The left picture of Figure 5 shows the equilibrium state of the mobility of grains in a dense granular system under shear, which almost half of the rows are moving one way, half moving the other way, with the shear band being formed at the very center. At this center area, the velocity is randomically distributed and its module is almost zero, resulting in a shear band. In this figure, for visualization purpose, the size of the vectors are the same, the colors are used to represent their norm. In this example, the samples are originally with an unknown noise. The middle and the right pictures shows the filtered vector field by the gaussian and by the random walks method. The gaussian filter almost removes the shear band, while the random walk stresses it.

Smooth Particle Hidrodynamics (SPH) has been recognized as a flexible mesh free method for computational fluid dynamics simulations [22]. In such method the fluid is modeled as a collection of particles, which move under the influence of hydrodynamic and external forces. Each portion of fluid is represented by a particle with attributes, among which the velocity vector. We finally checked our method on simulation models of a two-dimensional granular slide deformation on a slope and its impact into a water body [21], such data is available from SPHERIC [23]. First our method is tested on an landslide measured by PIV methods (Figure 6) and then it is again tested on a SPH simulation data (Figure 7). We can see on both cases that the random walk matches the global behaviour, sketched on (Figure 7).

6 Conclusions

This work proposed a 2D vector field denoising technique based on random walks, whose main characteristic is that it preserves coherent discontinuities while removing noise under the vanishing-mean *per continuous region* model. To do so, a suitable similarity function with weighted additive distance for the pair point/vector was proposed. Although the method has been proposed for 2D, it can be easily extended to higher dimensions by proposing adequate similarity functions. We show several applications of the method to PIV images, SPH and granular flows.

For the future, the authors plan to continue this work implementing a 3D vector field denoising algorithm and also plan to develop new algorithms for vector field segmentation based on random walks.

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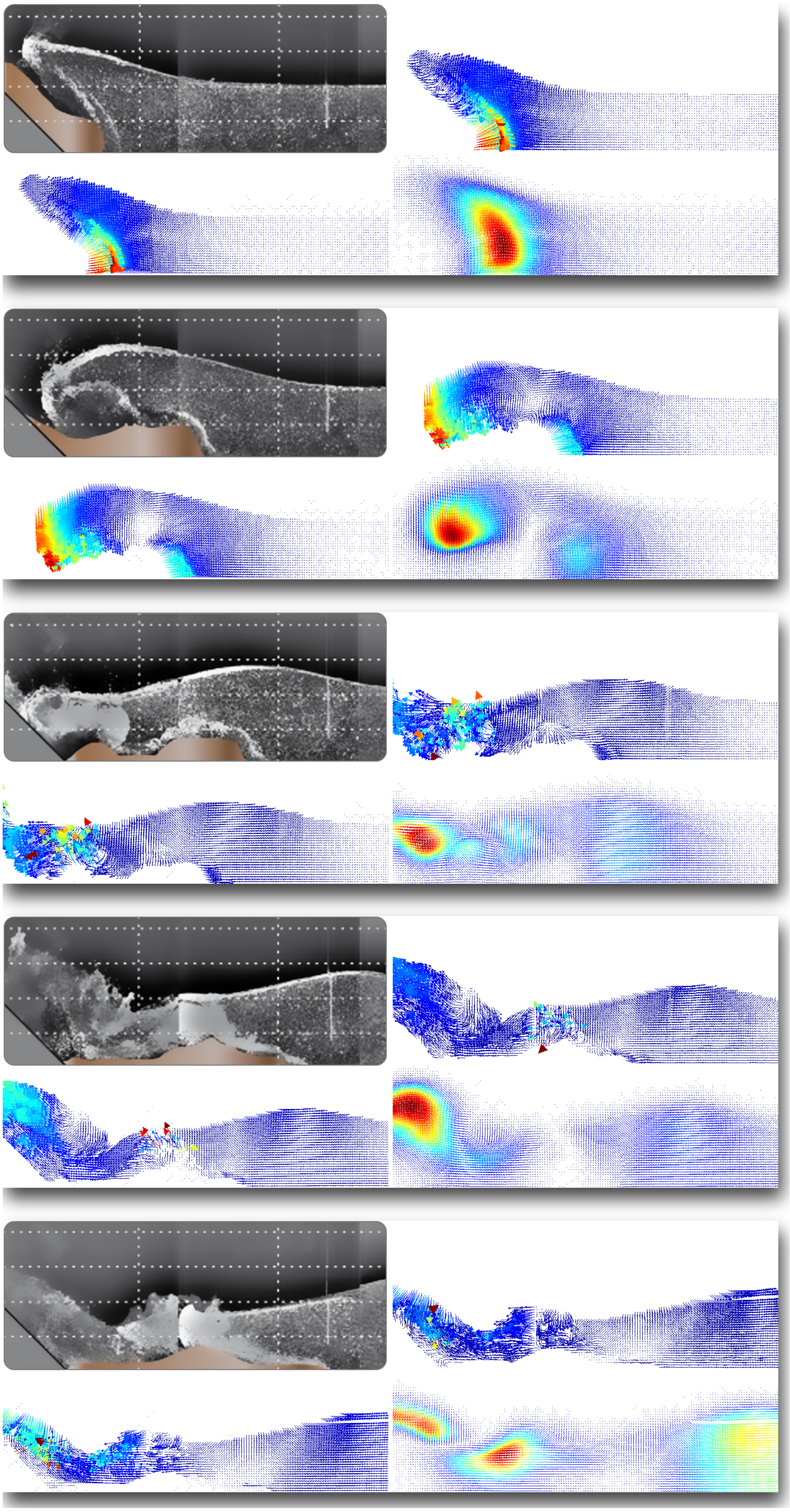

Figure 6: *Landslide in 5 steps (each block): PIV measure (top left), simulated SPH vector field (top right). The bottom left and right pictures show the random walks and the gaussian filtered vector fields, respectively. The gaussian method oversimplifies the model.*

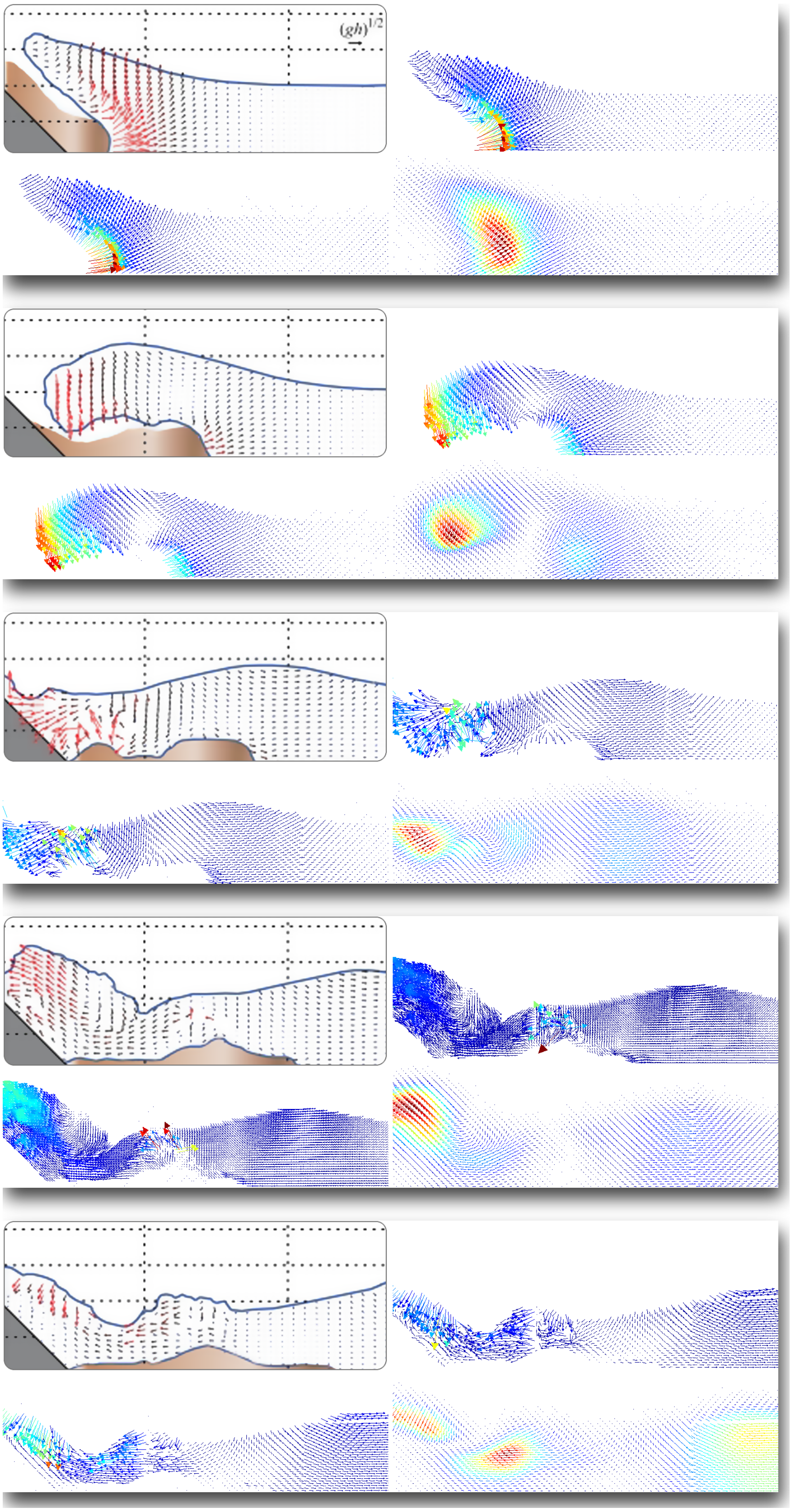

Figure 7: *Sketched model of the landslide of Figure 6 (top left) that corresponds to the vector field on the top right rendered with a third of the samples. The bottom left and right pictures show the random walks and the gaussian filtered vector fields, respectively.*

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